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ERRATA.

Vol. VII (Jan. 1898), p. 50, line 7, of Stoney's article on "Atmospheres upon Planets and Satellites," *for* "the two satellites of Jupiter," *read* "the new satellite of Jupiter."

Vol. VIII (Oct. 1898), p. 144, line 4, of equation (11) of Campbell's article on "The Mills Spectrograph of the Lick Observatory, *for* $+ Dx^2$, *read* $+ 3\,Dx^2$.

hypotheses, whose incorrectness I could not have demonstrated without its aid.

But the formula is a cumbersome one. I do not know how many coördinate numbers may be necessary for an ideal chemistry to indicate all the properties of any particular substance, especially to compute the wave-length of every line of its spectrum, but that eight or even six constants of the formula for a series should be independent of one another, I do not believe. However, as long as I lack all knowledge of general relations suitable to reduce the law of series into a simpler form, I shall decidedly prefer a formula which satisfies the observations by means of many constants to one which, with only three or four constants, is unable to represent the observations approximately within the limits of their mean errors.

But unfortunately, not many spectra are rich enough in lines to permit the application of so cumbersome an apparatus of calculation, and at the same time not so complicated that it is impossible to separate their series. And of these only a few are sufficiently well known. My knowledge of the literature of this special subject being rather small, the only data I have found which are suitable for the calculation are the excellent measures of the carbon and cyanogen band-spectra by Messrs. Kayser and Runge.[1]

On these I have tried my formula, and computed the constants for a considerable number of series, of which I now publish only the part concerning the third band of the carbon spectrum. For the spectrum of cyanogen a photograph of Mr. Rydberg's was measured at the Copenhagen Observatory while I was engaged upon the preliminary computations, and these new measures promise to be valuable supplements to the measures of Kayser and Runge.

The carbon band ought not to wait for the cyanogen, though it requires further measurements before its calculation can be terminated definitively. The measures of Kayser and Runge have proved excellent in permitting a nearly complete resolution

[1] *Abhandlungen d. K. Akad. d. W., Berlin*, 1889, Anhang, p. 1.

of the band into its series. But as a little uncertainty remains in the computation of each series, it is possible that a revision of their old photographs (as to the existence of some lines probably overlooked) and the addition of the intensities of the lines might afford an essential improvement. I wish, however, to appeal to all observers by pointing out that here is a field where small special improvements in measuring give hopes of considerable results. A complete resolution of the heads themselves of this band, which is the radical remedy for the present uncertainty, may require an extraordinary dispersion, but every small increase in dispersion, if it only result in the resolution of a few of the coincident lines in Kayser and Runge's tables, especially near the heads of the band, may also possibly furnish the computer with the means of deciding among certain hypotheses now too equally probable. On the other hand, prolonged exposure of the photographs may reveal the existence of some important lines, or parts of series, in the faint half of the band (near the fourth band) which may have been too faint to appear on the photographs of Kayser and Runge.

The third band of the carbon spectrum contains not only the two heads indicated by Kayser and Runge, but certainly also two others, and probably many with rapidly decreasing intensity. The first head, $\lambda = 5165$, is very intense, the second, $\lambda = 5129$, rather faint, the third, $\lambda = 5100$?, the fourth, $\lambda = 5067$, and that which is probably the fifth, $\lambda = 5050$, show on Kayser and Runge's photograph no head-like appearance; most likely only the series from the first three heads are sufficiently intense to render visible their isolated lines, while in the others we see only those which are strengthened by coincidences.

The parts of the band belonging to each of these heads seem to consist of five or probably six complete series, or rather of two groups, each of three (or two) series, arranged in some analogous manner. The series of a group proceed with a pronounced tendency to parallelism, and in hardly less degree the groups themselves, also. Far from the heads this parallelism is most pronounced, and but few coincidences occur; but those

that occur continue for a considerable distance. Near the heads the coincidences set in, and soon become very complicated on account of the large number of series whose heads tend to coincide in the great common head of the band. These coincidences between the heads of the series cannot be determined by means of Kayser and Runge's photographs. A line, $\lambda = 5166.25$, which falls near the first great head, may possibly represent the head of a series; otherwise the only trace of multiplicity in the heads found by Kayser and Runge is that the second line of the band is marked "strong." Nor have I been able to decide by my computations, whether the five or six heads of series very nearly coincide in the common head, or whether their wave-lengths differ from one another by an Ångström unit or even more. This doubt is the principal cause of my request for new observations.

It is common to all the series of the carbon band that, as their phases are all nearly equal to $\frac{1}{4}$, their branches are very well separated. Indeed the phase is in general not exactly $\frac{1}{4}$. The halting progress of the complete series, the regular succession of longer and shorter intervals, has already struck Kayser and Runge, and made them complain of the slight regularity of a series and of the variability of its intervals.

In one of the groups of the first common head there are only two series, α and β, but α is much stronger than β, and in the other series I believe it is irresolvably double from head to tail. The β series is also a very close companion to α. Near the head, where they are most widely separated, Kayser and Runge invariably unite the measures of α and β with a $\{$ as a sign of imperfect resolution. The coincidences set in with that of $\beta-16$[1] and $\alpha-16$, and apparently continue with rapid decrease of the intensity of the joint $\alpha\beta$ series until the lines of the positive branch disappear at $\alpha\beta+43$, and the last of the negative at $\alpha\beta-47$.

The other group of the first head, consisting of the series γ,

[1] β-16 is understood to mean the sixteenth line of the negative branch of the β series.

δ and ϵ, is a no less conspicuous feature, mentioned by Kayser and Runge, and described by them as a series of triplets occurring in the intervals of $\alpha\beta$. The γ series has constantly the shortest wave-length, δ the middle, and ϵ, with a somewhat greater distance, the longest wave-length. The middle series δ is first separated from the strong α and β headlines at $\delta+7$, then ϵ, while γ has no isolated line before $\gamma+11$. There are no later coincidences with $\alpha\beta$, but soon γ begins to coincide with δ (from $\gamma+17$ and $\delta+17$), and at last the coincident $\gamma\delta$ series also coincides with ϵ (from $\gamma\delta+29$ and $\epsilon+29$). By their junction these series acquire a great intensity, greater and much more permanent than that of $\alpha\beta$; they are seen by Kayser and Runge to the end of the band.

The second problem in this spectrum is to determine whether these coincidences remain irresolvable up to the end of the band. From Kayser and Runge we learn only that after apparent coincidence the lines cannot approach very rapidly, for in one instance, $\gamma\delta\epsilon-35$, Kayser and Runge have actually resolved the triplet, and seen the (ϵ?) line isolated; but with this exception it must be admitted that their measures show no trace of a division or of the separation of a diverging lateral branch. I have serious reason to believe, however, that such a separation takes place. Where the three series are isolated, and also after the coincidence of γ and δ, their phases are very nearly the same number, 0.244 (decidedly different from 0.250). But we observe with some surprise that, shortly after the total coincidence, the values of the phase begin to vary gradually, so that at last it attains the value $c=0.235$, which is beyond doubt really different from 0.244.

That the phase must remain constant in each series is my fundamental hypothesis. This, in fact, is not contradicted by this variability in the phase of something that manifestly is not a series but a combination of series. Consequently I have sought to make use of this phenomenon as an indication of the distribution of the single series in the compound. It is obvious that a greater variation of the phase of the compound series

must lead to the supposition of a considerable difference between the intensities of the branches of some of the coincident series. For the sake of computation I have formulated the necessary supposition as the following special hypothesis: The lines seen at the end of the band only belong to the two extreme branches $\gamma - n$ and $\epsilon + n$ of the six branches, so that they are here isolated again; both branches of the δ series have, I think, become invisible to Kayser and Runge about $\delta \pm 50$; even before the positive branch of γ and the negative branch of ϵ have been lost sight of, say at $\gamma + 40$ and $\epsilon - 40$. Evidently this hypothesis requires confirmation by further observations. A prolonged exposure of the photographs may settle this question. The hypothetical lateral branches must be some 0.2 or 0.3 Ångström unit distant from the known main branches, consequently no greater dispersion is wanted, but only a greater intensity.

Among the feebler series of the second head we again find the same duplicities or triplicities, but only so much wider that they do not appear by a simple inspection of the photographs. To one group belong at least two series, ζ and ξ, of which (because of the great difficulties arising principally from the coincidences with the strong series from the first head) I have been able to compute only the ζ series, while the existence of the ξ series is evinced by too few isolated lines to permit an independent computation. I have found it only by a geometrical representation of the differences between the computed ζ lines and all those observed by Kayser and Runge. The ζ series itself may be double. Beyond $\zeta - 37$ my ζ series loses its negative branch; the lines marked $\zeta - 36$, $\zeta - 39$, and the following ones in the catalogue of comparison below belong probably to a series ζ', intervening with ζ in a similar manner as γ with ϵ, whose existence is indicated also in the positive branch by some cases of apparent duplicity.

To the other group of the second head belong two computed series, η and θ, and a third series ι, traced out in the same manner as ξ by means of graphical comparison of η and θ with all the observed lines. These series, η and θ, coincide with $\eta\theta - 32$,

and are lost sight of shortly after; traces of η? may perhaps be found as far as $\eta \pm 46$.

As to the third head, I know neither the exact value of its maximum wave-length nor the number of series issuing from it. By means of the graphical process, described below, it is easy to prove the existence of two such series, κ and λ. Of these λ can be computed, a fairly sufficient number of isolated (?) lines being observed by Kayser and Runge; but such a computation must be immature as yet, because a great number of additional lines are lacking in Kayser and Runge's tables.

Before giving the numerical results of my computation, however, I shall set forth some particulars of the operations. Indeed, the unraveling and computing of a series of a band spectrum differs in more than one regard from common practice in astronomical computations.

As soon as we have succeeded in discovering the first series of a band spectrum and in calculating the formula, $\lambda = f(n+c)^2$, it is possible to arrange the operations always necessary for its improvement in a way which gives hope that other series may be detected at the same time. With this formula a table of the function $f(m^2)$ is to be computed, say for the whole numbers of m; by interpolation in such a table the values of $f(n+c)^2$ can be found and compared with the observed values of λ, corresponding to the supposed series, all in the ordinary way. Nor is it too much to ask that in a somewhat difficult matter of this kind the computer shall not satisfy himself by the immediate inspection of the catalogue of differences Obs.—Comp., but also illustrate them by means of the ordinary construction, putting m as the abscissa and $\lambda_{obs} - f(m+c)^2$ as the ordinate of a point for each observation.

In our case the labor of interpolation with respect to the phase can be spared; on the other hand it may be very useful to extend the comparison of the observed values, without regard to their relation to the supposed series, to all observed lines, or practically to those differing from $f(m^2)$ by less than a convenient arbitrary quantity (*e.g.*, the maximum of $f(m^2) - f(m+1)^2$).

For every abscissa m, all the differences $\lambda_{obs}-f(m^2)$ are to be laid down as ordinates in the construction of points of observation. It may also be useful to lay down on each ordinate the points corresponding to $f(m+r)^2-f(m)^2$ within the arbitrary limit. A curve joining the points for the same r then represents both the abscissa and the wave lengths of a series with the phase $=0$, but in all other respects similar to the supposed series.

The total figure formed by these curves has, for all band-spectra, the shape of an onion, cut through longitudinally. The curves all pass very nearly and with inflexion through the zero point of the system of coördinates or the head of the series; at some distance from this point they attain their maximum distance, and then approach again. The space between any two consecutive curves, the section of a single layer of the onion, contains one and only one point of observation for each observed line of the part of the spectrum in which the supposed series is found.

Being a two-dimensional representation of the spectrum, each of these spaces must show any of its series as a curve passing through the observed points of its ordinates; in particular, the supposed series employed in the construction ought to represent its two branches as two veins of the layer, running through its whole length symmetrically to its limits, so that on the two layers next to the abscissa its geometrical representation becomes very well fitted for graphical adjustment, with these special advantages that the two branches can be treated separately, that the value of the phase can be found repeatedly by the proportions of sections of each ordinate, that its constancy can be tried, and the phase finally can be eliminated in the formation of normal values for the subsequent computation.

The other series of the same spectrum are represented in a somewhat similar manner; only, in general, they may traverse the limiting lines of our onion figure and be more or less difficult to perceive. But as the adjacent series, following one another in a compound band, always seem to obey some continuous law, it may be expected that they only differ by small quantities,

and that their representations consequently do not differ greatly from those of the supposed series.

So in the carbon band; the series ϵ, which passes through the whole band, may be employed for comparison. Any of our formulæ for this series may be used. We may with 6^{mm} as unit for m on the abscissa and 15^{mm} representing an Ångström unit on the ordinates, obtain a fairly good representation, not only of the two branches of the ϵ series itself, but also of the δ and γ series which are represented by nearly parallel curves, only starting from somewhat distant heads. Moreover, the α and β series are still well represented, showing only a small inclination towards the three just mentioned. The series of the second head of the band experience greater distortion ; near their heads, particularly, they cross the other series and the abscissa with nearly perpendicular transverse curves. Somewhat farther off, where these series no longer coincide, the inclinations of the curves become gradually smaller, and finally some of their curves may also be detected immediately by inspection.

Choosing one of these discovered series, say ζ, as the fundamental of a new geometrical representation, especially for the examination of the series from the second head, and repeating these operations as far as possible, we can resolve the whole band into its series, in a methodical manner, fairly applicable to most of the band-spectra. In such a way the series ξ, ι, κ, and λ are actually discovered.

Moreover, this method supplies another advantage. This kind of induction is a somewhat hazardous one, something like the deciphering of a cryptograph. Only the final complete concordance affords security; in all the preparatory stages great mistakes threaten us, the worst mistake being to pass from one series into another coincident series, particularly if the coincidence is prolonged and more like a tangency than a simple intersection. For the criticism necessary to avoid such mistakes, we can scarcely be provided with any better instrument than our graphical representation illustrating on a single sheet all the series in question.

The constants of the computed series will not be published in a form corresponding directly to equation (1). For the computation I have with some little advantage put

$$\frac{1}{\lambda-\mu}=\phi=\frac{v_0+v_1(n+c)^2+\quad\ldots\quad v_r(n+c)^{2r}}{p_0+p_1(n+c)^2+\quad\ldots\quad 1.(n+c)^{2r}}, \qquad (2)$$

μ being an estimated value of the wave-length λ_0 of the head, but this form is also unsuitable for publication. But knowing only very imperfectly the characteristic details of this application of our formula, I have greatly hesitated in my choice and ultimately preferred the form

$$\lambda=\lambda_0-\kappa\left(\frac{n+c}{10}\right)^2.\frac{1+s_1\left(\frac{n+c}{10}\right)^2+\quad\ldots\quad+s_{r-1}\left(\frac{n+c}{10}\right)^{2r-2}}{1+t_1\left(\frac{n+c}{10}\right)^2+\quad\ldots\quad+t_r\left(\frac{n+c}{10}\right)^{2r}}, \qquad (3)$$

because it brings into relief the natural constants λ_0 and κ, showing the frame of the head, and moreover because it fairly illustrates the degree of convergence obtained. It may be remarked, however, that all the special values for s_i show a tendency to equality with t_i; taking this peculiarity into consideration, a further development,

$$\lambda=\lambda_0-\frac{\kappa}{\left(\frac{10}{n+c}\right)^2+\psi},$$

is indicated, where ψ is a function of the same class as λ, only with $r-1$ instead of r; if ψ be constant, the formula becomes identical with that of Professor Pickering, being thus the first step of a development in continuous fraction.

The results of my computations are as follows :

Series α		Series β	
$c=$	0.266	$c=$	0.266
$\lambda_0=$	5165.1733	$\lambda_0=$	5165.5911
$\kappa=$	11.53776	$\kappa=$	13.17201
$t_1=$	+4.523456	$t_1=$	+4.437308
$s_1=$	+5.049774	$s_1=$	+4.342970
$t_2=$	+0.1273115	$t_2=$	+0.1248869
$s_2=$	+0.1278293	$s_2=$	+0.1098562
$t_3=$	+0.000615434	$t_3=$	+0.000603713

The α and β constants are not found independently. The phase c has been determined from the coincident part of the series. Only two normal values for each series have represented isolated lines, and five common normal values for the coincident part were regarded as belonging to means $\frac{1}{3}(2\alpha + \beta)$ with double weights for the α series. Thus only two constants, v_0 and p_0 (with $\mu = 5165$), are determined separately for α and β, the other constants of formula (2) being common to both series. There are reasons to suppose that this choice, made in order to save labor, has not been fortunate as to the best representation of observations.

Series γ, I		Series γ, II		Series δ	
$c =$	0.245	$c =$	0.255	$c =$	0.2445
$\lambda_0 =$	5163.7023	$\lambda_0 =$	5164.4046	$\lambda_0 =$	5164.9634
$\kappa =$	13.73249	$\kappa =$	13.03697	$\kappa =$	15.28366
$t_1 =$	+0.150106	$t_1 =$	+0.005907	$t_1 =$	+0.668854
$s_1 =$	+0.140146	$s_1 =$	+0.000001	$s_1 =$	+0.579184
$t_2 =$	−0.0017029	$t_2 =$	+0.0000241	$t_2 =$	+0.0028975
$s_2 =$	−0.0022202				
$t_3 =$	−0.000008955				

Series ϵ, I		Series ϵ, II	
$c =$	0.2435	$c =$	0.2565
$\lambda_0 =$	5166.2211	$\lambda_0 =$	5164.9003
$\kappa =$	17.85338	$\kappa =$	16.75553
$t_1 =$	+1.161822	$t_1 =$	+0.584760
$s_1 =$	+0.861099	$s_1 =$	+0.465199
$t_2 =$	−0.0186694	$t_2 =$	−0.0101763
$s_2 =$	−0.0178138	$s_2 =$	−0.0104149
$t_3 =$	−0.000104691	$t_3 =$	−0.000059560

Coincidences with the α and β series have forced me to take the first normal values for γ, δ, and ϵ so far from the heads that the number of lines between the heads and the nearest normal value cannot be fixed with accuracy. Both for γ and ϵ two hypotheses (I and II) concerning this number have been tried (by addition of a line in the case of γ, II, and by removing a line in that of ϵ, II). The most important results of these

alterations are found in the variations of λ_0 (0.7023 for γ and 1.3208 for ϵ) but these heads being invisible on Kayser and Runge's photographs, the question cannot be decided at present by that criterion. For the rest the two formulæ for ϵ do not show any real difference, both representing very well the totality of observed lines, and if the second formula for γ is certainly inferior to the first, the reason is only that it has been computed with $r=2$ (without s_2 and t_3), because by putting $r=3$ I met with one of those pleasant changes of sign in both the denominator and numerator of the formula. Until this question is settled, however, by new observations I am inclined to accord the leading place to γ, I, and ϵ, I, for the following secondary reasons, or perhaps presumptions: If the phases are for γ, $c=0.245$, for δ, $c=0.2445$, and for ϵ, $c=0.2435$, the three series may have really identical phases; not so with 0.255, 0.2445, and 0.2565. To this I may add that only in the combination γ, I, δ, and ϵ, I, each of the lines of the series corresponds naturally to one, and only to one, line of each other series, and in such a way in the long coincidences of these series that the coincident lines are also corresponding lines.

Series ζ	Series η	Series θ
$c = 0.241$	$c = 0.250$	$c = 0.233$
$\lambda_0 = 5130.5489$	$\lambda_0 = 5128.6955$	$\lambda_0 = 5129.4943$
$\kappa = 12.59299$	$\kappa = 8.83519$	$\kappa = 9.28698$
$t_1 = -0.047702$	$t_1 = +1.516465$	$t_1 = +1.288308$
$s_1 = -0.053423$	$s_1 = +2.026739$	$s_1 = +1.655164$
$t_2 = +0.0029518$	$t_2 = +0.0046593$	$t_2 = +0.0045931$
$s_2 = +0.0032713$		
$t_3 = +0.000017036$		

To these series, belonging to the second head of the band, the same remarks apply, only to a greater extent than for γ, δ, and ϵ. Two or more lines should perhaps be added or taken away near their heads.

Upon the whole it must be evident that these systems of constants are by no means to be considered as definitive and reliable for speculations regarding the true properties of the law

of spectral series. The main interest of my computations is not to be found in these constants, but in the tables of computed wave-lengths founded upon them. While my work in other points may call for many revisions, these tables will be useful for comparisons with the observations, present or future, in order to demonstrate the existence of all the more remarkable series of this spectrum.

Of these tables, the two tables for the more problematic formulæ γ, II and ϵ, II are here given directly, but only for the whole value of $n+c$:

$n+c$	λ (γ, II)	λ (ϵ, II)	$n+c$	λ (γ, II)	λ (ϵ, II)
0	5164.404	5164.900	32	5035.791	5028.049
1	64.274	64.733	33	27.958	20.022
2	63.883	64.233	34	19.925	11.801
3	63.231	63.408	35	11.697	03.388
4	62.319	62.266	36	03.277	4994.789
5	61.148	60.821	37	4994.668	86.006
6	59.717	59.083	38	85.875	77.041
7	58.027	57.065	39	76.902	67.904
8	56.080	54.776	40	67.754	58.597
9	53.875	52.225	41	58.434	49.121
10	51.414	49.419	42	48.947	39.483
11	48.698	46.362	43	39.298	29.686
12	45.727	43.058	44	29.491	19.735
13	42.504	39.510	45	19.530	09.634
14	39.030	35.720	46	09.421	4899.389
15	35.306	31.690	47	4899.168	89.003
16	31.334	27.421	48	88.776	78.481
17	27.115	22.914	49	78.251	67.828
18	22.652	18.171	50	67.597	57.048
19	17.946	13.193	51	56.819	46.147
20	13.000	07.982	52	45.923	35.131
21	07.815	02.539	53	34.913	24.004
22	02.395	5096.867	54	23.795	12.772
23	5096.740	90.967	55	12.574	01.442
24	90.855	84.842	56	01.256	4790.020
25	84.741	78.494	57	4789.845	78.515
26	78.401	71.925	58	78.347	66.935
27	71.838	65.139	59	66.767	55.292
28	65.056	58.138	60	55.112	43.601
29	58.057	50.925	61	43.386	31.882
30	50.844	43.504	62	31.595	20.166
31	43.421	35.878	63	19.744	08.504

The other eight are employed in the formation of the following catalogue of differences, "Observation—Computation," and may

easily be reconstructed by its means with three correct decimal places of the Ångström unit, and in their interpolated form.

In this catalogue, the first two columns contain the observations of Kayser and Runge, their remarks being given with the abbreviations used by themselves; s = strong, d = double, u = indistinct, dgr = dark ground, and $\{$ for imperfect separation. The catalogue has been completed with the hypothetical lines required by my formulæ, which are wanting, however, in Kayser and Runge's memoir; these insertions are marked with hp = hypothetical, and are conspicuous also by having three decimal places, while Kayser and Runge give only two. In the third column are given the designations of the computed line or lines as to their series, and for each of these lines (after a), the difference Obs.—Comp. The line

$$s, d\ 5133.79 \mid \alpha - 16,\ +.032;\ \beta - 16,\ -.048$$

for instance, indicates that the strong double line measured by Kayser and Runge at wave-length 5133.79 is a compound of the sixteenth lines of the negative branches of series α and series β; that α differs towards the violet side by 0.032, and β towards the red side by 0.048. Both the phases being $c = 0.266$, the tabular values are for $\alpha : n+c = -15.734$, $\lambda = 5133.758$, and for β: $n+c = -15.734$, $\lambda = 5133.838$. A fourth column is reserved for special remarks.

A statistical enumeration shows a total number of 524 lines in this catalogue, of which 479 are measured by Kayser and Runge, while 45 hp lines are hypothetical. Probably some of these are owing to errors in my interpretations of the behavior of the series, especially in the neighborhood of the heads; but having found more or less pronounced traces of most of these lines on the enlarged photograph, which accompanies the measures of Kayser and Runge, I must regard my hypotheses as previously corroborated, and attribute these discordances principally to accidental omissions by Kayser and Runge, which give hope that new observations, even though not necessarily more refined ones, will add a considerable number of new lines for the basis

of future computations. Indeed, ten years ago Kayser and Runge could not feel as great a call as we do now to extend the measures to the faintest lines, and to resolve as many duplicities as possible.

Nevertheless, some few lines are really missing in the observations, or only visible with very little intensity, in comparison to their neighbors in the series. Thus, for instance, the line 5048.978, whose absence is expressly mentioned by Kayser and Runge; on the Atlas photograph only a very faint line is visible at the wave-length where a strong triple line, $\gamma\delta\epsilon - 30$, is expected. Similar absences are found at 4922.236 for $\gamma\delta\epsilon + 44$ and 4891.583 for $\delta\epsilon + 47$, and $\zeta - 46$. In other spectra also similar omissions are remarked, and it has struck me that such an omission is relatively frequent for coincident lines, while I have never met with it with isolated lines. Without a thorough knowledge of the very fundamental law of the genesis of spectra, this phenomenon cannot be explained; but I am inclined to believe that, in analogy with the behavior of interfering rays of light, while coincident lines as a rule strengthen the common intensity, a very close coincidence may eventually result in a mutual counteracting and loss of intensity,

Among the lines measured by Kayser and Runge, 79 do not correspond to any of my eight computed series. So far the resolution of the spectrum into its series is not complete. But taking into account the results of my above-mentioned graphical construction by means of the table for the ζ series, I must conclude that the measured spectrum can contain but very few foreign lines. Upon the strength of this construction, 65 lines must be attributed to certain branches of certain series. A critical revision, based upon computations of these ξ, ι, κ, and λ series may probably exclude some of these lines; if so, they may, like the other 14 lines marked *N3* or μ? or ν?, most likely be attributed to series from the third head other than κ and λ, or to the feeble series from the fourth and fifth head, or like the line 5155.25, represent the survivors of the second carbon band within the domain of the third band.

CATALOGUE OF COMPARISON: OBSERVATION-COMPUTATION.

Observed by Kayser and Runge		Indication of series, and difference of wave-length	Special remarks
hp	5166.210	$\epsilon+0$	
hp	66.118	$\epsilon-1$	
hp	65.946	$\epsilon+1$	
hp	65.675	$\epsilon-2$	
I. head	65.30	$\beta+0,-.281$; $\beta-1,-.219$; $\beta+1-.079$; $\epsilon+2,-.036$	
s	65.12	$\alpha+0,-.047$; $\alpha-1,+.004$; $\alpha+1,+.127$; $\beta-2,-.075$	
	64.84	$\alpha-2,+.014$; $\beta+2,-.077$; $\delta+0,-114$; $\delta-1,-.035$; $\delta+1,+.114$; $\epsilon-3,-.054$	
	64.59	$\alpha+2,+.019$	
	64.46	$\beta-3,-.152$; $\delta-2,-.033$; $\epsilon+3.+.065$	
	64.28	$\alpha-3,-.006$; $\delta+2,+.083$	
	64.04	$\beta+3,-.156$	The first lines of β evidently require a negative correction
	63.87	$\alpha+3,-.025$; $\delta-3,+.059$; $\epsilon-4,+.076$	
	63.62	$\beta-4,-.150$; $\gamma+0,-.074$; $\gamma-1,-.004$	
	63.49	$\alpha-4,+.000$; $\gamma+1,+.000$; $\delta+3,+.121$	
d	63.16	$\beta+4,-.058$; $\gamma-2,-.120$; $\epsilon+4,+.010$	
	62.96	$\alpha+4,+.000$; $\gamma+2,-.051$; $\delta-4,+.127$	
	62.60	$\beta-5,-.072$; $\gamma-3,-.061$	
	62.41	$\alpha-5,-.023$; $\gamma+3,+.152$; $\delta+4.+160$; $\epsilon-5,+.010$	Perhaps $\gamma+3$ and $\delta+4$ ought to be referred to an hp line 62.254
u	61.95	$\beta+5,-.033$	
	61.77	$\alpha+5,+.005$; $\gamma-4,+.000$; $\epsilon+5,+.150$	
u	61.40	$\beta-6,+.082$; $\delta-5,-.168$?	
	61.23	$\gamma+4,-.003$	
	61.08	$\alpha-6,-.036$	
	60.79	$\delta+5,-.058$; $\epsilon-6,+.058$	
	60.43	$\beta+6,-.063$; $\gamma-5,-.174$	
	60.31	$\alpha+6,+.002$	
	59.92	$\gamma+5,-.015$; $\delta-6,-.105$; $\epsilon+6,+.096$	
	59.66	$\beta-7,-.048$	
	59.50	$\alpha-7,-.036$	
u	59.10	$\gamma-6,-.068$; $\delta+6,-.071$	
	58.69	$\beta+7,-.058$; $\epsilon-7,-.113$	
	58.58	$\alpha+7,-.008$	
u	58.17	$\gamma+6,-.197$; $\delta-7,-.040$	
	57.79	$\beta-8,-.054$; $\epsilon+7,+.016$	
	57.65	$\alpha-8,-.043$; $\gamma-7,+.186$	
u	57.24	$\delta+7,+.014$	
	56.71	$\beta+8,-.039$	
	56.61	$\alpha+8,+.002$; $\gamma+7,+.080$; $\epsilon-8,-.017$	
u	56.17	$\delta-8,+.039$	
	55.70	$\beta-9,-.026$	
	55.56	$\alpha-9,-.032$; $\gamma-8,+.070$; $\epsilon+8,+.082$	
u	55.25	N_3	From the second band?
u	55.07	$\delta+8,+.051$	
	54.49	$\beta+9,-.005$; $\gamma+8,+.064$	
	54.35	$\alpha+9,-.018$; $\epsilon-9,+143$	Only from about here, where most of the lines become isolated, a closer agreement with the observations can fairly be expected
u	53.82	$\delta-9,+.028$	
	53.32	$\beta-10,-.033$	
	53.21	$\alpha-10,-.022$; $\gamma-9,-.041$	

CATALOGUE OF COMPARISON: OBSERVATION — COMPUTATION—*cont'd.*

Observed by Kayser and Runge		Indication of series, and difference of wave-length	Special remarks
	5152.97	ε +9,+.027	
u	52.56	δ +9,+.004	
	51.97	β +10,—.018; γ +9,—.088	
	51.87	α +10,—.003	
	51.57	ε —10,+.022	
	51.22	δ —10,+.021	
	50.73	β —11,+.002; γ —10,—.018	
	50.61	α —11,—.007	
	50.20	ε +10,+.032	
	49.83	δ +10,—.009	
u	49.33	β +11,+.102; γ +10,—.096	
s	49.14	α +11,+.018	
	48.65	ε —11,—.002	
	48.36	δ —11,+.006	
u	47.89	β —12,+.040; γ —11,—.093	
s	47.73	α —12,—.018	
	47.15	ε +11,—.005	
	46.85	δ +11,—.022	
	46.52	γ +11,—.013	
	46.23	β +12,+.014	
s	46.13	α +12,+.012	
	45.52	ε —12,+.002	
	45.27	δ —12,+.010	
	44.98	γ —12,+.020	
	44.72	β —13,—.001	
s	44.62	α —13,—.006	
	43.91	ε +12,+.005	
	43.64	δ +12,—.016	
	43.37	γ +12,—.012	
	42.98	β +13,+.026	
s	42.89	α +13,+.028	
	42.15	ε —13,+.004	
	41.93	δ —13,+.012	
	41.69	γ --13,+.013	
	41.37	β —14,+.028	
s	41.26	α —14,+.008	
	40.41	ε +13,—.007	
	40.18	δ +13,—.016	
	39.95	γ +13,—.026	
	39.49	β +14,+.048	
s	39.39	α +14,+.034	
	38.56	ε —14,+.025	
	38.34	δ —14,+.007	
	38.13	γ —14,—.010	
	37.72	β —15,+.006	
s	37.62	α —15,—.010	
	36.71	ε +14,+.020	
	36.47	δ +14,—.020	
	36.31	γ +14,—.005	
	35.70	β +15,+.017	
s	35.63	α +15,+.029	
	34.70	ε —15,+.014	

CATALOGUE OF COMPARISON: OBSERVATION — COMPUTATION—*cont'd.*

Observed by Kayser and Runge		Indication of series, and difference of wave-length	Special remarks
	5134.53	δ—15,+.028	
	34.34	γ—15,—.010	
s, d	33.79	α—16,+.032; β—16,—.048	
	32.74	ε+15,+.014	
	32.52	δ+15,—.021	
	32.40	γ+15,—.002	
s, d	31.68	α+16,+.082; β+16,+.004	
	30.62	ε—16,+.022	
	30.46	δ—16,+.030; ζ+0,—.082; ζ—1,—.017	The Atlas photograph of K. & R. agrees well with the assumption of a head of a single series at 30.5, the interval between δ—16 and γ—16 being darker, and particularly one small line being visible in the interval between γ—16 and α—17, otherwise very clear
	30.32	γ—16,+.010; ζ+1,—.035; ζ—2,+.159	
hp	29.918	ζ+2	
s	29.67	α—17,+.029; β—17,—.047; ζ—3,+.078	
II. head	29.36	θ+0,—.129; θ—1,—.080; θ+1+.008	
	29.20	ζ+3,—.028; θ—2,—.001	
	28.93	θ+2,—.093	
d. gr.	28.72	ζ—4,—.052; η+0,+.028; η—1,+.074; θ—3,—.045	The head of the η series at 287 agrees with the d. gr of K. & R. But an interchange of the heads of ζ. η, and θ is possible
	28.51	ε+16,—.012; η+1,—.046; η—2,+.089; θ+3,+.019	
	28.23	γ+16,—.010; δ+16,—.121; ζ+4,—.057; η+2,—.008	
	28.02	η—3,+.016; θ—4,—.099	
	27.73	ζ—5,+.028; η+3,+.011; θ+4,—.012	
	27.38	α+17,+.029; β+17,—.045; η—4,+.001	
	27.26	θ—5,+.012	
	27.03	ζ+5,—.066; η+4,+.046	
	26.88	ξ+? ι—?	The ι and ξ series are not computed
	26.73	θ+5,—.033	
hp	26.531	η—5	
	26.30	ε—17,+.027; ζ—6,—.081	
s	26.13	δ—17,+.013; θ—6,—.013	
	26.04	γ—17,+.017; η+5,+.020	
	25.71	ζ+6,+.054	
	25.53	η—6,+.083; θ+6,—.015	
s	25.30	α—18,+.021; β—.18,—.052	
	24.90	ξ—? ι+?	
	24.82	ζ—7,+.008; η+6,+.006; θ—7,+.025	
s	24.11	ε+17,+.029; η—7,—.009; θ+7,+.027	
s	23.87	γ+17,+.038; δ+17,—.050; ζ+7,—.097	
	23.34	η+7,—.022	
	23.21	θ—8,+.010	
s	22.88	α+18,+.020; β+18,—.050; ζ—8,—.114	
	22.46	η—8,—.083	
	22.36	θ+8,—.012	
s	21.76	ε—18,+.050; ζ+8,—.270; η+8,+.098	Perhaps rather hp 22.030 for ζ+8
s, d	21.52	γ—18,+.030; δ—18,—.044; θ—9,+.164	
	20.71	α—19,+.036; β—19,—.034; ζ—9,—.220 η—9,—.009	Perhaps rather hp 20.930 for ζ—9
	20.39	θ+9,—.022	
u	19.72	ζ+9,—.127; η+9,+.007	

CATALOGUE OF COMPARISON: OBSERVATION—COMPUTATION—*cont'd.*

Observed by Kayser and Runge		Indication of series, and difference of wave-length	Special remarks
	5119.40	$\epsilon+18,-.003$	
s, d	19.21	$\gamma+18,+.030;\ \delta+18,-.041;\ \theta-10,-.053$	
	18.85	$\zeta-10,+.229;\ \eta-10,+.204$	N 3
s	18.17	$\alpha+19,+.043;\ \beta+19,-.025;\ \theta+10,-.033$	
	18.08	$\iota-?$	
	17.38	$\zeta+10,-.041;\ \eta+10,-.137$	
	16.93	$\epsilon-19,+.018;\ \theta-11,+.007$	
s, d	16.74	$\lambda-19,+.026;\ \delta-19,-.034$	
	16.30	$\eta-11,-.027$	
s	15.84	$\alpha-20,+.010;\ \beta-20,-.057;\ \zeta-11,-.228;\ \theta+11,+.092$	Perhaps rather hp 16.068 for $\zeta-11$
	14.99	$\eta+11,-.085$	
hp	14.751	$\zeta+11$	This ζ line seems to be visible on K. & R.'s Atlas photograph; its observation and the resolution of some of the neighboring ζ coincidences are important
	14.48	$\epsilon+19,-.010$	
s	14.31	$\gamma+19,+.024;\ \delta+19,-.035;\ \theta-12,-.027$	
	13.76	$\eta-12,-.003$	
s	13.17	$\alpha+20,+.015;\ \beta+20,-.050;\ \zeta-12,-.102;\ \theta+12,+.121$	
	12.41	$\eta+12,+.019$	
	11.87	$\epsilon-20,-.008;\ \zeta+12,+.031$	
s	11.71	$\gamma-20,+.012;\ \delta-20,-.037$	
	11.42	$\theta-13,-.088$	
s	10.77	$\alpha-21,+.024;\ \beta-21,-.041;\ \eta-13,-.189$	
u	10.10	$\zeta-13,-.137;\ \theta+13,-.007$	
	09.79	$\iota-?$	
	09.35	$\epsilon+20,+.009;\ \eta+13,-.117$	
s	09.17	$\gamma+20,+.016;\ \delta+20,-.033$	
u	08.45	$\zeta+13,-.239;\ \theta-14,+.012$	Perhaps rather hp 08.689 for $\zeta+13$
s	07.97	$\alpha+21,+.025;\ \beta+21,-.039;\ \eta-14,+.055$	
	07.67	$\xi-?$	
d	06.98	$\zeta-14,+.016;\ \theta+14,+.055$	
	06.60	$\epsilon-21,-.009$	
s	06.44	$\gamma-21,-.004;\ \delta-21,-.045;\ \eta+14,+.136$	
	06.00	$\xi+?$	
	05.44	$\alpha-22,+.014;\ \beta-22,-.049;\ \zeta+14,+.139$	
u	05.11	$\theta-15,-.019$	
	04.67	$\eta-15,+.035$	
	03.95	$\epsilon+21,-.010$	
s	03.80	$\gamma+21,+.016;\ \delta+21,-.028$	
	03.43	$\zeta-15,-.024;\ \theta+15,-.079$	
	03.17	$\iota-?$	
	02.93	$\eta+15,+.023$	
s	02.53	$\alpha+22,+.029;\ \beta+22,-.032$	
	01.58	$\zeta+15,-.098;\ \theta-16,-.006$	
	01.10	$\epsilon-22,-.010;\ \eta-16,-.021$	
s	00.95	$\gamma-22,-.006;\ \delta-22,-.042$	
s	5099.89	$\alpha-23,+.017;\ \beta-23,-.043;\ \zeta-16,+.181;\ \theta+16,+.036$	
	99.27	$\eta+16,-.006$	
	98.34	$\epsilon+22,-.007$	
s	98.19	$\gamma+22,+.009;\ \delta+22,-.032$	
u	97.80	$\zeta+16,-.022;\ \theta-17,-.008$	The indistinctness of

CATALOGUE OF COMPARISON: OBSERVATION — COMPUTATION—*cont'd.*

Observed by Kayser and Runge		Indication of series, and difference of wave-length	Special remarks
u	5097.51	ι+?	this and the following lines may indicate the approximate position of the III head
u	97.36	η−17,−.014	
s	96.84	α+23,+.016; β+23,−.043	
u	95.98	θ+17,+.011	
hp	95.734	ζ−17	
	95.36	ε−23,−.019; η+17,−.055	
s	95.22	γ−23,−.015; δ−23,−.049	
	94.83	ξ+?	
s	94.13	α−24,+.042; β−24,−.017	
	93.74	ζ+17,+.005; θ−18,−.060	
u	93.45	η−18,+.051	
u	92.88	ξ−?	
	92.52	ε+23,+.014	
s	92.36	γ+23,+.012; δ+23,−.027	
	91.85	θ+18,−.005	
	91.51	ζ−18,−.018	
u	91.29	η+18,−.035	
s	90.94	α+24,+.022; β+24,−.035	
u	90.51	ξ+?	
	89.43	ε−24,+.008; ζ+18,+.009; θ−19,−.134	
s	89.29	γ−24,+.006; δ−24,−.028; η−19,+.095	
	88.55	ξ−?	
	88.11	α−25,+.034; β−25,−.022	
	87.53	θ+19,+.017	
	87.09	ζ−19,−.007	
	86.91	η+19,−.099	
	86.43	ε+24,−.009	
s	86.31	γ+24,+.024; δ+24,−.015	
	85.12	θ−20,+.018	
s	84.80	α+25,+.015; β+25,−.041; ζ+19,−.081; η−20,+.033	
u	83.93	ξ−?	
	83.24	ε−25,+.001	
s	83.08	γ−25,−.026; δ−25,−.060; θ−20,+.134	
u	82.35	ζ−20,−.091; η+20,−.119	
s	81.86	α−26,+.022; β−26,−.033	
	81.42	ξ+?	
	80.45	θ−21,+.034	
	80.15	ε+25.+.003; ζ+20,+.034; η−21,+.035	
s	80.03	γ+25,+.032; δ+25,−.009	
s	78.44	α+26,+.012; β+26,−.043	
	78.16	θ+21,+.003	
	77.70	η+21,−.006	
	77.52	ζ−21,−.043	
	76.83	ε−26,−.002	
s	76.70	γ−26,−.006; δ−26,−.041	
	75.42	α−27,+.043; β−27,−.011; θ−22,−.090	
	75.03	ζ+21,−.103; η−22,−.212	N 3
	73.64	ε+26.+.006	
	73.53	γ+26,+.042; δ+26,−.001	
	73.16	θ+22,+.012	
	72.61	η+22,−.113	

CATALOGUE OF COMPARISON: OBSERVATION—COMPUTATION—*cont'd.*

Observed by Kayser and Runge		Indication of series, and difference of wave-length	Special remarks
u	5072.48	ζ − 22, + .014	
s	71.88	α + 27, + .030; β + 27, — .024	
d	70.46	θ — 23, + .076	
	70.20	ε — 27, — .007; η — 23, + .050	
s	70.08	γ — 27, — .004; δ — 27, — .042	
	69.86	ζ + 22, — .072	
s	68.73	α — 28, + .034; β — 28, — .020	
	68.28	λ — ?	The λ series belonging to the third head is not computed
	67.91	θ + 23, — .012	
u	67.59	η + 23, + .068	
hp	67.151	ζ — 23	
	66.91	ε + 27, + .008	
s	66.81	γ + 27, + .050; δ + 27, + .005	
	66.46	λ + ? μ?	The μ lines belong probably to some uncomputed series from the IV head
	66.32	μ?	
s	66.07	ξ + ? μ?	
	65.56	μ?	
	65.41	μ?	
	65.18	μ?	
s	65.09	α + 28, + .036; β + 28, — .016; θ — 24, + .047	
hp	64.841	η — 24	
u	64.59	ζ + 23, + .075	
	64.32	ι + ? μ?	
	63.67	ξ — ? μ?	
	63.39	ε — 28, + .024	
hp	63.265	γ — 28(+ .021); δ — 28, (— .020)	Overlooked by K. & R.; very strong on their Atlas photograph
	62.46	θ + 24, — .020	
hp	62.106	η + 24	
	61.81	α — 29, + .010; β — 29, — .042	
	61.53	ζ — 24, — .096	
	59.94	ε + 28, — .015	
	59.85	γ + 28, + .036; δ — 28, — .013	
u	59.34	η — 25, + .023; θ — 25, — .147	
	58.91	ζ + 24, + .024	
s	58.06	α + 29, + .016; β + 29, — .035	
	56.80	θ + 25, — .026	
	56.30	ε — 29, — .008; η + 25, — .176	
s	56.21	γ — 29, + .020; δ — 29, — .024	
u	55.88	ζ — 25, — .006	
s	54.73	α — 30, + .040; β + 30, — .011	
	54.37	λ + ?	
	53.66	η — 26, + .079; θ — 26, — .060	
	53.14	ζ + 25, + .094	
s	52.75	γ + 29, + .094; δ + 29, + .042; ε + 29, — .045	
s, u	50.86	α + 30, + .037; β + 30, — .012; η + 26, + .226; θ + 26, — .101	
	49.89	ζ — 26, — .048	
s	49.68	λ + ? ν?	The ν lines belong probably to some uncomputed series from the V head
	49.52	ν?	
hp	48.978	γ — 30(+ .054); δ — 30(+ .008); ε — 30, (— .063)	Remarkable absence of 3 coincident lines. Instead of the expected strong line, the Atlas photograph has only a faint trace of a line
	48.61	ξ + ? ν?	
	48.46	ν?	
	48.27	ν?	

CATALOGUE OF COMPARISON: OBSERVATION—COMPUTATION—*cont'd.*

Observed by Kayser and Runge		Indication of series, and difference of wave-length	Special remarks
u	5047.68	$\eta-27,+.046$; $\theta-27,-.064$	
	47.41	$\alpha-31,+.038$; $\beta-31,-.011$	
	47.16	$\zeta'+?$ $\lambda-?$ $\nu?$	The ζ' lines seem to indicate a duplicity of the ζ series
	47.02	$\zeta+26,+.022$	
s	45.39	$\gamma+30,+102$; $\delta+30,+.047$; $\epsilon+30,-.036$	
	44.87	$\theta+27,-.019$	
hp	44.582	$\eta+27$	
u	43.81	$\zeta-27,+.024$	
u	43.42	$\alpha+31,+.026$; $\beta+31,-.023$	
s	41.47	$\gamma-31,+.020$; $\delta-31,-.030$; $\epsilon-31,-.096$; $\eta-28,-.009$; $\theta-28,-.092$	
u	40.86	$\zeta+27,+.113$	
	40.54	$\iota+?$	
s	39.88	$\alpha-32,+.032$; $\beta-32,-.016$	
u	38.60	$\theta+28,-.012$	
	38.22	$\eta+28,-.104$	
s	37.82	$\gamma+31,+.108$; $\delta+31\ +.048$; $\epsilon+31,-.032$	
	37.57	$\iota-?$ $\lambda-?$	
	37.42	$\zeta-28,-.009$	
u	35.79	$\alpha+32,+.027$; $\beta+32,-.020$	
u	35.14	$\eta-29,+.022$; $\theta-29,-.037$	
	34.46	$\zeta'+?$	
	34.27	$\zeta+28,-.023$	
s	{ 33.84	$\gamma-32,+.069$; $\delta-32,+.015$; $\epsilon-32,-.048$	
d	{ 33.68	$\kappa+?$	The uncomputed κ series belongs to the third head; the duplicity of 33.68 is probably due to $\gamma-32$
	33.08	$\xi-?$	
s	32.18	$\alpha-33,+.058$; $\beta-33,+.011$; $\theta+29,+.048$	
d	31.91	$\eta+29,+.051$	
hp	30.871	$\zeta-29$	
	30.48	$\kappa-?$	
s	30.05	$\gamma+32.+116$; $\delta+32,+.051$; $\epsilon+32,-.024$	
	29.60	$\lambda+?$	
u	28.54	$\eta-30,-.013$; $\theta-30,-.050$	
	27.94	$\alpha+33,+.008$; $\beta+33,-.037$	
	27.65	$\zeta+29,+.012$	
s, d	25.92	$\gamma-33,+.030$; $\delta-33,-.029$; $\epsilon-33,-.087$	
u	25.49	$\theta+30,+.037$	
hp	25.195	$\eta+30$	
	{ 24.22	$\alpha-34,+.021$; $\beta-34,-.024$	
s	{ 24.09	$\zeta-30,-.023$	
s	22.07	$\gamma+33,+.114$; $\delta+33,+.045$; $\epsilon+33,-.026$	
	21.72	$\eta-31,-.067$; $\theta-31,-.085$	
	20.89	$\zeta'+?$ $\lambda-?$	
	20.79	$\zeta+30,+.004$	
	19.87	$\alpha+34,-.034$; $\beta+34,-.079$	
u	18.58	$\theta+31,+.003$	
hp	18.329	$\eta+31$	
s, d	17.83	$\gamma-34.+.018$; $\delta-34,-045$; $\epsilon-34,-.100$	
	17.28	$\iota-?$	
	17.13	$\zeta-31,-.030$	
	16.12	$\alpha-35,+.038$; $\beta-35,-.005$	
u	15.29	$\lambda-?$	

CATALOGUE OF COMPARISON: OBSERVATION — COMPUTATION—*cont'd.*

Observed by Kayser and Runge		Indication of series, and difference of wave-length	Special remarks
u	5014.84	η—32,+.019; θ—32,+.015	
s	13.89	γ+34,+.106; δ+34,+.034; ε—34,—.036; ζ+31,+.151	
u	12.42	ξ—? λ+?	
	11.66	α+35,—.026; β+35,—.068; θ+32+.154	
hp	11.265	η+32	
	10.03	ζ—32,+.017	
	{ 09.62	ε—35,—.038	
s	09.53	γ—35,—.010; δ—35,—.076	
	09.18	λ—?	
	07.82	α—36,+.046; β—36,+.003; η—33,+.160; θ—33,+.168	
u	07.27	κ—?	
	06.50	ζ+32,+.000	
	06.24	ι+? λ+?	
s, d	05.55	γ+35,+.133; δ+35,+.058; ε+35,—.013	
	04.37	θ+33,+.126	
hp	04.006	η+33	
	03.20	α+36,—.078; β+36,—.121	
d	02.68	ζ—33,+.005	
s, d	01.09	γ—36,+.014; δ—36,—.055; ε—36,—.110	
	00.32	η—34,+.016; θ—34+.031	
	4999.91	λ+?	
	99.65	N 3	For the lines marked N 3 no special indication is possible, but they may belong to the series from third head
	99.32	α—37,+,039; β—37,—.003	
u	99.09	ζ+33,+.019	
s, d	96.99	γ+36,+.129; δ+36,+.050; ε+36,—.024; θ+34,+.196	
hp	96.554	η+34	
u	95.16	ζ—34,+.011	
u	94.68	α+37,—.008; β+37,—.050	
	93.39	λ+?	
	92.89	η— 35,+.133; θ+35,+.151	
s, d	92.44	γ—37,+.013; δ—37,—.058; ε—37,—.113	
	91.50	ζ+34,+.044	
	91.12	ι+?	
	90.64	α— 38,+.034; β—38,—.008	
u	90.12	ξ—? λ—?	
hp	89.035	η+35 (+.123); θ+35,(—.123)	The θ series is vanishing, the η series perhaps also, though the graphical construction seems to indicate a prolongation of both branches. The following η lines are given here only for use in a future investigation upon this question
s	88.27	γ+37,+.150; δ+37,+.068; ε+37,—.004	
	87.44	ζ—35,+.001	
u	86.70	λ+?	
	85.96	α+38,+.040; β+38,+.000	
hp	85.013	η—36 (—.008); θ—36,(+.008)	
s	83.62	γ—38,+.026; δ—38,—.046; ε—38,—.102; ζ+35,—.037	
	81.79	α —39,+.034; β—39,—.006	
hp	81.211	η+36 (+.128); θ+36,(—.128)	
s	79.36	γ+38,+.160; δ+38,+.077; ε+38,+.002; ζ—36,—.186	The negative branch of ζ seems to vanish
	76.97	α+39,—.006; β+39,—.046; η—37,—.128; θ—37,—.120	

CATALOGUE OF COMPARISON: OBSERVATION — COMPUTATION—*cont'd.*

Observed by Kayser and Runge		Indication of series, and difference of wave-length	Special remarks
u	4975.69	ζ+36,+.012	
s	74.58	γ—39,—.002; δ—39,—.076; ε—39,—.135	
	73.69	η+38?	
	72.78	α—40,+.048; β—40,+.008	
	71.54	ζ—37,+.064	
s	70.25	γ+39,+.148; δ+39,+.064; ε+39,—.016	
hp	68.992	η—38	
	67.84	α+40,—.024; β+40, -.062	
	67.53	ζ+37,+.006	
s	65.39	γ—40,—.004; δ—40,—.080; ε—40,—.144	
hp	64.871	η+38	
	63.60	α—41,+.058; β—41,+.021	
hp	63.233	ζ—38	
	63.02	ι—?	
s	60.96	γ+40,+.129; δ+40,+.046; ε+40,—.040; η—39,+.255?	
	59.19	ζ+38,—.009	
	58.59	α+41,+.005; β+41,—.032	
	58.16	λ+?	
	57.73	ξ—?	
	57.42	η+39,+.926?	
s	56.08	γ—41,+.042; δ—41,—.031; ε—41,—.102	
	54.70	ζ—39,—.120?	ζ—39 and the following ζ lines marked ? do not really belong to the negative branch of ζ; probably the duplicity indicated by the ζ' lines of the positive branch will explain the anomalies of these lines
	54.25	α—42,+.063; β—42,+.026	
hp	52.239	η—40?	
s	51.50	γ+41,+.109; δ+41,+.026; ε+41,—.068	
	50.69	ζ+39,—.015	
	50.20	ι+?	
	49.14	α+42,—.006; β+42,—.042	
hp	47.940	η+40?	
s	46.46	γ—42,—.053; δ—42,—.124; ε—42,—.204	
	46.08	ζ—40,—.161?	
	44.69	α—43,+.016; β—43,—.021; η—41,+1.092	
	42.94	λ+?	
	42.62	N 3	
	41.92	γ+42,+.133; δ+42,+.054: ε+42;—.051; ζ+40,—.129	
	40.90	κ+?	
hp	39.212	η+41?	
	37.37	ζ—41,—.133?	
	36.83	γ—43,+.003; δ—43,—.064; ε—43,—.156	
	35.11	α—44,+.102; β—44,+.066	
hp	34.783	η—42?	
	33.27	ζ+41,+.035	
	32.18	γ+43,+.158; δ+43,+.084; ε+43,—.043	
hp	30.312	η+42?	
	28.52	ζ—42,—.089?	
	26.96	γ—44,—.022; δ—44,—.082; ε—44,—.189	
	25.79	η—43?	
hp	25.204	α—45 (+021); β—45,(—.011)	
	24.87	κ+?	

CATALOGUE OF COMPARISON: OBSERVATION — COMPUTATION—*cont'd.*

Observed by Kayser and Runge		Indication of series, and difference of wave-length	Special remarks
	4924.28	ζ + 42, +.011	
hp	22.236	γ + 44, +.134; δ + 44, (+.068); ε + 44, (—.068).	Absence of coincident lines
hp	21.244	η + 43 ?	
hp	19.566	ζ — 43 ?	
	18.05	η — 44, + 1.402	
	16.96	γ — 45, —.024; δ — 45, —.074; ε — 45, —.200	
	15.16	α — 46, —.074; β — 46, —.108; ζ + 43, +.004	
	14.63	λ - - ?	The α and β series are totally lost
	12.23	γ + 45, +.199; δ + 45, +.144; ε + 45, —.012; η + 44, +.220 ?	
hp	10.379	ζ — 44 ?	
hp	07.332	η — 45 ?	
	06.86	γ — 46, +.023; δ — 46, —.014	A considerable number of the last lines belonging to the positive branch of γ and to the negative branch of ε, though certainly invisible, have hitherto been taken in consideration of future observations; now the δ series also fades away
	05.88	ζ + 44, —.021	
	05.42	N3	
hp	02.614	η + 45 ?	
	01.96	δ + 46, +.108; ε + 46 —.072	
	00.90	ζ — 45, —.153 ?	
	4899.98	η — 46, + 2.125 ?	
	97.56	λ — ?	
	96.52	γ — 47, —.026; δ — 47, —.046; ζ + 45. +.010	
	93.72	λ + ?	
hp	93.057	η + 46 ?	
hp	91.583	δ + 47, (+.109); ε + 47, (—.098); ζ — 46; (—.010)	Absence of coincident lines
	90.89	η — 47, + 2.671	See remark to 4989.035
	87.01	ζ + 46, +.023	
s	86.14	γ — 48, +.023; δ — 48, +.026	
	85.64	N	
	85.05	λ + ?	
hp	82.005	ζ — 47 ?	
	81.19	ε + 48, —.003	
	77.33	ζ + 47, —.010	
	75.51	γ — 49, —.042; δ — 49, —.012	
hp	72.295	ζ — 48 ?	
	70.58	ε + 49, +.008	
	67.52	ζ + 48, —.052	
	64.86	γ — 50, +.000; δ — 50, +.066	
hp	62.467	ζ — 49 ?	
	59.88	ε + 50, +.056	
	58.55	N3	
	57.68	ζ + 49, —.009	
	55.95	κ + ?	
s	54.11	γ — 51, +.067	
	53.67	λ — ?	
	52.44	ζ — 50, —.085 ?	
s	48.93	ε + 51, —.023	
	47.66	ζ + 50, —.035	
	43.11	γ — 52, +.001	

CATALOGUE OF COMPARISON: OBSERVATION—COMPUTATION—*cont'd.*

Observed by Kayser and Runge		Indication of series, and difference of wave-length	Special remarks
	4842.31	$\zeta - 51, -.168$?	
s	37.99	$\epsilon + 52, +.025$	
	37.59	$\zeta + 51, -.008$	
	32.80	$\kappa -$?	
	32.13	$\gamma - 53, -.068$; $\zeta - 52, -.197$?	
	27.38	$\zeta + 52, -.020$	
	26.87	$\epsilon + 53, +.004$	
	25.88	$\lambda -$?	
	21.80	$\zeta - 53, -.279$?	
	20.93	$\gamma - 54, +.022$	
	17.14	$\zeta + 53, +.035$	
	15.66	$\epsilon + 54, +.000$	
	11.99	N3	
	11.50	$\zeta - 54, -.237$?	
s	09.63	$\gamma - 55, -.026$	
hp	06.720	$\zeta + 54$	
s	04.35	$\epsilon + 55, -.004$	
	01.03	$\zeta - 55, -.277$?	
	4798.79	$\kappa +$?	
s	98.32	$\gamma - 56, +.011$	
	96.24	$\zeta + 55, -.009$	
	92.92	$\epsilon + 56, -.036$	
	86.88	$\gamma - 57, +.004$	
	85.63	$\zeta + 56, -.064$	The last trace of the ζ series
	81.46	$\epsilon + 57, -.013$	
	79.44	$\kappa +$?	
	75.32	$\gamma - 58, -.044$	
	72,18	$\lambda +$?	
	69.87	$\epsilon + 58, -.044$	
	63.86	$\gamma - 59, +.078$	
	58.33	$\epsilon + 59, +.041$	
	52.06	$\gamma - 60, -.082$	
	46.55	$\epsilon + 60, -.063$	Some more lines of the fourth band seem to belong to the γ and ϵ series

Of the 433 computed lines, no less than 229 are in the catalogue represented as isolated lines. In consideration of the great number of series, and the complicated structure of the spectrum, this proportion must be regarded as very satisfactory. The inevitable drawback, owing to coincidences with the ζ', ξ, ι, κ, λ, and other not yet computed series, is of no great importance, because of the small intensities of these series. If this number of isolated lines had been uniformly distributed, the

exact computation of all constants would have been an easy matter. But while the interval between 5154 and 5130 abounds in isolated lines, very useful for the special investigations of the series of the first head, the other parts of the band are rather poor in isolated lines.

Only in the case of isolated lines can we properly indicate the precision of the measures by their mean errors. Thus only by means of this interval 5154–5130 an approximate calculation of the mean error of a single observation by Kayser and Runge to $\pm$ 0.020 has been possible by means of our catalogue of differences (Obs.—Comp.). This value accords very well with the statements of the observers themselves. Under the most favorable circumstances indeed their measures are so accurate that they deserved to be given with three decimal places instead of two. It is not only in the hope of very accurate future observations that I have given my tables and comparisons with a greater number of decimal places, but also in respect to the measures of Messrs. Kayser and Runge.

In the cases of feebler and indistinct lines, no doubt the mean errors are greater, and as Kayser and Runge very seldom have resolved duplicities with less than 0.1 difference in wavelength, we can understand that faint lines with supposed differences of 0.15, or even more from strong ones, may easily have been overlooked. I confess, however, that not a few such lines, which I have found it more convenient to identify with Kayser and Runge lines, perhaps ought to have been mentioned as *hp* lines in my catalogue.

COPENHAGEN OBSERVATORY,
April 1898.

THE SPECTRA AND PROPER MOTION OF STARS.

By W. H. S. Monck.

In January 1893 I contributed to *Astronomy and Astro-Physics* an article on the spectra and proper motion of stars, and the same number contained one by Mr. Maclair Boraston dealing, at least partially, with the same subject. Five years have naturally led to further researches on the subject, and the views which I then expressed need some modification, though I think they were substantially correct.

In the first place the use of more powerful spectroscopes has led Professor Pickering to alter somewhat the classification of spectra which he adopted in the *Draper Catalogue*. The types of spectra which he now adopts as regards the vast majority of the stars are A, B, F, G, K and M, with intermediate spectra designated in some such manner as A5F or F2G. He agrees with the view which I then expressed in placing the type B above A in respect of brilliancy, so that the first two letters of the alphabet are transposed in his list. I divided these stars (except those of the type M) into three classes, Sirian, Capellan, and Arcturian: but I left out the stars described as G in the *Draper Catalogue* because I was doubtful as to which of the two latter classes they belonged to. There is now no doubt that they are Capellans, for most of the stars classed as E in the *Draper Catalogue* become G in a more powerful spectroscope. This of course would make some changes in my figures; and I also obtained, partly from published sources and partly from private communications, for which I have to thank Professor Pickering, a number of corrections in the *Draper Catalogue*. I was likewise able to use a fuller catalogue of stars with large proper motion than I believe had hitherto been available—I mean that of M. Bossert of the Paris Observatory, which (including the appendix) contains 2675 stars, all of which have proper motions of not less

than one-tenth of a second per annum in one or other of the elements.

The small proper motion of the Sirian stars compared with those of other types being known and acknowledged, I did not think it necessary to ascertain the corrected number of Sirians in the *Draper Catalogue*. It cannot be less than 5000. But it was otherwise with those of the Capellan and Arcturian types, because it was necessary to ascertain whether the preponderance of the former among the stars of large proper motion was or was not due to their greater frequency in the sky. My count may not be perfectly accurate even on the information now attainable, but I think it is so substantially. I made the Arcturian stars 2386 against 2293 Capellans. I think I may safely say that the latter are not more numerous than the former, and I have little doubt that they are less so. I was not, however, led to Mr. Boraston's conclusion that these two classes of stars are distributed in a similar manner over the sky. Following the order of right ascension as in the *Draper Catalogue*, the first hour showed a preponderance of Arcturians, but then there was a decided preponderance of Capellans until about the $9^h\ 30^m$, up to which point they outnumbered the Arcturians by about 160. Then there was another change, and from this to the 24th hour the Arcturian majority was about 250. There were several pages of the catalogue in which the stars of one class were more than twice as numerous as those of the other. I did not, however, attempt to trace the laws of distribution more minutely.

With this preface I give the results of my comparison of M. Bossert's catalogue with the *Draper Catalogue*, utilizing also a few spectra of Southern stars that have been published by Professor Pickering. The result is:

Sirian stars, - - - - - -	225
Capellan stars, - - - - - -	461
Arcturian stars, - - - - - -	366

Intermediate types I reckoned as ½ to each, merely omitting the fraction at the end of the result (strictly, I made 225½ Sirian). The Capellan stars with large proper motion thus

exceed the Arcturian in the proportion of fully five to four, while they are probably less numerous on the whole. They exceed the Sirians in the proportion of more than two to one, while they are less than half as numerous on the whole.

Stars of the type M seem to possess considerable proper motion according to this comparison. There are ninety-eight of them in the *Draper Catalogue* and twenty in that of Bossert. The number, however, is rather too small for a final decision, especially as the M is marked with a query in a large proportion of the cases. Stars with peculiar spectra — not reducible to any of the foregoing heads — seem, generally speaking, to have very small proper motions.

As to the figures which I have given, it will be noted that M. Bossert includes in his catalogue stars with less proper motion than any other catalogue of stars with *large* proper motion that I had hitherto examined. Of course the lower we descend the larger will be the proportion of Sirian stars and the more nearly will the Capellans and Arcturians become equal, or rather tend to exhibit a slight excess of the latter. Had M. Bossert stopped at one-fifth instead of one-tenth of a second, the distinction between the three classes would have been more strongly marked.

Though somewhat doubtful as to the positions of stars of the types G and M, my present arrangement in order of brilliancy (and consequently of average distance for stars of the same magnitude) would be B, A, K, M, F, G. If this order is correct, it seems clear that no continuous gradation of spectra can be traced through it. Arcturian stars are not Capellans which have cooled down to a lower stage. If they were so, then proper motions would, on the average, be greater, not less than those of the uncooled Capellans. Cooling would reduce the light of the star without affecting its proper motion. Consequently the cooled star would, on the average, have the greater proper motion, the magnitudes (*i. e.* quantity of light) being supposed equal. Are the Capellan stars cooled-down Sirians? The difference in the amount of their average proper motions is start-

ling. But then Professor Pickering has, on fuller examination, discovered many intermediate types, and as far as I am at present able to form an opinion, these intermediate types exhibit intermediate proper motions. I do not think we are as yet in a position to form any confident opinion. But an examination of binary stars with reliable orbits confirms my conclusion that the Arcturian stars are not cooled-down Capellans and that on the supposition that these two types represent different stages of star-life, the Capellan, not the Arcturian stars must represent the later stage.

ON THE RELATIVE INTENSITIES OF THE LINES IN THE SPECTRUM OF THE ORION NEBULA.

By C. RUNGE.

ON a visit to the Lick Observatory in September 1897 I was allowed, through the kindness of Professor Campbell, to confirm his observations of the differences in the relative intensities of the lines in different parts of the Orion nebula (see *Astronomische Nachrichten*, No. 3471). To these observations Professor Scheiner raises the objection that the differences are not real, but merely physiological effects due to the peculiar constitution of the human eye (*A. N.*, No. 3476). He very rightly insists that these physiological effects ought to be taken into account in a discussion of observations of this kind. But, as I believe I am able to show, he overrates the physiological effect in assuming that it suffices to explain the whole observed differences.

The physiological effect (the so-called Purkinje phenomenon) is this: Two objects emitting light of different colors do not maintain the same apparent relation of intensities when their real intensities are altered in equal proportions. A bright red object, for instance, of apparently the same brightness as a blue object, appears very much darker than the latter on reducing the real intensities of both in equal proportions. To keep the apparent intensities equal, the light of shorter wave-length must be reduced very much more than the light of longer wave-length. This reduction may be determined in quantity and thus furnishes a measure of the physiological effect. A. König has investigated the subject very thoroughly.[1] If we call B_{λ} and $B_{\lambda'}$ the apparent intensities of two objects at medium brightness emitting light of wave-lengths λ and λ', and if we call b_{λ} and b_{λ} the apparent intensities of the same objects at low bright-

[1] A. KÖNIG, "Abhängigkeit der Farben und Helligkeitsgleichungen von der absoluten Intensität," *Sitzungsber. d. K. Akad. d. W., Berlin*, July 1897. See also, "Helligkeitswerth der Spektralfarben," *Festschrift zum 70. Geburtstage von H. von Helmholtz*, by the same author.

ness after reducing the real intensities in equal proportions, then the value of $\frac{b_\lambda}{b_{\lambda'}} : \frac{B_\lambda}{B_{\lambda'}}$ measures the effect in question. This value, as A. König finds, does not practically change when the medium brightness or the low brightness is changed within considerable limits, always supposing that the real intensities of both colors keep the same relative intensity. It is also independent of the relation of the real intensities. With the color comparing instrument devised by Helmholtz and improved by A. König two semicircular fields of about 4° apparent magnitude can be made to emit light of any two wave-lengths. The intensity of each field can be altered separately by turning one of a pair of Nicols, and the intensity of both fields can be altered simultaneously in the same proportion by turning one of a pair of Nicols in the eyepiece. The angle between the planes of polarization of the two Nicols determines the amount of reduction of real intensity, the real intensity being proportional to the square of the cosine.

Through the kindness of Professor A. König I was allowed the use of his instrument to measure the Purkinje effect with the colors of the three nebular lines 4861, 4959, 5007. Comparing $\lambda = 4861$ and $\lambda' = 5007$ I first regulated the intensities so that both fields appeared to be of equal medium brightness ($B_\lambda = B_{\lambda'}$). By turning the Nicol in the eyepiece both intensities were reduced in the same proportion to low brightness (about $\frac{1}{100}$ of the medium intensity). The field $\lambda = 4861$ then appeared decidedly brighter than the other and was reduced to equal apparent intensity by turning a Nicol through a certain angle. The two positions of the Nicol were read. Then the Nicol in the eyepiece was turned back and the brightness again increased from low to medium. The fields equal at low brightness appeared unequal at medium brightness, the shorter wavelength this time giving the weaker field. The intensity of this field was then again increased to equal apparent intensity by turning the Nicol. The two positions of the Nicol were read. We may shortly state the proceedings thus:

(1) $B_\lambda = B_{\lambda'}$, $b_\lambda > b_{\lambda'}$, the proportion $\frac{b_\lambda}{b_{\lambda'}}$ was measured.

(2) $b_\lambda = b_{\lambda'}$, $B_{\lambda'} > B_\lambda$, the proportion $\frac{B_{\lambda'}}{B_\lambda}$ was measured.

Two sets of five settings each gave for $B_\lambda = B_{\lambda'}$, $\frac{b_\lambda}{b_{\lambda'}} = 1.81$ and 1.74 (mean = 1.78), and two other sets of five settings each gave for $b_\lambda = b_{\lambda'}$, $\frac{B_{\lambda'}}{B_\lambda} = 1.78$ and 1.73 (mean = 1.76). Both cases are contained in the equation

$$\frac{b_\lambda}{b_{\lambda'}} : \frac{B_\lambda}{B_{\lambda'}} = 1.8$$

or

$$\frac{b_\lambda}{b_{\lambda'}} = 1.8 \, \frac{B_\lambda}{B_{\lambda'}}.$$

I may therefore say that on reducing the brightness from a medium to a low degree the wave-length 4861 gained on 5007 so that the relative intensity was nearly doubled. Professor A. König, who had the kindness to repeat the measurements for me, did not find the effect so strong for his eye. His determinations were:

(1) $$B_\lambda = B_{\lambda'}, \ \frac{b_\lambda}{b_{\lambda'}} = 1.57 \text{ and } 1.38$$

(2) $$b_\lambda = b_{\lambda'}, \ \frac{B_{\lambda'}}{B_\lambda} = 1.54 \text{ and } 1.47$$

$$\text{mean } \frac{b_\lambda}{b_{\lambda'}} = 1.49 \, \frac{B_\lambda}{B_{\lambda'}}$$

The effect seems to decrease when the image of the field is formed on parts of the retina away from the fovea.

According to these results we should indeed expect the nebular lines 4861 and 5007 to alter their apparent relative intensity in more or less bright parts of the nebula, even if there were no real alteration of the relative intensity. $\frac{B_{4861}}{B_{5007}}$ might, according to my measurements, apparently increase in weaker parts of the nebula, but it would not increase more than 1.8 times, unless there was a real increase of relative intensity.

Now I estimated $\frac{B_{4861}}{B_{5007}}$ in brighter parts of the nebula as between $\frac{1}{3}$ and $\frac{2}{5}$ and in a weaker part of the nebula as equal to 10. The increase was therefore between twenty-five and thirty times. I do not place much confidence in the exact values of my estimations; but I have no doubt that a change in relative intensity, which I estimated to be in the proportion 1:25, cannot have been as small as 1:2.

In order to see if my estimation of the relative intensity 2:5 and 10:1 somewhat agreed with the measurements of the instrument, I tried by estimation to set the two fields in the colors $\lambda = 4861$ and 5007 to these relative intensities and afterwards measured them. The estimated relative intensity 2:5 I found 2:4, and the estimated relative intensity 10:1 I found 8.5:1. However, I did not satisfy myself how far this agreement was due to chance.

Another proof that the differences in the relative intensities of the nebular lines really exist, is furnished by comparing the lines 4861 and 4959. Near the trapezium they appear equally bright. In the neighborhood of Bond 734 the line 4861 is well discernible (10 times brighter than 5007), while 4959 could no longer be seen. Now I have made one field in the color-comparing apparatus emit light of wave-length 4861 and the other light of wave-length 4959 of apparently equal brightness. On lowering the intensities of both fields by turning the nicol of the eyepiece the Purkinje effect was plainly visible, 4861 appearing a little brighter. On lowering the intensity still more, so that the two fields were just visible, the effect was lost, and then the two fields vanished simultaneously. On increasing the intensity they again appeared simultaneously. This is not reconcilable with the observations of the two nebular lines 4861 and 4959, unless there is a real difference in the relative intensities. The proof is less objectionable than the first, as no estimation of relative intensities enters into it.

Professor Scheiner has observed the Purkinje effect with the colors of $H\alpha$ and $H\beta$. That is altogether a different thing.

According to A. König's measurements[1] the relative apparent intensity $\frac{H\beta}{H\alpha}$ increases about 1000 times on reducing the real intensities in equal proportions from medium to low brightness. The Purkinje effect is therefore about 600 times stronger than with the colors $H\beta$ and 5007. I agree with Professor Scheiner that the apparent absence of $H\alpha$ in the spectrum of many nebulæ may be due to this purely physiological cause.

One might increase the evidence of real differences in the relative intensities of the nebular lines. I do not think that it would give any serious difficulty to measure the relative intensities of the nebular lines instead of estimating them. And then instead of measuring the Purkinje effect as I have done with semicircular fields, it would be safer to measure it directly with the lines of the brighter portion of the nebula and to compare the difference of relative intensities with the difference observed on shifting the slit to other parts of the nebula.

TECHNISCHE HOCHSCHULE,
Hannover, May 1898.

[1] *Sitzungsber. d. K. Akad. d. W., Berlin*, July 1897, p. 881.

THE ECHELON SPECTROSCOPE.[1]

By A. A. Michelson.

The resolving power of a diffraction grating is proportional to the product of the total number of lines by the order of the spectrum observed. But little effort seems to have been made to make a decided step in the direction of increasing the *order* of the spectrum observed, and this is doubtless because for a grating acting by *opacity* the brightness of the spectrum diminishes very rapidly as the order increases. This difficulty has been successfully overcome by ruling the lines in such a way as to concentrate the greater proportion of light in one spectrum, but so far as I am aware such attempts have been limited to the first, second or third spectrum and the results even here are somewhat fortuitous.

It seems nevertheless quite possible to construct gratings which shall throw a quite large proportion of the light in very high orders of spectra — say the hundredth — in which case the grating space must be of the order of a hundred waves or say twenty to the millimeter, instead of a thousand. The lines would have to be drawn with no more accuracy than before, and the grating could be completed in a very short time and temperature changes would have a much smaller effect than at present.

It may be that there are more serious practical difficulties in the way of such a ruling as is represented in Fig. 1 than would be anticipated. Especially may this be true if the greater part of the light is to be returned in the direction from which it came; for that the grooves must be correspondingly deep, and the grating space would vary with the depth.[2] Fig. 1 at once suggests a possible method of effecting the same result, by

[1]The first part of this paper was published as a preliminary notice in the *Am. Jour. Sci.* for March.

[2]This question will receive a practical test as soon as a ruling machine now under construction is completed.

building up the steps by equal thicknesses of optical glass. Here the difficulty, even supposing the optical work to be

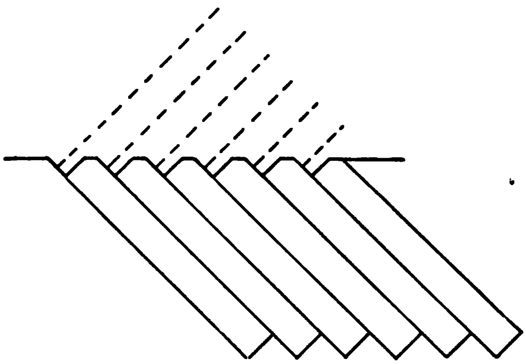

FIG. 1.

practically perfect, would be the joining of the separate plates in such a way as to have always the same distance between them.

By using the same arrangement for transmission instead of reflection this difficulty is avoided—and there remains absolutely nothing but the difficulty of making a considerable number of plane-parallel plates of the same thickness—to an order of accuracy only one-fourth that required in the former arrangement, or even one-tenth of this if the other medium be water or oil instead of air.

Probably the surprising thing is the smallness of the number of plates required to give results which are comparable with those of the best gratings. This can be shown as follows: Let abd (Fig. 2) be one step in the series of plates and let $ab = s$ and $bd = t$. If m is the order of the spectrum observed, $m\lambda = \mu \,.\, bd - ac$ or

$$m\lambda = \mu t - t \cos\theta + s \sin\theta$$

$$\frac{d\theta}{d\lambda} = \frac{m - t\dfrac{d\mu}{d\lambda}}{t \sin\theta + s \cos\theta}$$

$$\frac{d\theta_1}{dm} = \frac{\lambda}{t \sin\theta + s \cos\theta}$$

and if $\delta\theta$ is the displacement corresponding to $\delta\lambda$ and $\delta\theta_1$ that corresponding to $\delta m = 1$, assuming Cauchy's formula

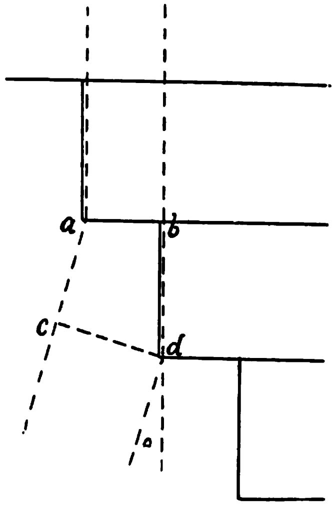

FIG. 2.

$\mu = a + b/\lambda^2$, and taking the approximate value of $m = (\mu - 1)\, t/\lambda$, we have

$$\delta\theta/\delta\theta_1 = [(\mu - 1) + 2(\mu - a)]\frac{t}{\lambda} \cdot \frac{\delta\lambda}{\lambda}.$$

For flint glass the coefficient of t/λ is approximately equal to unity; so that if $\frac{\delta\lambda}{\lambda} = .001$, as in the case of the two sodium lines, and $t = 5^{mm} = 10000\,\lambda$, then $d\theta = 10\, d\theta_1$, that is, the sodium lines would be separated by ten times the distance between the spectra.

The resolving power of such a combination is mn, exactly as in the case of gratings; so that with but *twenty* elements 5^{mm} thick and hence $m = 5000$ the resolving power would be 100000, which is about that of the best gratings.

The experiment was actually tried with but *seven* elements, placed between a collimator and an observing telescope and the collimator slit illuminated with light from a sodium flame.

The images were so distinct that the broadening of the lines could be very easily detected, and the Zeeman effect was readily observed when the sodium flame was placed in a magnetic field.

It is important to note that the resolving power is independent of the number of plates but depends only on the total thickness, and the only advantage in a large number of elements is the greater separation of the spectra. The overlapping of spectra is doubtless a disadvantage, which however could be overcome by a preliminary analysis; and for the examination of single lines and especially in the investigation of effects of broadening, shifting, or doubling of lines, the method seems especially well adapted.

An echelon spectroscope of twenty elements, the essential parts of which were constructed in the Ryerson Physical Laboratory, is represented in Fig. 3.

Fig. 3.

C is the collimator and *T* the observing telescope; *E* the echelon consisting of twenty plates, each 18^{mm} thick, and diminishing in width from 22^{mm} to 2^{mm}, so that the width of the elementary pencils is 1^{mm}, and the successive retardations are of the order of twenty thousand waves.

From what precedes it follows that practically all the light may be concentrated in one spectrum, so that the only losses are those due to the reflections and absorptions, and these are much less than in either gratings or prisms of equal resolving power. The intensity is given by the formula

$$I = \left[\frac{\sin \pi \frac{s}{\lambda} \theta}{\pi \frac{s}{\lambda} \theta} \right]^2$$

or by the curve Fig. 4.

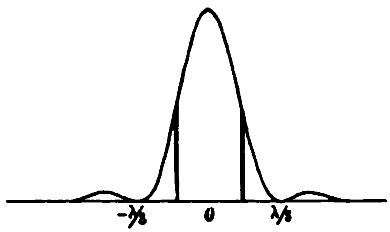

FIG. 4.

All the light is practically included between the deviations $-\frac{\lambda}{s}$ and $\frac{\lambda}{s}$. But the distance between successive spectra is $\frac{\lambda}{s}$, so that in general there will be *two* spectra visible. By slightly inclining the echelon however, the directions of the spectra may be varied so that one falls at O, where the intensity is a maximum, while the adjoining ones disappear.

The theoretical resolving power of the twenty element echelon is 15000×20 or 300000; and the results which are given below show that this limit is nearly reached in practice.

The overlapping of the spectra is overcome by a direct vision prism of moderate dispersion, but the distance between the spectra is so small in comparison with the dispersion of the echelon that the spectrum of the source under examination must consist of rather fine lines if overlapping is to be avoided. Thus, in the present case the range of wave-length corresponding to two successive spectra is $\frac{\delta\lambda}{\lambda} = \frac{2\lambda}{t} = \frac{1}{15000}$; that is,

the lines to be examined must have a total width of less than one-fifteenth of the distance between the sodium lines.

This caused some difficulty in the examination of certain phenomena, as for instance, the Zeeman effect, and it will doubtless be an improvement to have three times the number of elements of one-third the thickness—for then with equal resolving power, the range would be three times as great.

The increased number of the plates will of course increase the losses by reflections, and the degree of accuracy in working the plates must also be correspondingly increased. Thus for twenty plates the retardations must differ by less than one-twentieth of a wave, while with sixty the difference must be less than a sixtieth of a wave.

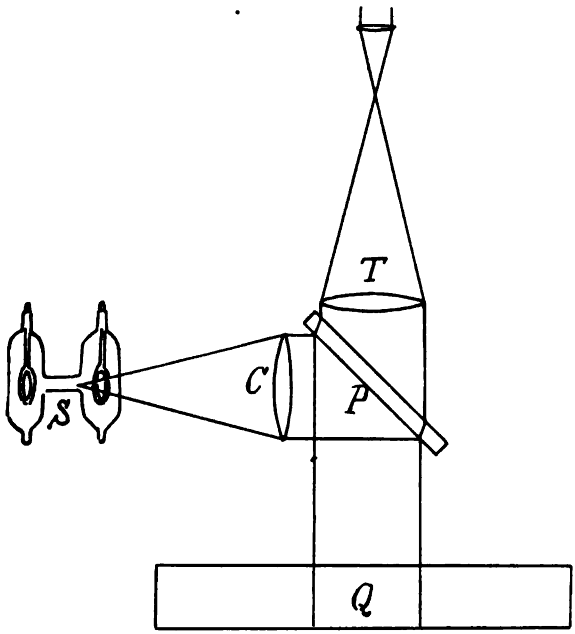

FIG. 5.

The method of testing the plate (from which the elements are afterwards cut) is shown in Fig. 5.

S is an end-on vacuum tube containing mercury vapor

which is illuminated by the spark of an induction coil. The green radiation is so much brighter than the others that it is not necessary to use a spectroscope. The light, after being brought to an approximately parallel beam by the collimator *C*, is reflected from the lightly silvered plate *P* to the plate *Q* to be tested. From both surfaces of *Q* it is returned normally, and part passing through *P*, is examined by a low-power telescope *T*.

When the two surfaces of *Q* are nearly parallel, circular interference fringes begin to appear, which become more clearly defined as the accuracy of parallelism increases. At this stage the plate *Q* is moved so that all parts of the surface are examined in turn, and any residual errors of parallelism are at once detected by a contraction or expansion of the circles. The surface is locally corrected till these disappear. Under ordinary circumstances it is quite easy to distinguish a difference in phase of one-twentieth, which would correspond to a difference of retardation of the transmitted light of one-eightieth of a wave, so that there would be no serious difficulty in constructing an echelon of eighty or even a hundred elements.

In order to test the practical efficiency of the instrument, a somewhat extended investigation of the Zeeman effect was undertaken.

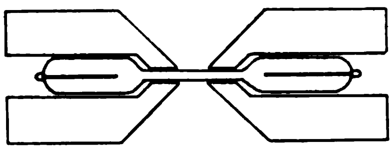

FIG. 6.

As has been already indicated in a previous paper,[1] in order to observe the complete analysis of the radiations in a magnetic field, it is necessary that these should be as nearly homogeneous as possible, and this is the case only when the radiations take place under low pressure, so that the substance must be placed in a vacuum tube from which the air is exhausted to 5–10 ^{mm} residual pressure, and illuminated by the electric dis-

[1] *Phil. Mag.*, September 1892.

charge. If the discharge takes place in a direction perpendicular to the field there is a lateral displacement of the discharge current, which interferes seriously with the result when the field is strong. In order to avoid this difficulty the pole pieces were arranged as in Fig. 6.

In the case of non-volatile substances the self-induction spark, between terminals of the substance to be examined, was used. The arrangement is shown in Fig. 7.

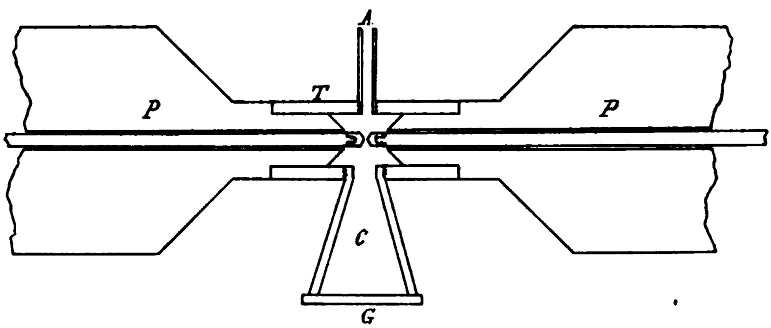

FIG. 7.

The pole-pieces are bored axially to receive the rods *r r*, to the ends of which the terminals are screwed.[1] The brass tube *T* is fitted air-tight by melted beeswax to the pole-pieces, and communicates by the tube *A* with a mercury pump, and a glass plate *G* closes the conical tube *C*. One of the rods *r* is fitted with a screw so that the corresponding terminal may be adjusted midway between the pole-pieces. The other rod is given a rapid oscillatory motion by a small electric motor, the joint being closed by a rubber tube.[2]

Under these circumstances a spark of considerable intensity is produced at every break of contact between the terminals with a storage battery of ten cells. The beeswax joints permit

[1] By making the rods and one of the terminals of iron, the field may be increased several times.

[2] This arrangement was only partially successful. The spark in the magnetic field was frequently too faint to observe, though occasionally the reverse was true.

an exhaustion to within a few millimeters of mercury, which may be maintained as long as necessary.

The observations completely confirm the experiments made by the method of visibility curves.[1] In particular the distribution for the cadmium lines is as represented in Fig. 8.

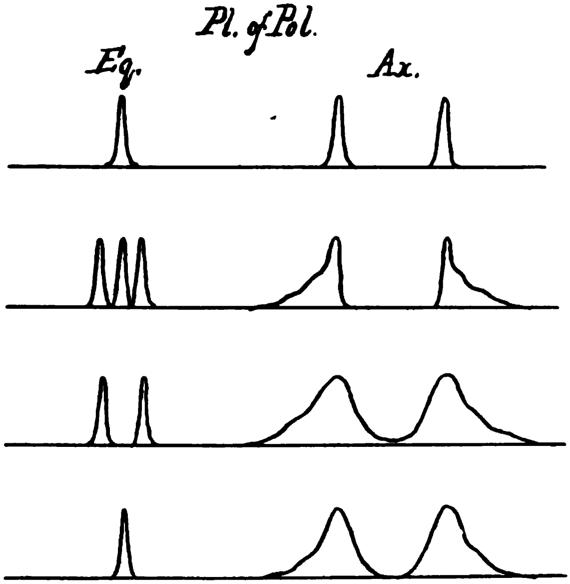

Fig. 8.

In addition to the lines previously classified[2] the following are added:

Gold, yellow line, class II.
Gold, green line, class I.
Silver, yellow line, class I.
Silver, green line, class I.
Copper, yellow line, class IV.
Copper, green lines, class I.

[1] This Journal, February 1898. The "outer lines" of the Zeeman triplet show indications of complex structure, but were actually resolved in the case of the green mercury line only; the difficulty arising from the overlapping of adjacent spectra when the field was sufficiently powerful to separate the elements of the group.

[2] *Ibid.*

Magnesium, green line (5183), class III.
Magnesium, green line (5172), class II.
Magnesium, green line (5167), class I.
Manganese, green (5340), class IV.
Argon, red line, class I.
Tin, red line (6450), class II.
Tin, yellow line (5798), class I.
Tin, yellow line (5587), class I.
Tin, yellow line (5564), class I.
Iron, yellow lines, class I.
Iron, blue lines, class I.

The central line of the iron triplets are all broadened, especially the blue lines.

A very remarkable effect is observed in the case of the yellow copper line. The line without the field is a close double, the distance being one-one hundred and fiftieth of the distance between the D lines, or 0.04 A. U. As the field increases the lines merge together without broadening, and with a strong field there is but a single very narrow line.

The behavior of the yellow-green line of manganese is even more striking. The line is a quadruple line, just resolvable. In a weak magnetic field the light accumulates at the center of the group, the lines becoming indistinct and merging together. In a strong field the quadruple band is reduced to a single fine line at the center of the group.

These two cases are the only ones of this character thus far observed, though doubtless a systematic search will reveal many others. They may be considered as a new type of line to be added to the three classes previously described.

Another interesting case is that of sodium. Without the field each component is a close double,[1] the distribution being that represented in Fig. 9*a*.

With a strong magnetic field and plane of polarization equatorial, the distribution is represented in Fig. 9*b*.

In *a* the elements of D_2 are about two-thirds as far apart as D_1, while in *b* the reverse is the case.

[1] This had been pointed out before. See *Phil. Mag.*, September 1892.

Finally, the distance between the components in *a* is a function of the density of the sodium vapor, increasing from zero for low densities to about one-sixtieth of D_1–D_2 for the greatest

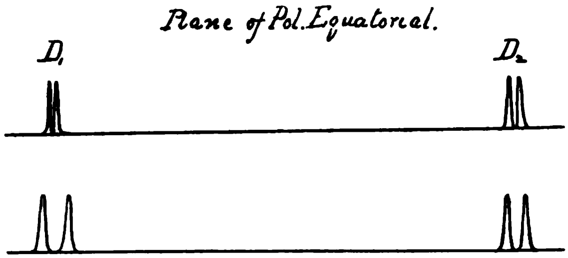

FIG. 9.

density obtained without broadening, and finally obliterating the lines.

For the investigation of very close lines the present instrument has not quite sufficient resolving power. For instance, it has been shown[1] that the green mercury line is a complex system whose principal components are less than 0.01 A. U. apart, while the theoretical limit of resolution of the echelon used is about twice as great.

A new echelon is now under construction which will have a resolving power about five times as great as the present instrument, and this will amply suffice for the analysis of the closes groups that are likely to occur.

[1] *Phil. Mag.*, September 1892.

NOTES ON THE ZEEMAN EFFECT.

By J. S. Ames, R. F. Earhart and H. M. Reese.

In verifying the Zeeman Effect, so-called, or the modification of radiations in the ether produced by the magnetic field, we have observed certain variations from the phenomena as originally described which seem worthy of note. Up to the present time we have studied the spark-spectra of iron, cadmium, zinc, and magnesium. The investigation is by no means completed; but it may be worth while to give a brief account of our work, and to defer until later a fuller discussion.

The apparatus used was the small concave grating spectroscope of the Physical Laboratory of the Johns Hopkins University, which has a radius of curvature of about twelve feet. The grating is ruled with 15,000 lines to the inch, and is five inches in width. The magnetic field was produced by a common type of electro-magnet, but we made no attempt to measure the intensity of the field, because our object was not to establish numerical relations. The field, however, was strong enough to produce a separation in the case of the "magnetic triplet" of over one-tenth of an Ångström unit. Our method of studying the effect was to place the spark gap between the poles of the magnet, introduce between the spark and the slit a Nicol's prism and a quartz lens, to photograph the resulting spectra along the middle of the photographic plate, having the Nicol's prism placed with its principal plane perpendicular to the field of force, then turning the Nicol's prism through 90° and at the same time rotating a horizontal shutter which is placed in front of the photographic plate, to thus expose the two edges of the plate to the new radiation coming through the Nicol's prism. By this method we secure on the same plate the components of the vibrations polarized along the lines of force and those polarized at right angles to them.

In the case of iron, we have studied the spectrum from wave-

length 3500 to wave-length 4400, and have made a careful investigation of all the lines in this region. With certain exceptions presently to be noted, all the lines are influenced in the way discovered by Zeeman. In particular, when the radiation at right angles to the magnetic field is studied, each line in the spectrum is in general broken up into three, the central component being plane polarized with its vibrations along the lines of force, the two side components being plane polarized at right angles to these, their vibrations being at right angles to the field of force.

We have observed, however, that three lines, of wave-length 3587.13, 3733.47, and 3865.67, are affected in the opposite way; that is, the line is a triplet when viewed at right angles to the magnetic field, but the central component is so polarized that its vibrations are at right angles to the field, and the two side components have their vibrations along the field. Two lines, one at 3722.72, the other at 3872.64, are so modified as to be quadruplets, the central component, which has its vibrations along the lines of force, being a close double. Several lines, among which are those at 3746.06, 3767.34, 3850.12, and 3888.67, have, so far as our indications go, no modification produced whatever. There are several lines concerning which we have doubt, but the majority of the other lines are certainly modified in the way described by Zeeman. (Several of the above phenomena have been observed by other investigators, notably Deslandres and Cornu.)

We have observed, also, that the separation of the side components of the triplets seemed to be irregular. In studying this, however, we noticed that there were certain lines in which the separation was nearly the same, but much greater than that of other lines whose separations seemed to be quite closely alike; that is, the lines of the iron spectrum, as studied, seemed to break up into two classes in each of which the "magnetic separation" is the same, but in the one set much greater than in the other. On tabulating these lines belonging to the two sets it was at once observed that there was a striking agreement with

the sets into which the iron spectrum breaks up when studied with reference to the shift produced by pressure.

In the article by Dr. W. J. Humphreys in this JOURNAL, October 1897, there is given a list of eighteen lines in the spectrum of iron which have a moderately small shift, whereas seven have an abnormally large shift. We found, with no exception, that those lines which had a great magnetic shift were those which had a great pressure shift. (Our attention was called in this investigation to a slight error in Dr. Humphrey's paper which he was able to correct at once from his laboratory note-book. The line at 4236.112 should be erased from those lines which have a small shift, and among the lines which have a large shift the one at wave length 4271.934 should read 4271.325.)

This agreement between the separation or change produced by a magnetic field and by pressure direct is most interesting and will undoubtedly lead to a better understanding of both phenomena.

In studying the spectrum of cadmium we noticed that all lines belonging to the second subsidiary series which we could investigate had magnetic separations nearly the same but much greater than the magnetic separation produced in those lines of the first subordinate series which we could study. This action is perfectly in accord with the phenomenon noted above in the case of iron.

Our investigation of the spectra of zinc and magnesium is not yet complete enough to warrant us in making any statement.

JOHNS HOPKINS UNIVERSITY,
May 1898.

THE STRUCTURE OF THE SHADING OF THE H AND K AND SOME OTHER LINES IN THE SPECTRUM OF THE SUN AND ARC.

By L. E. JEWELL.

SEVERAL years ago, while examining a series of photographs of the solar spectrum, made by Professor Rowland in 1888 and 1889, I discovered one plate on which the shading of the H and K lines (due to calcium vapor) was broken up into bands or series. In each case the bands began at the center of the shaded lines and extended outward, the distance apart of the component lines of the series increasing as the distance from the center increased. In each case the series were perfectly symmetrical about the centers of H and K, and the individual lines or components of the series somewhat nebulous, while nearly all other small lines in the same region were sharp, clear lines. All other plates available were carefully examined without the presence of series being indicated. Possibly a very few of the stronger lines in the series may be present upon most plates, but these lines may be merely approximately coincident, and not connected with the series. A great many plates of this region of the spectrum have been taken by myself during the last few years, over the center of the Sun, at the Sun's limb, over Sun-spots, and under various conditions; but the presence of these series has never been certainly indicated. It is probable that the principal reason for this is that all the negatives taken during several years past have been upon commercial plates, which are coarse grained, thus masking the faint lines of the series, while the plate upon which they are shown is almost grainless.

There are several plates taken in the third order spectrum which show the shading of some of the strongest of the iron lines broken up into series; but the component lines are faint, nebulous, and close together. There is also suspicion of the partial resolution of the shading of the strongest lines of some

other elements, but it is less certain. All plates taken of the arc-spectrum of the calcium lines H, K, and *g* ($\lambda = 3933.809$, 3968.620, and 4226.892), have been carefully examined without any result until the 11th of last March, when a plate taken under somewhat special conditions showed the shading broken up into series.

I had for a number of years suspected that the shading of lines was produced by the overlapping of many series all converging towards the principal line, but that under ordinary circumstances it is impossible to separate any series from the others (a possible exception occurs in the shaded lines of tin, which are plainly resolved into series).

It was thought probable that the distance apart of the lines in any particular series depended upon the degree of crowding of the molecules or density of the gas producing the shaded lines, and consequently the damping of the original vibration.

In both the electric arc and the Sun's atmosphere we probably have layers of metallic vapors of varying degrees of density, so that the combination will generally produce a smooth shading, gradually increasing in intensity towards the central line.

In the case of the solar spectrum plate under consideration, it was thought that the slit of the spectroscope probably covered a region of the Sun's atmosphere where the principal layer of calcium gas was of a particular density; and, being thus to a large degree isolated, it was able to produce its characteristic series. This idea is somewhat confirmed by the fact that the general shading of H and K on the plate is unusually weak. These particular conditions would probably be most likely to occur over faculæ.

On March 11, having occasion to photograph the arc-spectrum of calcium at λ 4000, I took several plates under widely different conditions. One plate shows the shading of H on the red side quite distinctly broken up into series similar to those of the solar spectrum plate mentioned. The resolution into series on the violet side is less distinct, while the shading of K shows the resolution better upon the violet than upon the

red side, though not so distinctly as upon the red side of H. The resolution of the shading of *g* into series is uncertain. If the lines of the series are present they are so nebulous and indistinct as to not be easily recognized. The resolution into lines is hardly perceptible close to the principal line, but is fairly distinct about three Ångström units from H. In the solar spectrum plate the lines of the series are much more distinct nearer the central line (about 1 or 2 Ångström units away) and become too faint to follow at any great distance. They are much more distinct each side of K than around H.

In the arc-spectrum plate it is also to be noted that the lines of the series are reversed or absorption lines instead of emission lines, although at some distance away from the central lines it is probable that the series are continued as faint emission lines.

The arc-spectrum plate showing these series was obtained by using an extremely powerful direct electric current, allowing it to act for a short time before throwing the image of the arc upon the slit of the spectroscope, and then exposing for three or four seconds only. Under these conditions the calcium was rapidly volatilized and the highly heated vapor formed a much more extended atmosphere around the poles than with a weaker current, and it is also possible that the conditions throughout the larger part of the arc were more uniform than under ordinary circumstances.

JOHNS HOPKINS UNIVERSITY,
May 25, 1898.

Minor Contributions and Notes.

". . . . TEACH ME HOW TO NAME THE . . . . LIGHT."

It would be a convenience if a name were chosen for the as yet undiscovered gas, which is suggested by the typical bright nebular lines, as a principal constituent of the nebulæ. Sir William Huggins has used occasionally the term *nebulum*. Independently, Miss Agnes Clerke has made the suggestion to me of *nebulium* as an appropriate term, which, "though not unobjectionable from an etymological point of view, is on all fours with *coronium*." If, however, the Greek nomenclature adopted for *helium* and *argon* is to be followed, the term *nephelium* or *nephium* may be suggested as suitable;—for, probably, *asterium* would be thought too general in its meaning. It is most desirable that the name chosen should be one universally acceptable to astrophysicists, and so exclusively adopted. Hence, this note.

Margaret L. Huggins.

ASTRONOMICAL AND PHYSICAL CONFERENCE AT THE HARVARD COLLEGE OBSERVATORY.

As the conference held at the Yerkes Observatory in October 1897 was successful in bringing together a large number of astronomers and physicists, whose contributions in the form of papers and discussions rendered the proceedings of great interest and value, it was felt that an attempt should be made to provide for a repetition of the conference in the summer of 1898, with a view to its continuance in future years. A preliminary inquiry, directed to those who attended the previous conference, and to certain others who were considered likely to accept, brought a large number of satisfactory replies. The writers were practically unanimous in favoring a continuation of the conferences, and definite acceptances were so numerous as to insure the success of a second conference at the Yerkes Observatory. It was found, however, that the geographical position of the Observatory would prevent many persons in the eastern states from coming. No practical way of continuing the meetings and at the same time avoiding this

difficulty presented itself until Professor Pickering offered to hold the 1898 conference at the Harvard College Observatory, beginning on Thursday, August 18, and continuing until the following Saturday. The advantages of this plan are so obvious that a large attendance may confidently be expected. On Monday, August 21, the Fiftieth Anniversary Meeting of the American Association for the Advancement of Science opens in Boston, and members of the conference can thus be present on this important occasion. By joining the American Association those who are not already members can take advantage of the low rates offered by the railroads. It is hoped that the attendance of the conference may greatly exceed that of last October, and that the meetings may be so successful as to warrant their repetition in future years.

G. E. H.

VARIABLE STARS OF SHORT PERIOD.[1]

WHOEVER will make a careful examination of the brightness of a large number of stars either in the sky, or better, as photographed upon different plates, will be impressed with the vast number which show no perceptible variation. The discovery of variable stars is greatly aided when we are able to make a suitable selection for examination, either from their spectra or from their presence in clusters. Visually, we can never be sure that all the variables in a given region have been found, however carefully we may study them. Photography brings this problem more nearly within our reach, and a partial solution of it is illustrated in the accompanying figure. A photographic telescope was constructed having as an objective a Cooke Anastigmatic Lens, with an aperture of $2^{cm}.6$, and a focal length of $33^{cm}.3$. This telescope was mounted equatorially and the lens was alternately exposed and covered for intervals of exactly 10 and 50 minutes by an electrical attachment. The polar axis of the mounting was displaced and the rate of the driving clock was increased, so that the successive images should be slightly separated. An 8×10 photographic plate was exposed in this instrument on April 21, 1898, and eight successive images were obtained, the Greenwich Mean Times of the middle of the exposures being $13^h\ 49^m$, $14^h\ 49^m$, $15^h\ 49^m$, $16^h\ 49^m$, $17^h\ 49^m$, $18^h\ 48^m$, $19^h\ 48^m$, and $20^h\ 48^m$. The plate covered a region

[1] *Harvard College Observatory Circular* No. 29.

about 33° square, whose center was R. A.$=1^h\ 2^m$, Dec. $=+76°.6$. The images of the stars in the corners of the plate were sufficiently good when visible to show very slight variations in light, but owing to their increased size the faintest stars were not shown. The greatest

FIG. 1.

loss amounted to about one magnitude. If now any variable star having a period of less than fourteen hours was contained in this region it is probable that at least one maximum and one minimum would be photographed. The figure represents a portion of the plate described above, enlarged ten times to a scale of $60'=0^{cm}.1$, and covers about one square degree. It therefore represents one-thousandth of the entire plate, the size of which on this scale would be two meters, or nearly seven feet square. The entire sky, from the north to the south pole, could be covered by forty such plates, and it is proposed to do this as soon as the best method of taking the plates has been determined. The arrow indicates the variable star U Cephei, and its photometric magnitudes at the times the eight images were taken were 7.5, 8.1, 8.9, 9.1, 9.1, 8.3, 7.6, and 7.2. The three stars above it are $+81°30$, $+81°27$, and $+81°29$, which have the photometric magnitudes 7.9, 8.5, and 8.6. To separate the successive images various methods have been tried. The best of these seems to be stopping the

driving clock for a few seconds every hour. By the above plan we hope to secure a complete list of all variable stars of short period brighter than the ninth magnitude at maximum whose variation exceeds half a magnitude, and whose period is less than a day. Doubtless many other variable stars of longer period, and stars of the Algol type may also be incidentally found.

EDWARD C. PICKERING.

MAY 21, 1898.

THE SUPPOSED VARIABLE STAR, Y AQUILAE.[1]

MEASURES to determine the light curves of variable stars of short period, north of declination $-40°$, are now in progress with the meridian photometer. Four sets of four settings each are ordinarily made when the star to be observed is about half an hour east of the meridian, and again about an hour later. These measures are repeated on twenty or thirty nights. The principal error is that due to the unequal transparency of the air in different portions of the sky, the stars compared being often far apart. The accidental errors of measurement are small, owing to the number of settings. Smooth light curves have been found for all the stars thus measured, with the exception of $+10°3787$. The designation Y Aquilae was given to this star by Mr. S. C. Chandler, and in his catalogue of variable stars he states that it varies from magnitude 5.3 to 5.7 in a period of 4.986 days. Also, that it was "Suspected by Gould, confirmed by Chandler, 1894; also by Yendell." It will be noticed that the period is so nearly five days that for several months the same phase will recur at about the same hour angle, thus permitting errors to occur in visual observations by Argelander's method, such as are mentioned in *Circular* No. 23, and which led to such wholly erroneous conclusions in the case of U Pegasi.

The star $+10°$ 3787 was observed with the meridian photometer on nineteen nights, from August 25 to October 13, 1897. Placing together the observations having the same phase, we find, corresponding to the phases $0^d.0$, $1^d.0$, $2^d.0$, $3^d.0$, and $4^d.0$, the mean residuals -0.09, 0.00, $+0.02$, $+0.02$, and $+0.03$. We might infer a variation with a range of a tenth of a magnitude, but the first value, -0.09, depends on observations on a single night, and the range of the other four is only 0.03. This star is No. 39, of the standards selected by

[1] *Harvard College Observatory Circular* No. 30.

the observers at Potsdam, and they state that from their observations they find no confirmation of the variation suspected by Dr. Gould. Mr. Chandler, however (*Astron. Jour.*, 14, 135), states that these observations, fifty-seven in number, confirm the period of variation that he has found, although indicating that the range of variation is less. A reduction of these observations made here, however, leads to the same conclusion as that found at Potsdam. The actual number of observations of this star made at Potsdam appears to be seventy five, not fifty-seven, forty-nine of them in determining the light of the standards, and twenty-six in the supplementary zones. Grouping them according to phase, we obtain the residuals —0.07, —0.02, +0.01, 0.00, and —0.16, corresponding to the phases $0^d.6$, $1^d.6$, $2^d.6$, $3^d.6$, and $4^d 6$. The last residual is reduced to —0.12 if we reject the observations on one night, and to —0.09, if we reject those made on two nights, and use only the observations from which the light of the fundamental stars was determined. To decide whether such residuals as these are due to accident, the Potsdam observations were again grouped, assuming a period of six days instead of five. The mean residuals corresponding to the phases 0, 1, 2, 3, 4, and 5 days then become —0.03, —0.19, —0.14, —0.02, +0.01, and 0.00. The preponderance of negative residuals is caused by the supplementary observations, which indicate that the star was slightly brighter than when the standards were originally measured. Grouping the observations with the meridian photometer in the same way, we obtain the mean residuals 0.00, 0.00, —0.07, —0.02, —0.02, and +0.07. In both cases, therefore, a period taken at random indicates variability more clearly than that hitherto assumed. Mr. Yendell (*Astron. Jour.*, 14, 160), confirms visually the variability of this star.

The accuracy of the observations described in *Circulars* Nos. 23 and 25, seems to afford a conclusive test of the variability of this star. Accordingly, comparisons were made by Mr. Yendell with the photometer attached to the 15-inch equatorial telescope on May 9, 10, 13, 14, 21, and 22, 1898. Eighty settings were made each night. The comparison star was $+10^\circ$ 3784. The mean differences of magnitude were 3.63, 3.64, 3.66, 3.66, 3.66, and 3.62, and the average deviations from their means, of the five groups of sixteen settings each on the different nights, were ±0.022, ±0.028, ±0.020, ±0.012, ±0.030, and ±0.012. The first and fourth nights have nearly the same phase, $0^d.8$. Combining these in one group, and placing the others in the order of

phase, we have the phases, $0^d.8$, $1^d.8$, $2^d.8$, $3^d.9$, and $4^d.8$, and the corresponding residuals, 0.00, 0.00, —0.02, +0.02, and —0.02. A positive residual, as usual, denotes that the star was faint. The average value of these residuals is ± 0.012, and the range 0.04.

The three series of photometric observations discussed above, therefore, fail to show any evidence of variation, since deviations of a tenth of a magnitude, except in the last series of measures, may be ascribed to errors of observation. Since it is impossible to prove that the light of a star never changes, this star may still be an Algol variable with a short time of variation, or the period may be entirely wrong.

EDWARD C. PICKERING.

MAY 25, 1898.

ELECTION OF EDWIN BRANT FROST AS PROFESSOR OF ASTROPHYSICS AT THE YERKES OBSERVATORY.

THE staff of the Yerkes Observatory will soon be materially strengthened by the addition of Edwin Brant Frost, Director of the Shattuck Observatory of Dartmouth College, who has been elected Professor of Astrophysics in the University of Chicago. A gift of $15,000 to the University has been recently made by Miss Catherine W. Bruce for the special purpose of providing for this appointment. Professor Frost expects to devote special attention to a study of stellar spectra with the forty-inch telescope. He will also continue to assist in the editorial work of the ASTROPHYSICAL JOURNAL.

G. E. H.

REVIEWS.

Sur une nouvelle Mèthode de Spectroscopie Interferentielle. A. PEROT et CH. FABRY, *Comptes Rendus*, t. CXXVI, p. 34.

Étude de quelques Radiations par la Spectroscopie Interferentielle. A. PEROT et CH. FABRY, *Comptes Rendus*, t. CXXVI, p. 407.

THE interferential spectroscope described in these two papers consists of two plane parallel glasses enclosing a film of air between them. Suitable mountings allow the distance between the glasses, and consequently the thickness of the air film to be varied from nearly zero to any desired amount.

When illuminated by a beam of monochromatic light the film of air produces the usual interference phenomena of thin plates. Such plates give rise to two complementary sets of interference fringes, the reflected and the transmitted, the latter being the one utilized by Perot and Fabry.

Airy's well-known formula for the transmitted system is

$$I = \frac{a^2 (1-e^2)^2}{(1-e^2)^2 + 4\, e^2 \sin^2 \left(\frac{\pi}{\lambda} V\right)},$$

where I is the illumination, a^2 the intensity of the incident light, and e the reflection coefficient of the surfaces bounding the film. The intensity of a bright fringe is then a^2, while that of a dark one is

$$\frac{a^2 (1-e^2)^2}{(1+e^2)^2},$$

and their ratio is consequently

$$\frac{(1+e^2)^2}{(1-e^2)^2}.$$

In case the film is air and the bounding surfaces glass, at normal incidence $e = 0.2$, and we obtain as the ratio of the intensity of a bright fringe to that of a dark band

$$\left(\frac{1+.04}{1-.04}\right)^2 = 1.18.$$

The contrast would then be small and the fringes would be lost in the general illumination. If, however, the glasses bounding the air film are lightly silvered on their inner surfaces, e may become as great as 0.87 and we get for the ratio 49. In this case the fringes are very prominent, the dark ones being nearly black. It is under these conditions that the fringes have been employed in the interferential spectroscope.

The incident beam is made slightly convergent and the transmitted system of fringes is viewed through a telescope focused for nearly parallel rays. The surfaces of the glasses being parallel to each other, with a convergent beam the fringes are concentric circles, the common center being the normal to the film passing through the axis of the observing telescope.

In Airy's formula the incident light is considered to be strictly monochromatic. It is easy to see, however, that if the incident beam contains light of various wave-lengths, each kind of light will give rise to an independent system of fringes, which may or may not coincide. For instance if the incident beam consists of light of two very nearly equal wave-lengths, there will be two systems of fringes, which will coincide when the distance between the glasses is small, and hence will appear to the observer as a single system. When, however, by a gradual separation of the glasses a considerable difference of path is attained, the coincidence will no longer exist and both sets of fringes will be visible.

In order to measure the relative wave-lengths of the two kinds of light, the difference of path is so adjusted that the bright fringes of one system coincide exactly with the dark ones of the other system. If p is the number of the fringe at which this takes place, λ and $\lambda + \Delta\lambda$ the wave-lengths of the incident light, and e the distance between the surfaces, we evidently have

$$(p + \tfrac{1}{2})\,\lambda = p\,(\lambda + \Delta\lambda) = 2\,e,$$

whence

$$\frac{\Delta\lambda}{\lambda} = \frac{1}{2p} = \frac{\lambda + \Delta\lambda}{4e}.$$

Therefore by observing the order of the fringe at which the bright fringes of one system coincide in position with the dark ones of the other system we obtain the value of $\frac{\Delta\lambda}{\lambda}$.

The authors show that in favorable cases, *i. e.*, those in which the component lines are very narrow, two lines whose distance apart is only $\frac{1}{1000}$ of that between the D_1 and the D_2 lines may be separated.

Perot and Fabry have applied their instrument to the study of certain radiations which have previously been analyzed by Michelson with the interferometer. Their results are in some cases quite different from those of Michelson; for instance, where Michelson finds the green thallium line ($\lambda = 5349$) to consist of four lines, Perot and Fabry find it to be a triplet, the distances between the components agreeing however with the values given by Michelson.

In the case of the blue cadmium line ($\lambda = 4800$) which Michelson decides to be a double, Perot and Fabry find it to be a triplet, consisting of a principal radiation with a companion on each side at distances of 0.08 Ångström units.

In order to explain these differences Perot and Fabry call attention to the fact that in the visibility curve of Michelson, which is given by $V^2 = \frac{C^2 + S^2}{P^2}$, C and S are indeterminate, and can thus be chosen in any manner whatever so as to satisfy the equation of the visibility curve. In order to determine them uniquely a second condition is necessary.

While this statement is perfectly true in the most general case, yet in those cases where the distribution is symmetrical the terms expressed by S vanish, $V = \frac{C}{P}$, and the criticism does not apply. It has been shown that in the radiations thus far examined, the distribution of light in the ultimate lines agrees with Maxwell's formula for the velocity distribution in a gas and is therefore symmetrical.

In order to gain an idea as to the effectiveness of the new instrument it seems most natural to compare its performance with that of the interferometer.

The interferential spectroscope has the advantage of showing directly the structure of a given radiation by a simple inspection of the system of fringes at considerable differences of path, and therefore does not depend upon a series of estimates of visibilities, as is the case with the interferometer. Each fringe is in fact a true spectrum of the source, and the conditions are precisely the same as those existing in the spectra obtained by the use of a grating possessing a small number of lines, but with which spectra of a very high order can be observed.

Furtbermore, in order to show that a given radiation is complex it is not necessary to attain such great differences of path as are necessary in the interferometer.

From the visibility curve obtained with the interferometer it is impossible to say whether a companion line is situated on the red or on the violet side of the principal line, while with the interferential spectroscope this is at once revealed by the way in which the doubling takes place.

The visibility curve, however, allows the determination of the breadth of each component line with a considerable degree of accuracy, while the interferential spectroscope as used by its inventors does not. This is quite an important advantage that the interferometer possesses, and it will no doubt be recalled that measurements of this sort have given us probably the most direct proof of the kinetic theory of gases.

Another advantage possessed by the interferometer is that the difference of path may be made zero, and yet the distance between the semi-silvered glass and the two mirrors is considerable, thus rendering the instrument very suitable for investigations concerning refractive indices, etc.

In conclusion it may be said that Messrs. Perot and Fabry deserve the thanks of physicists for placing in their hands an instrument of such power and efficiency, and if the problem of the mechanism of radiation is ever solved it seems that the new instrument will take an important part in the solution.

A. St. C. D.

NOTICE.

The scope of the ASTROPHYSICAL JOURNAL includes all investigations of radiant energy, whether conducted in the observatory or in the laboratory. The subjects to which special attention will be given are photographic and visual observations of the heavenly bodies (other than those pertaining to "astronomy of position"); spectroscopic, photometric, bolometric and radiometric work of all kinds; descriptions of instruments and apparatus used in such investigations; and theoretical papers bearing on any of these subjects.

In the department of *Minor Contributions and Notes* subjects may be discussed which belong to other closely related fields of investigation,

Articles written in any language will be accepted for publication, but unless a wish to the contrary is expressed by the author, they will be translated into English. Tables of wave-lengths will be printed with the short wave-lengths at the top, and maps of spectra with the red end on the right, unless the author requests that the reverse procedure be followed. If a request is sent *with the manuscript* one hundred reprint copies of each paper, bound in covers, will be furnished free of charge to the author. Additional copies may be obtained at cost price. No reprints can be sent unless a request for them is received before the JOURNAL goes to press.

The Editors do not hold themselves responsible for opinions expressed by contributors.

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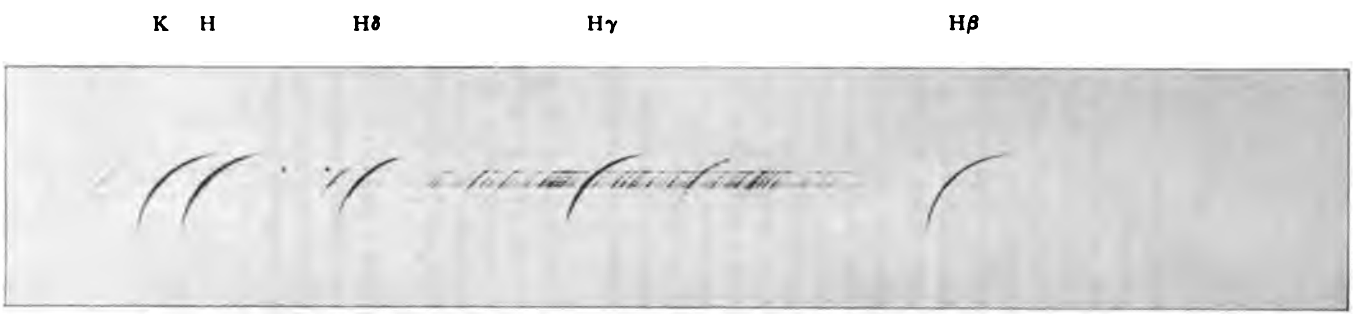
All papers for publication and correspondence relating to contributions and exchanges should be addressed to *George E. Hale, Yerkes Observatory, Williams Bay, Wisconsin, U. S. A.*

PLATE I.



SPECTRUM OF THE "FLASH" PHOTOGRAPHED BY PROFESSOR K. D. NAEGAMVALA
AT THE ECLIPSE OF JANUARY 21, 1898.

THE

ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY
AND ASTRONOMICAL PHYSICS

VOLUME VIII   AUGUST 1898   NUMBER 2

SOME OBSERVATIONS ON STELLAR MOTIONS IN THE LINE OF SIGHT MADE AT THE EMERSON McMILLIN OBSERVATORY.

By H. C. Lord.

In a paper read before the conference of astronomers held at the opening of the Yerkes Observatory, an abstract of which was published in this Journal, **6**, 5, I gave an account of the work undertaken at the Emerson McMillin Observatory upon stellar motions in the line of sight. The observations there published seemed to indicate that the instrument was in satisfactory adjustment and that the accuracy reached was about all that could be expected from the instrumental power at my command. I also called attention to the fact that there were some twenty to twenty-five stars of type II*a* given in the Draper Catalogue, whose velocities had, so far as I know, not been determined, and which were nevertheless bright enough to be observed with the instruments here. Upon my return from Williams Bay, systematic work was begun upon these stars. As this work will require some time for its completion it has seemed to me advisable, as the work upon the separate stars shall be completed, to publish the results in advance of a general discussion. These results are not definitive, but any subsequent changes will not materially alter the values given.

The instrument used has already been described. One important change should be noted; upon the frame of the spectroscope, as far removed from the prisms as possible and at points where the heat of the prisms would be lost through conduction to the heavy metal parts of the telescope, are placed coils of wire, connected through a switch to four cells of storage battery. These coils are of such resistance as to allow a current of from five to seven amperes to flow through them. The whole spectroscope is also inclosed in a tight fitting covering of two thicknesses of felt and one of black velvet. A thermometer is placed in the prism box, the bulb not touching the metal and the stem projecting through the felt and velvet cover. This thermometer is read from time to time during the exposure, and as soon as the mercury starts to fall the current is turned on the coils for from 60 to 120 seconds. After a few minutes the mercury will rise to its normal point. The temperature lag is very great, but a little practice enables the observer to allow for it, and the temperature of the thermometer can easily be kept within ½° F. while the temperature on the outside falls from five to ten degrees. Though this may not represent the temperature of the prisms exactly, yet since the thermometer is placed in the air surrounding them, I feel confident that they must be kept at a fairly constant temperature.

The hydrogen tube is placed on an arm which can be swung over and about eight inches in front of the slit, directly in the cone of rays.

The method of taking the photographs is as follows. The telescope is set on the star, the hydrogen tube placed in position and exposure on the tube of from ten to twenty seconds is given. The tube is then removed and the exposure on the star given, after which the tube is replaced and a second exposure of the same length as the first given on the artificial line.

The method of measuring the plates is substantially that given in the Potsdam Publications, Vol. 7, for the type II*a* stars, except that in every case the plates were measured in two positions under the microscope, once with the red end on the right and the solar standard below the star spectrum, and once

with the red end on the left and the solar spectrum above the star spectrum. The solar standard No. 454 was used in all these observations. This spectrum is about four times as wide as the star spectra but much narrower than the solar standard used in the earlier observations above quoted.[1]

On this plate 1^{mm} corresponds to a velocity 1443^{km} per second. The velocity is also computed for each line compared. The separate results together with the collected means are given below. Plate No. 438 has been rejected from the means. This plate was so badly underexposed that I rejected it twice before measuring but at last was induced to try it. I find in the record of the observations that at the end of the exposure the objective was badly dewed and hence have decided to reject this value at least for the present. On account of the large difference between Red End Right and Red End Left in Plate 488 it was remeasured. The small value -5.2 might have been due to recording the setting on the artificial $H\gamma$ for that on the star $H\gamma$ and *vice versa*. The mean of the two values is adopted.

As a further check on the work, the observations on ι Aurigae and ϵ Geminorum were reduced, regarding the solar parallax and the star's velocity with respect to the Sun as unknown. The values found are for ι Aurigae: $V_s + 21.4$, $\pi = 9''.2$ and for ϵ Geminorum: $V_s + 12.0$, $\pi = 10''.1$. This agreement of these values of V_s thus found with those previously obtained and that of π with the true value would seem entirely satisfactory.

It is as yet too soon to discuss the probable error of these observations, but from the thirty-six residuals (rejecting plate 438) I find that the probable error of a single observation is about 2^{km} per second, which corresponds to a displacement on the photographic plate of only $0^{mm}.0018$.

The faintest star yet photographed is ι Aurigae, with a photographic magnitude of 4.87. The Draper Catalogue gives

[1] In the abstract above quoted 1^{mm} displacement on the plate is said to correspond to a velocity of $143^{km}.7$ per second. This is an error. It should have been 1437^{km} per second.

OBSERVATIONS FOR MOTION IN THE LINE OF SIGHT.

Plate number	Star	Date	Exposure time	Observed velocity: Red end right	Observed velocity: Red end left	Observed velocity: Mean	Reduction to the Sun	Velocity reduced to the Sun
441	ι Aurigae	Nov. 20, 1897	75^m	+12.7	+ 5.2	+ 8.9	+ 7.9	+16.8
443	"	" 23, "	80	+21.2	+16.2	+18.7	+ 6.9	+25.6
461	"	Jan. 8, 1898	80	+40.0	+41.4	+40.7	−16.1	+24.6
465	"	" 23, "	75	+47.9	+45.8	+46.9	−21.9	+25.0
467	"	" 24, "	70	+44.3	+40.7	+42.5	−22.4	+20.1
468	"	" 27, "	75	+40.1	+38.4	+39.2	−23.5	+15.7
471	"	Feb. 1, "	75	+51.1	+45.8	+48.4	−25.0	+23.4
473	"	" 3, "	75	+43.3	+39.0	+41.2	−25.6	+15.6
475	"	" 6, "	75	+52.2	+47.2	+49.7	−26.3	+23.4
480	"	" 25, "	80	+48.9	+46.0	+47.4	−29.2	+18.2
							Mean	+20.8
438*	ε Geminorum	Nov. 17, 1897	80	+11.9	+11.6	+11.7	+19.9	[+30.7]
438*	"	" 17, "	80	+ 8.9	+10.9	+ 9.9		
442	"	" 20, "	75	− 4.2	− 9.5	− 6.9	+18.8	+11.9
444	"	" 23, "	90	− 2.3	− 9.7	− 5.9	+17.7	+11.8
462	"	Jan. 8, 1898	90	+16.7	+11.8	+15.2	− 5.2	+10.0
469	"	" 27, "	75	+32.2	+25.9	+29.0	−15.0	+14.0
472	"	Feb. 1, "	75	+31.7	+28.0	+29.9	−17.0	+12.9
474	"	" 3, "	75	+28.7	+25.6	+27.1	−17.9	+ 9.2
476	"	" 6, "	75	+37.0	+34.1	+35.5	−19.0	+16.5
485	"	Mar. 13, "	70	+45.9	+41.7	+43.8	−28.7	+15.1
489	"	" 14, "	75	+46.2	+44.0	+45.1	−28.8	+16.3
							Mean	+13.1
486	ε Leonis	Mar. 13, 1898	50	+29.2	+24.4	+26.8	−16.0	+10.8
490	"	" 14, "	45	+24.1	+23.4	+23.8	−16.4	+ 7.4
496	"	" 25, "	45	+28.1	+26.1	+27.1	−20.6	+ 6.5
500	"	" 31, "	45	+34.3	+25.8	+30.1	−22.7	+ 7.4
504	"	April 7, "	45	+34.7	+28.5	+31.6	−24.8	+ 6.8
							Mean	+ 7.8
487	ψ Urs. maj.	Mar. 13, 1898	70	+18.3	+ 9.9	+14.1	−10.3	+ 3.8
491	" "	" 14, "	70	+16.2	+ 9.0	+12.6	−10.7	+ 1.9
494	" "	" 24, "	70	+10.8	+10.2	+10.5	−14.2	− 3.7
497	" "	" 25, "	70	+ 8.6	+ 6.5	+ 7.6	−14.5	− 6.9
501	" "	" 31, "	70	+15.6	+11.5	+13.6	−16.4	− 2.8
505	" "	April 7, "	70	+25.9	+20.7	+23.3	−18.4	+ 4.9
							Mean	− 0.4
488§	ε Virginis	Mar. 13, 1898	75	− 5.2	−16.1	−10.7	+ 7.9	− 5.1
488§	"	" 13, "	75	−11.6	−18.9	−15.3		
492	"	" 14, "	75	−15.8	−15.8	−15.8	+ 7.4	− 8.4
495	"	" 24, "	75	−13.5	−14.3	−13.9	+ 2.5	−11.4
498	"	" 25, "	50	− 8.5	− 9.6	− 9.0	+ 2.2	− 6.8
502	"	" 31, "	75	− 9.4	−13.8	−11.6	− 0.8	−12.4
506	"	April 7, "	75	− 4.8	− 6.9	− 5.8	− 4.5	−10.3
							Mean	− 9.1

* Plate 438 was rejected on account of underexposure.

§ Plate 488 was measured twice.

Velocities are given in kilometers per second. The + sign denotes recession; the — sign approach.

twenty-four stars north of the Equator classed as *K*, brighter than 4.89, whose velocities have not been determined at Potsdam, and eleven stars of types *M* and *Q*. These stars will be investigated as rapidly as possible, at least five satisfactory photographs of each star being secured.

EMERSON MCMILLIN OBSERVATORY,
Ohio State University, Columbus, Ohio.

ON THE SERIES SPECTRA OF OXYGEN, SULPHUR, AND SELENIUM.

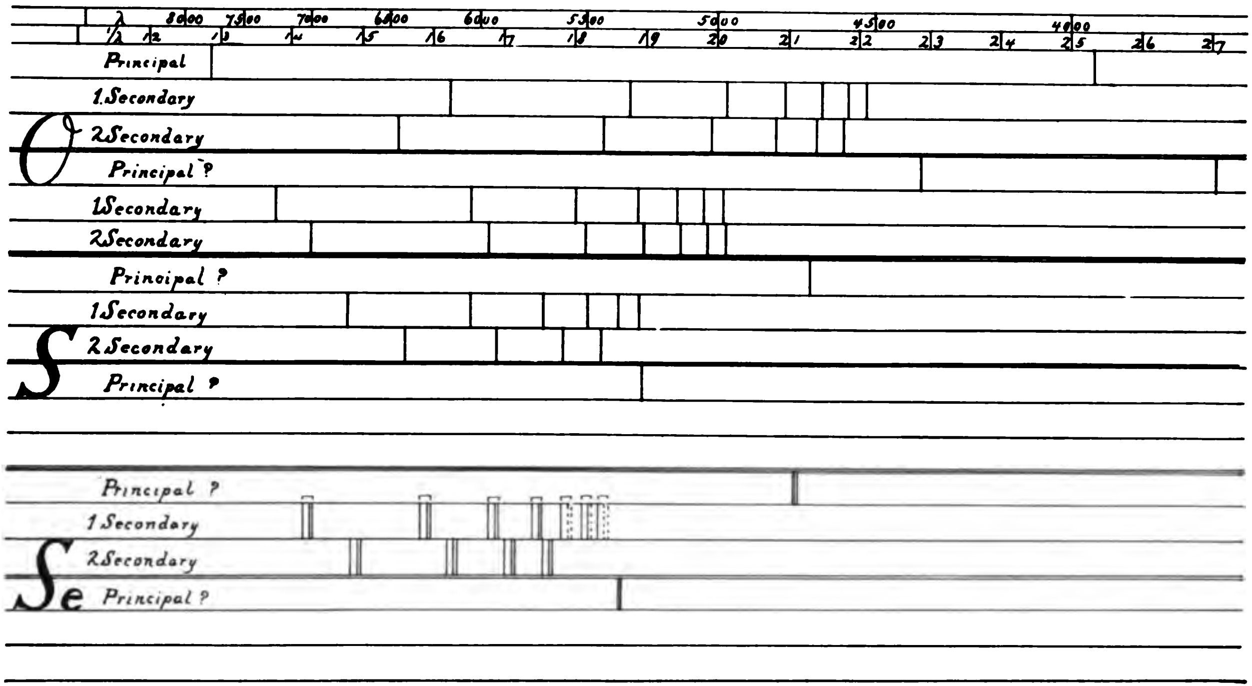
By C. RUNGE and F. PASCHEN.

I. OXYGEN.

WE may say that there are six different spectra of oxygen. First, there are two absorption spectra, as Janssen has shown. They differ in the variation of their intensity. One increases in intensity proportional to the density of the gas, the other proportional to the square of the density. Then there are four different emission spectra of oxygen in the vacuum tube, as Schuster has shown. A continuous spectrum, a band spectrum near the cathode, the spark spectrum, and the "compound line spectrum." We propose to alter the name of the compound line spectrum, and to call it series spectrum, because we have found that the lines are distributed in "series" similar to those that had been found in the spectra of many other elements. This is the spectrum with which the present investigation deals. At the same time we have found that sulphur and selenium, under similar conditions, also emit series spectra, which seem to be analogous to the series spectrum of oxygen. For producing the spectra we used the arrangement devised by Paalzow and H. W. Vogel.[1] Both sides of the vacuum tube are separately connected with the pump. Both connections pass through U tubes, one containing sulphuric acid and electrodes to generate oxygen, the other containing a solution of potassium bichromate in concentrated sulphuric acid to keep off the impurities of the grease of the stopcocks. The same arrangement was used for the investigation of sulphur and selenium, with the sole addition of a wider part at one end of the tube, which contained H_2SO_4 or H_2SeO_4 (see Fig. 1). The electrodes were made of platinum foil.

[1] PAALZOW and H. W. VOGEL, *Wied. Ann.*, 13, 336, 1887.

PLATE II.



SERIES SPECTRA OF OXYGEN, SULPHUR AND SELENIUM.

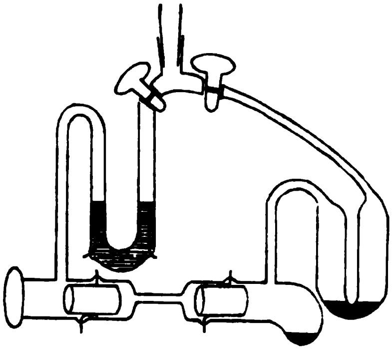


FIG. 1.

OXYGEN.

Wave-length	Mean error	Number of determinations	Intensity	Remarks
2883.95	0.01	4	6	
3692.586	0.003	3	7	
3823.714	0.018	2	7	
3825.22	0.04	3	6	
3830.41	0.07	3	2	
3947.480			10	The differences between the wave-lengths of this triplet are more accurate than the absolute wave-lengths.
3947.661	0.010	3	7	
3947.759			4	
3954.775	0.020	3	8	
4217.25	0.04	4	4	
4222.94	0.01	8	5	
4233.48	0.03	9	7	
4368.466	0.010	3	10	
4522.95				Double.
4523.70				
4576.97	0.11	2	2	Double.
4577.84	0.06	2	3	
4589.16	0.06	2	2	Double.
4590.07	0.06	2	3	
4654.41		1	2	
4654.74		1	3	
4655.54	0.03	4	4	
4672.93	0.06	3	3	Double.
4673.88	0.04	4	3	
4772.72	0.02	2	3	
4773.07	0.05	2	4	
4773.94	0.01	6	5	
4801.98	0.14	2	2	
4802.38	0.10	2	3	
4803.18	0.03	8	4	
4967.58	0.04	3	4	

OXYGEN—*continued.*

Wave-length	Mean error	Number of determinations	Intensity	Remarks
4968.04	0.04	3	5	
4968.94	0.01	3	6	
4973.05		1	1	Diffuse.
4979.73		1	1	Diffuse.
5018.96	0.04	2	3	
5019.52	0.07	2	4	
5020.31	0.01	13	5	
5037.34	0.5	3	2	Diffuse.
5047.88	0.1	4	2	Diffuse.
5130.70	0.07	9	3	
5146.23	0.03	3	5	
5275.25	0.04	7	4	
5299.17	0.03	3	5	
5329.162			6	
5329.774	0.01	3	7	
5330.835			10	
5405.08	0.10	6	3	
5408.80	0.04	7	4	
5410.97	0.03	7	3	
5435.371			5	
5435.968	0.015	3	6	
5437.041			8	
5512.92	0.02	3	5	Double, weak component on the more refrangible side.
5555.16	0.04	3	6	Double, weak component on the more refrangible side.
5750.82	0.04	2	5	
5958.75	0.02	6	6	Double, weak component on the more refrangible side.
5992.67		1	3	
5995.70	0.15	3	4	
6046.336	0.03	6	2	
6046.564			7	There is a weak line at 6107 (intensity 2) that we did not measure.
6156.198			7	
6156.993	0.018	7	8	
6158.415			10	
6256.81	0.05	1	1	
6261.68	0.03	1	3	
6264.78	0.09	1	1	
6267.06	0.04	1	1	
6453.900			6	
6454.756	0.015	5	7	
6456.287			9	
7002.48	0.13	4	4	There are weak lines at 7157, intensity 1, and 7474, intensity 1, that we did not measure.
7254.32	0.13	1	2	
7772.26	0.07	5	10	
7774.30	0.15	3	8	The mean error 0.15 refers to the mean of the two wave-lengths. Their distance is less accurate, as the lines were not easily seen separated.
7775.97			6	

The triplet in the extreme red part near 7770 is probably the strongest of the whole spectrum, considering how little sensitive the eye is for these rays. A singular interest is attached to it, as the three components coincide with three solar lines of equal relative intensities.[1] Jewell has observed them with high and low Sun. After first suspecting them to be water-vapor lines of terrestrial origin, he later on came to the conclusion that they are probably true solar lines.[2]

There are thirteen triplets of similar appearance, whose wave-numbers show the same differences. They decrease in intensity with decreasing wave-length, and form two "secondary series," as will be shown later on. The wave-numbers given in the following table are the reciprocals of the wave-lengths reduced to vacuum, and the decimal is placed so that we have the number of waves per centimeter in vacuo.

TRIPLETS.

Wave-length	Wave-number	Difference	Wave-length	Wave-number	Difference
6456.287	15484.57		4803.18	20813.84	
6454.756	15488.25	368	4802.38	20817.30	346
6453.900	15490.30	205	4801.98	20819.04	174
6158.415	16233.52		4773.94	20941.31	
6156.993	16237.28	376	4773.07	20945.12	381
6156.198	16239.38	210	4772.72	20946.66	154
5437.041	18387.34		4673.88	21389.62	
5435.968	18390.97	363	4672.93d	21393.97	435
5435.371	18392.99	202			
			4655.54	21473.88	
5330.835	18753.65		4654.74	21477.57	369
5329.774	18757.39	374	4654.41	21479.09	152
5329.162	18759.54	215			
			4590.07	21780.17	
5020.31	19913.63		4589.16d	21784.49	432
5019.52	19916.78	315			
5018.96	19918.99	221	4577.84	21838.35	
			4576.97d	21842.50	415
4968.94	20119.50				
4968.04	20123.14	364	4523.70	22099.71	
4967.58	20125.01	187	4522.95d	22103.37	366

[1] RUNGE and PASCHEN, "Oxygen in the Sun," this JOURNAL, 4, December 1896.
[2] JEWELL, this JOURNAL, 5, February 1897 and 6, November 1897.

In the four weakest triplets the two weaker components could not be measured separately. In the four strongest triplets the distances of the components could be determined with our large concave grating of $6^m.5$ radius. They are therefore far more accurate than the differences in the other triplets. They give as the mean of the differences

3.70 and 2.08.

In the following table we give the wave-numbers of each triplet corrected to these values. The components are altered according to their weights in such a manner that the center of gravity remains unaltered. The weights of the three components were assumed equal to 3, 2, 1, which very nearly corresponds to the accuracy of their determination.

WAVE-NUMBERS.

Wave-number	Corrected wave-number	Difference	Corresponding deviation of wave-length
15484.57	15484.56	−0.01	0.004
15488.25	15488.26	+0.01	−0.004
15490.30	15490.34	+0.04	−0.017
16233.52	16233.55	+0.03	−0.011
16237.28	16237.25	−0.03	+0.011
16239.38	16239.33	−0.05	+0.019
18387.34	18387.30	−0.04	+0.012
18390.97	18391.00	+0.03	−0.009
18392.99	18393.08	+0.09	−0.027
18753.65	18753.68	+0.03	−0.009
18757.39	18757.38	−0.01	+0.003
18759.54	18759.46	−0.08	+0.023
19913.63	19913.38	−0.25	+0.063
19916.78	19917.08	+0.30	−0.076
19918.99	19919.16	+0.17	−0.043
20119.50	20119.44	−0.06	+0.015
20123.14	20123.14	0.00	0.000
20125.01	20125.22	+0.21	−0.052
20813.84	20813.66	−0.18	+0.042
20817.30	20817.36	+0.06	−0.014
20819.04	20819.44	+0.40	−0.093

WAVE-NUMBERS—(*continued.*)

Wave-number	Corrected wave-number	Difference	Corresponding deviation of wave-length
20941.31	20941.28	−0.03	+0.007
20945.12	20944.98	−0.14	+0.032
20946.66	20947.06	+0.40	−0.091
21473.88	21473.78	−0.10	+0.022
21477.57	21477.48	−0.09	+0.020
21479.09	21479.56	+0.47	−0.102

The corrections are throughout within the errors of observation. As to the four triplets where the two weaker components could not be measured separately, the difference between the wave-number of their mean and that of the strongest component ought to be 4.74. But as the two weaker components differ in intensity, the measurements do not give the mean, but a wave-length nearer to the stronger of the two. From photographs of the other triplets where the two weaker components are not separated, we concluded that the measurements ought to give 4.41 instead of 4.74. They actually give the differences 4.35, 4.32, 4.15, 3.66. The corresponding corrections of the wave-lengths would be 0.007, 0.008, 0.027, 0.078 Ångstöm units. Thus the corrected wave-numbers of the strongest component are:

21389.59
21780.13
21838.22
22099.33

The distribution of the thirteen triplets over the spectrum is most regular, as may be seen from Plate II. They form two series ending at the same place, similar to the two series of triplets in the spectra of magnesium, calcium, strontium and of zinc, cadmium, mercury.[1]

The wave-numbers may be represented with considerable accuracy by the formulæ

[1] KAYSER and RUNGE, *Abhdl. d. K. Akad. d. W., Berlin*, 1891. RYDBERG, *Svenska vetensk. Akad. Handl.*, 23, No. 11.

$$A - \frac{B}{n^2} - \frac{C}{n^3} \text{ or } A - \frac{B}{n^2} - \frac{C}{n^4},$$

where A, B, C are constants and n stands for the values 3, 4, 5. . . . . The first formula is more accurate than the second. In both series the triplets decrease in intensity with increasing wave-number. But one of the series as a whole is decidedly stronger than the other.

STRONGER SERIES.

Strongest component, - -	23207.93	$-110387.7n^{-2}$	$-4814n^{-3}$
Middle component, -	23211.63	$-110387.7n^{-}$	$-4814n^{-3}$
Weakest component, - -	23213.71	$-110387.7n^{-2}$	$-4814n^{-3}$

n	Wave-number calculated	Wave-number observed	Obs.-calc.	Correspondence difference of wave length
4	16233.48	16233.52	+ 0.04	— 0.02
	16237.18	16237.28	+ 0.10	— 0.04
	16239.26	16239.38	+ 0.12	— 0.04
5	18753.91	18753.65	— 0.26	+ 0.07
	18757.61	18757.39	— 0.22	+ 0.06
	18759.69	18759.54	— 0.15	+ 0.04
6	20119.32	20119.50	+ 0.18	— 0.c4
	20123.02	20123.14	+ 0.12	— 0.03
	20125.10	20125 01	— 0.09	+ 0.02
7	20941.08	20941.31	+ 0.23	— 0.05
	20944.78	20945.12	+ 0.34	— 0.08
	20946.86	20946.66	— 0.20	+ 0.05
8	21473.72	21473.88	+ 0.16	— 0.03
	21477.42	21477.57	+ 0.15	— 0.03
	21479.50	21479.09	— 0.41	+ 0.09
9	21838.52	21838.35	— 0.17	+ 0.04
	21842.22	21842.50d		
	21844.30			
10	22099.24	22099.71	+ 0.47	— 0.10
	22102.94	22103.37d		
	22105.02			

WEAKER SERIES.

Strongest component,	23193.85	$-107567.1n^{-2}$	$-63108n^{-3}$
Middle component,	23197.55	$-107567.1n^{-2}$	$-63108n^{-3}$
Weakest component,	23199.63	$-107567.1n^{-2}$	$-63108n^{-3}$

n	Wave-number calculated	Wave-number observed	Obs.-calc.	Corresponding difference of wave-length
4	15484.84	15484.57	−0.27	+0.11
	15488.54	15488.25	−0.29	+0.12
	15490.62	15490.30	−0.32	+0.13
5	18386.30	18387.34	+1.04	−0.31
	18390.00	18390.97	+0.97	−0.29
	18392.08	18392.99	+0.92	−0.27
6	19913.71	19913.63	−0.08	+0.02
	19917.41	19916.78	−0.63	+0.16
	19919.49	19918.99	−0.50	+0.13
7	20814.61	20813.84	−0.77	+0.18
	20818.31	20817.30	−1.01	+0.23
	20820.39	20819.04	−1.35	+0.31
8	21389.86	21389.02	−0.24	+0.05
	21393.56	21393.97		
	21395.64			
9	21779.29	21780.17	+0.88	−0.19
	21782.99	21784.49		
	21785.07			

The first constant has very nearly the same value for both series. The difference 14.08 corresponds to 2.6 Ångström units. We may well assume that they really end at the same place. According to the notations of Rydberg, and of Kayser and Runge, they are, therefore, to be considered as "secondary series." The stronger one we denote as the first, the weaker one as the second secondary series.

Rydberg's formula

$$A - B\,(n + \mu)^{-2}$$

also applies with much the same accuracy as the formula

$$A - Bn^{-2} - Cn^{-3}$$

to represent the wave-numbers of the two series. We find for the strongest component

First secondary series: $23207.96 - 110396\ (n - 0.02148)^{-2}$
Second secondary series: $23200.63 - 109011.3\ (n - 0.24127)^{-2}$

The differences between the observed and calculated values are, in Ångström units :

First Secondary Series.	Second Secondary Series.
− 0.02	+ 0.05
+ 0.07	− 0.15
− 0.03	+ 0.03
− 0.04	+ 0.13
− 0.01	+ 0.04
+ 0.06	− 0.10
− 0.02	

Rydberg's formula agrees a little better with the observed values, especially for the weaker series. And the first constant differs less for the two series.

Rydberg has expressed the opinion that a connection exists between the second secondary series and the principal series (which hitherto has only been found in the spectra of the alkalis). It may be stated thus. If

$$A_a - B\ (n + \mu)^{-2} \qquad (a = 1,\ 2,\ 3)$$

be the formulæ for the three components of the second secondary series, $n = 2$ being the smallest value for which the formula is positive, and if the values A, σ_1, σ_2, σ_3, are calculated from the equations

(1) $$A_a = B\ (1 + \sigma_a)^{-2}$$
(2) $$A = B\ (1 + \mu)^{-2}$$

then the principal series is given by the formula

$$A - B\ (n + \sigma_a)^{-2} \qquad (a = 1,\ 2,\ 3;\ n = 1,\ 2,\ 3\ \ldots\ .)$$

The relative intensities of the three components for $a = 1, 2, 3$, should be the same in both series. If $a = 1$ corresponds to the

strongest component in the triplets of the secondary series, it also corresponds to the strongest component in the triplets of the principal series. It follows, therefore, that in the order of intensity the components are reversed. For if $A_1 < A_3$, then $A - B\ (1+\sigma_1)^{-2}$ is larger than $A - B\ (1+\sigma_3)^{-2}$.

To test Rydberg's suggestion, we calculated the first two triplets of the principal series and found the wave-numbers:

	$n = 1$	$n = 2$
Strongest component,	12042.39	24378.72
Middle component,	12038.69	24377.54
Weakest component,	12036.61	24376.87

If we take these calculated values to be only rough approximations, we may say that they correspond to the observed triplets near 7770 and 3947. These have the wave-numbers:

Strongest component,	12862.79	25325.59
Middle component,	12859.41	25324.43
Weakest component,	12856.65	25323.79

The differences in the wave-numbers of the components agree well enough with the differences of the calculated values to ascribe the deviations to errors of observation. In the red triplet the two weaker components give too large a difference; but this determination has little weight, as it was very difficult to see them with a slit narrow enough to separate them. But the mean of the two weaker components may be assumed to be determined more accurately, and here the distance from the strongest component has indeed very nearly the calculated value.

We therefore think that there is strong reason to believe these two triplets to be the first two members of the principal series. Rydberg's statements contain an approximation to the facts. But the statements are of unequal value. Equation (1) appears to be satisfied more closely than equation (2). Equation (1) may also be written thus:

$$A - A_a = A - B\ (1 + \sigma_a)^{-2},$$

that is to say, the difference between the limits of the principal and secondary series are equal to the wave-numbers of the first triplet of the principal series. In this way the law has been formulated by Schuster,[1] who discovered it independently, without being acquainted with Rydberg's memoir. It is interesting to note that the wave-numbers of the three Fraunhofer lines that seem to coincide with the lines of the first principal triplet show the calculated differences of wave-number more accurately than our measurements of the oxygen lines. According to the readings in Higgs' Atlas we obtain the differences 3.69 and 1.97, while the means of the secondary triplets give 3.70 and 2.08. The deviation 0.11 corresponds to 0.07 Ångström unit, and lies within the limits of error caused by the inaccuracy of the reading. This adds to the evidence that the Fraunhofer lines are oxygen lines.

The final proof that the principal series has been found would be given by observing the subsequent members. The next triplet is expected to be contracted to a width of 0.09 A. U. and to lie near 3334. The following triplet would then be still narrower and lie somewhere near 3117. These lines we have not found. But it may be that the slope of intensity, which in principal series appears to be steeper than in the secondary series, causes our exposures to be insufficient. From the intensity of the triplet 3947 we should have expected to find subsequent members of the series.

Besides the series of triplets we have found two other series of lines. Probably all these lines are close doubles; for the four brightest we have seen double, and we have measured the components of the brightest of these four. The weaker component is on the more refrangible side. We should expect the two components to have the same difference of wave-numbers in all the doublets. But we could not confirm it, because the weaker components were too weak to be measured.

[1] SCHUSTER, *Nature*, 55, 200.

STRONGER SERIES.

Wave-number = 21203.19 — 108793 n^{-2} — 39825 n^{-3}.

n	Wave-length observed	Wave-number calculated	Wave-number observed	Difference	Corresponding difference of wave-length
4..........	7254.32	13781.36	13781.15	—0.21	+0.11
5..........	6046.56	16532.87	16533.81	+0.94	—0.35
6..........	5555.16	17996.79	17996.36	—0.43	+0.13
7..........	5299.17	18866.82	18865.72	—1.10	+0.31
8..........	5146.23	19425.52	19426.38	+0.86	—0.23
9..........	5047.88	19805.44	19804.89	—0.55	+0.14
10..........	4979.73	20075.43	20075.93	+0.50	—0.12

WEAKER SERIES.

Wave-number = 21210.98 — 110329.5 n^{-2} — 2485 n^{-3}.

n	Wave-length observed	Wave-number calculated	Wave-number observed	Difference	Corresponding difference of wave-length
4..........	7002.48	14276.56	14276.78	+0.22	—0.11
5..........	5958.75	16777.92	16777.48	—0.44	+0.16
6..........	5512.92	18134.77	18134.27	—0.50	+0.15
7..........	5275.25	18952.11	18951.28	—0.83	+0.23
8..........	5130.70	19482.23	19485.20	+2.97	—0.78
9..........	5037.34	19845.48	19846.31	+0.83	—0.21
10..........	4973.05	20105.20	20102.89	—2.31	+0.57

Rydberg's formulæ for the same series are:

$$21205.56 - 109366.7\,(n - 0.16191)^{-2}$$
$$21211.11 - 110346.7\,(n - 0.01093)^{-2}$$

The agreement with the measured values is about the same. But it must be stated in favor of Rydberg's formula that the first constant as well as the second differs less for both series than in Kayser and Runge's formula.

The two series have apparently the same limit and must therefore be taken as a second pair of secondary series. It seems that there is a second principal series as well. If we calculate the formula

$$A - 109366.7\,(n + \sigma)^{-2}$$

according to Rydberg's law, from the formula of the secondary series

$$21205.56 - 109366.7\,(n - 0.16191)^{-2}$$

so that

$$21205.56 = 109366.7\,(1 + \sigma)^{-2}$$

and

$$A = 109366.7\,(2 - 0.16191)^{-2}$$

we obtain for $n = 1, 2, 3$, the wave-numbers

11165.10
22148.96
26375.15

The first number corresponds to a wave-length in the ultra-red part of the spectrum, where observations do not exist. The other two numbers are not very different from the wave-numbers of the strong lines 4368.466 and 3692.586.

Wave-number calculated:	22148.96	26375.15
Wave-number observed:	22885.23	27073.96
Difference:	736	699

In the first principal series the corresponding differences were 820 and 947. We venture to think that these two lines belong to a second principal series, although we did not succeed in finding the subsequent lines. At the same time it must be mentioned that the calculated values are deduced from the stronger one of the two secondary series, while according to Rydberg the weaker series ought to be taken.

Plate II is from a drawing of the six series. They show a striking similarity to the six series that make up the spectrum of clèveite gas,[1] as they form two sets of three series each, each set resembling the spectrum of one of the alkali metals In our paper on the spectrum of clèveite gas we had suggested that it consisted of two elements corresponding to the two sets

[1] C. RUNGE and F. PASCHEN, this JOURNAL, January 1896.

of three series. This inference can no longer be upheld; for the same argument would hold good for oxygen, for the duplicity of which there is no chemical evidence.

II. SULPHUR.

One end of the vacuum tube was widened and filled with concentrated sulphuric acid. The rest of the arrangement has been described above. We heated the sulphuric acid until it evaporated and a little of it condensed in and in front of the capillary tube. We then waited until the water vapor, generated at the same time, was again absorbed and in the meantime generated oxygen and drew it through the tube. Immediately afterwards a spectrum of sulphur appeared, which so far as we know, has not been observed heretofore. It was pure, when the tube was recently made with clean material. After a while the second spectrum of hydrogen appeared, probably on account of organic impurities dissolved in the sulphuric acid. The principal lines of the new spectrum, though weak, were also to be seen when sulphur was heated in the tube and oxygen added to it, and also when hyposulphuric acid was introduced into the tube. In these two cases the band spectrum of sulphur prevails and it seems that the new spectrum requires the presence of oxygen. It is best obtained from sulphuric acid and even then it is improved by an atmosphere of oxygen.

NEW SPECTRUM OF SULPHUR.

Wave-length	Mean error	Number of determinations	Intensity	Remarks
4694.357	0.03	3	10	The differences of the three wave-lengths are more accurate than the absolute values.
4695.690			8	
4696.488			6	
5278.31	0.08	8	3	The differences of the three wave-lengths are more accurate than the absolute values.
5278.81			5	
5279.19			6	
5287.88		1	2	
5290.89	0.03		4	Coincides with an impurity, which is perhaps due to oxygen.
5295.86	0.08	3	4	
5372.82	0.05	6	2	
5375.98	0.04	6	3	
5381.19	0.04	6	4	
5444.58	0.08	2	2	
5449.99	0.05	2	3	
5498.38	0.05	6	3	
5501.78	0.03	6	4	
5507.20	0.03	6	5	
5605.52	0.06	4	3	
5608.87	0.03	5	4	
5614.48	0.07	5	5	
5697.02	0.03	6	6	
5700.58	0.02	6	7	
5706.44	0.01	6	8	
5879.79	0.03	2	1	
5883.74	0.04	3	2	
5890.08	0.01	2	2	
6042.17	0.01	2	5	
6046.23	0.05	2	6	
6052.97	0.01	2	7	
6173.77	0.11	2	1	
6176.01	0.01	2	1	
6395.10		1	1	
6396.90		1	1	
6403.70	0.04	4	2	
6408.32	0.02	4	3	
6415.68	0.02	5	4	
6536.55	0.06	3	1	
6538.82	0.08	3	1	
6743.92	0.18	3	5	
6749.06	0.08	4	6	
6757.40	0.05	4	7	
7242			2	The limit of error may be as high as 2 Ångstr. units.

The greater part of the lines of this list may be arranged in two series of triplets, which show a perfect analogy to the triplets of oxygen. The differences of wave-numbers of the three components of each triplet are again the same for all triplets, as far as the accuracy of the determination goes.

TRIPLETS.

Wave-length	Wave-number	Difference	Remarks
6757.40 6749.06 6743.92	14794.58 14812.87 14824.16	18.29 11.29	
6415.68 6408.32 6403.70	15582.57 15600.47 15611.73	17.90 11.26	
6052.97 6046.23 6042.17	16516.32 16534.73 16545.85	18.41 11.12	
5890.08 5883.74 5879.79	16973.07 16991.36 17002.78	18.29 11.42	
5706.44 5700.58 5697.02	17519.28 17537.29 17548.26	18.01 10.97	
5614.48 5608.87 5605.52	17806.23 17824.04 17834.70	17.81 10.66	
5507.20 5501.78 5498.38	18153.09 18170.98 18182.22	17.89 11.24	
5449.99 5444.58	18343.64 18361.87	18.23	The third component was not seen.
5381.19 5375.98 5372.82	18578.17 18596.18 18607.12	18.01 10.94	
5295.86 5290.89 5287.88	18877.50 18895.24 18906.00	17.74 10.76	

The means of the differences are 18.15 and 11.13, taking account of the weights. We may now correct the components of each triplet, so that the corrected wave-numbers show these differences 18.15 and 11.13, while their center of gravity remains the same. The following list contains these corrected wave-numbers:

WAVE-NUMBERS.

Wave-number observed	Wave-number corrected	Difference	Corresponding deviation of wave-lengths	Mean error of the wave-length
14794.58	14794.63	+ 0.05	— 0.02	0.05
14812.87	14812.78	— 0.09	+ 0.04	0.08
14824.16	14823.91	— 0.25	+ 0.11	0.18
15582.57	15582.45	— 0.12	+ 0.05	0.02
15600.47	15600.60	+ 0.13	— 0.05	0.02
15611.73	15611.73	0.00	0.00	0.04
16516.32	16516.45	+ 0.13	— 0.05	0.01
16534.73	16534.60	— 0.13	+ 0.05	0.05
16545.85	16545.73	— 0.12	+ 0.04	0.01
16973.07	16973.12	+ 0.05	— 0.02	0.01
16991.36	16991.27	— 0.09	+ 0.03	0.04
17002.78	17002.40	— 0.38	+ 0.13	0.03
17519.28	17519.23	— 0.05	+ 0.02	0.01
17537.29	17537.38	+ 0.09	— 0.03	0.02
17548.26	17548.51	+ 0.25	— 0.08	0.03
17806.23	17805.84	— 0.39	+ 0.12	0.07
17824.04	17823.99	— 0.05	+ 0.02	0.03
17834.70	17835.12	+ 0.42	— 0.13	0.06
18153.09	18152.96	— 0.13	+ 0.04	0.03
18170.98	18171.11	+ 0.13	— 0.04	0.03
18182.22	18182.24	+ 0.02	— 0.01	0.05
18343.64	18343.66	+ 0.02	— 0.01	0.05
18361.87	18361.81	— 0.06	+ 0.02	0.08
18578.17	18578.04	— 0.13	+ 0.04	0.04
18596.18	18596.19	+ 0.01	— 0.00	0.04
18607.12	18607.32	+ 0.20	— 0.06	0.05
18877.50	18877.15	— 0.35	+ 0.10	0.08
18895.24	18895.30	+ 0.06	— 0.02	0.03
18906.00	18906.43	+ 0.43	— 0.12	— -

The two series appear to be secondary series. We have observed six triplets belonging to the stronger series, and only four of the weaker one. The wave-numbers are represented by the following formulæ :

FIRST SECONDARY SERIES.

Strongest line: $20086.89 - 109598\,n^{-2} - 113556\,n^{-3}$
Middle line: $20105.04 - 109598\,n^{-2} - 113556\,n^{-3}$
Weakest line: $20116.17 - 109598\,n^{-2} - 113556\,n^{-3}$

n	Wave-number calculated	Wave-number observed	Difference	Corresponding difference of wave-lengths
5	14794.52	14794.58	+ 0.06	— 0.03
	14812.67	14812.87	+ 0.20	— 0.09
	14823.80	14824.16	+ 0.36	— 0.16
6	16516.78	16516.32	— 0.46	+ 0.17
	16534.93	16534.73	— 0.20	+ 0.07
	16546.06	16545.85	— 0.21	+ 0.08
7	17519.13	17519.28	+ 0.15	— 0.05
	17537.28	17537.29	+ 0.01	0.00
	17548.41	17548.26	— 0.15	+ 0.05
8	18152.63	18153.09	+ 0.46	— 0.14
	18170.78	18170.98	+ 0.20	— 0.06
	18181.91	18182.22	+ 0.31	— 0.09
9	18578.06	18578.17	+ 0.11	— 0.03
	18596.21	18596.18	— 0.03	+ 0.01
	18607.34	18607.12	— 0.22	+ 0.06
10	18877.35	18877.50	+ 0.15	— 0.04
	18895.50	18895.24	— 0.26	+ 0.07
	18906.63	18906.00	— 0.63	+ 0.18

The formulæ of the first secondary series have been calculated by the method of least squares from the wave-numbers corrected to equal differences. The formulæ of the second secondary series have been calculated simply from the wave-numbers of the first three triplets corrected to equal differences.

For smaller values of n the formulæ of the first secondary series give two ultra-red triplets, of whose existence we know nothing. The second series also gives two triplets, of which one might just be visible being near 7700. We have looked for it, but could not detect it. However, it may well have escaped our attention, as the eye is not sensitive in this region, and the second series is the weaker one of the two.

SECOND SECONDARY SERIES.

Strongest line, $20078.37 - 108744.5\, n^{-2} - 18268\, n^{-3}$
Middle line, $20096.52 - 108744.5\, n^{-2} - 18268\, n^{-3}$
Weakest line, $20107.65 - 108744.5\, n^{-2} - 18268\, n^{-3}$

n	Wave-number calculated	Wave-number observed	Difference	Corresponding difference of wave-lengths
5	15582.45	15582.57	+0.12	−0.05
	15600.60	15600.47	−0.13	+0.05
	15611.73	15611.73	0.00	0.00
6	16973.12	16973.07	−0.05	+0.02
	16991.27	16991.36	+0.09	−0.03
	17002.40	17002.78	+0.38	−0.13
7	17805.84	17806.23	+0.39	−0.12
	17823.99	17824.04	+0.05	−0.02
	17835.12	17834.70	−0.42	+0.13
8	18343.56	18343.64	+0.08	−0.02
	18361.71	18361.87	+0.16	−0.05
	....	....	......	......

From the analogy with the spectrum of oxygen, we should expect a principal series of triplets, for which the differences of wave-numbers become smaller for smaller wave-lengths and for which the components are in the inverse order of intensity. According to Rydberg's law, found to give a rough approximation to the facts in the case of oxygen, we may calculate the formulæ for the principal series.

For Rydberg's form the formulæ of the second secondary series are:

Strongest line, $20078.95 - 108868.5\,(n - 0.07945)^{-2}$
Middle line, $20097.10 - 108868.5\,(n - 0.07945)^{-2}$
Weakest line, $20108.23 - 108868.5\,(n - 0.07945)^{-2}$

From these we obtain for the principal series:

Strongest line, $29515.56 - 108868.5\,(n + 0.32852)^{-2}$
Middle line, $29515.56 - 108868.5\,(n + 0.32747)^{-2}$
Weakest line, $29515.56 - 108868.5\,(n + 0.32683)^{-2}$

The triplet corresponding to $n = 2$ is in the ultra-red, but for $n = 3$ we get

		Difference
Strongest line,	19689.05	
Middle line,	19682.84	6.21
Weakest line,	19679.05	3.79

In the case of oxygen the values calculated in this manner showed only a rough approximation to the observed wave-numbers. The latter were considerably larger. But the differences of the three components agreed well with the observed differences. Bearing this in mind, it does not seem improbable that the strong triplet near wave-length 4695 is a member of the principal series:

	Wave-length	Wave-number	Difference
Strongest line,	4694.357	21296.32	
Middle line,	4695.690	21290.27	6.05
Weakest line,	4696.488	21286.66	3.61

As we have shown above, Rydberg's rule of calculating the formulæ of the principal series may be separated into two equations. One of them is only roughly satisfied, while the other is in close accordance with the facts. Schuster has formulated it thus: the difference between the wave-numbers of the units of the principal series and the secondary series is equal to the wave-numbers of the first member of the principal series. If we neglect the first equation and keep only the second, we have one constant of the formula at our disposal, which we may determine to make the observed triplet the second member of the principal series.

Thus we obtain the formulæ

Strongest line, $31123.00 - 108868.5\,(n+0.32852)^{-2}$
Middle line, $31123.00 - 108868.5\,(n+0.32747)^{-2}$
Weakest line, $31123.00 - 108868.5\,(n+0.32683)^{-2}$

These give for the following value of n the wave-lengths (in air):

3949.54, 3949.98, 3950.25

which, however, we have no more been able to observe than the corresponding triplet in the spectrum of oxygen. It may be

that the intensity in the principal series decreases so rapidly that this triplet becomes imperceptible.

It seems only probable that in the spectrum of sulphur the other three series of oxygen should also find their analogy. We have, however, not succeeded in finding them. Possibly the strong triple line near 5279 corresponds to the oxygen line 4368. The latter would then also have to be a triplet, but so close that it might be rather difficult to separate the components. For if we compare the differences of wave-numbers in the triplets of oxygen with those of sulphur we see that they increase about in the proportion of 1 to 5. If therefore the oxygen line 4368 corresponded to the sulphur triplet 5279, we would expect 4368 to be a triplet of one-fifth the width.

III. SELENIUM.

If instead of sulphuric acid we introduce concentrated selenious acid[1] into the vacuum tube and treat it in a similar way as described above, there appears a spectrum of selenium, which as far as we know has not been observed heretofore. Selenious acid must be heated somewhat stronger than sulphuric acid before the vapor condenses in the capillary tube and gives a bright spectrum. But the spectrum tube keeps better than that of sulphur. An atmosphere of oxygen seems to increase its visibility in the same way as that of sulphur. Many lines of the selenium spark spectrum appear at the same time with the new spectrum. This is also the case with oxygen and sulphur. We have observed many, though not all of the lines of the selenium spark spectrum found by Plücker and Hittorf. They are not contained in the following table. Our attention was principally directed to the new spectrum, which mainly consists of triplets. Sometimes for short periods this alone was visible together with the oxygen and hydrogen lines. But we could not keep the tube in this state. For this reason our photographs, which had

[1] We concentrated the commercial dilute selenious acid by letting it slowly evaporate, so that in vacuo it did not give off any water-vapor.

to be exposed for some time on account of the small intensity of some of the lines, do not show the spectrum free from impurities. Of the red lines we could only measure the stronger ones. The weaker ones are not contained in the following table except a few weak lines that are near to measured lines so that their wave-lengths could be estimated without micrometrical measurement.

NEW SPECTRUM OF SELENIUM.

Wave-length	Mean error	Number of determinations	Intensity	Remarks
4731.04	0.07	5	10	The differences between the wave-lengths of the components in these two triplets are more accurate than the absolute values.
4739.28		5	9	
4742.52		5	8	
5365.59	0.03	4	8	
5370.04		4	10	
5374.27		4	10	
5464.82	0.15	3	3	
5497.06	0.11	3	3	
5528.64	0.05	3	4	
5618.05	0.08	3	5	
5652.62	0.04	2	3	
5666.95	0.03	3	3	
5700.32	0.07	2	3	
5703.86	0.05	4	4	
5705.18	0.05	4	3	
5718.28	0.03	4	7	
5718.5			1	The distance from the preceding line was estimated.
5752.31	0.09	3	2	
5753.52	0.04	4	7	
5827.90	0.06	2	1	
5843.10	0.05	4	2	
5866.53	0.10	3	2	
5878.88	0.06	4	2	
5907.10	0.03	2	2	
5909.49	0.02	4	3	
5925.13	0.02	4	4	
5925.31			1	The distance from the preceding line was estimated.
5961.7			1	The distance from the following line was estimated.
5962.08	0.03	4	5	
6121.95	0.15	4	2	
6135.52	0.09	3	1	
6138.51	0.09	5	2	
6177.87	0.10	5	3	
6266.36	0.06	4	4	
6269.28	0.09	3	3	

NEW SPECTRUM OF SELENIUM (*continued*).

Wave-length	Mean error	Number of determinations	Intensity	Remarks
6283.54	0.06	3	2	
6284.19			1	The distance from the neighboring lines was estimated.
6284.51	0.07	4	3	
6325.4			1	The distance from the following line was estimated.
6325.81	0.03	4	6	
6679.72	0.10	4	5	
6699.78	0.06	5	6	
6701.29	0.03	2	1	
6746.65	0.07	5	6	
6831.28	0.24	3	5	
6990.96	0.06	6	4	Double ?
7010.84	0.12	5	5	
7014.24	0.09	5	3	
7062.14	0.06	5	5	

It is not impossible that some of the weaker lines contained in this list belong to the spark spectrum. The greater part form two series of triplets which do not appear in the spark-spectrum and are similar to the series of triplets in the spectra of oxygen and sulphur.

The constant differences between the wave-numbers of the components are not so easily shown as with the two elements of smaller atomic weight. It appears that some of the components of the triplets are accompanied by weak satellites, and that these, not the main lines, show the constant differences of wave-number. Something quite similar has been observed with the first secondary series in the spectra of calcium, strontium, zinc, cadmium, and mercury. The weak satellites are not so easily measured, and may sometimes escape the attention of the observer. We are therefore disposed to believe that the deviations from the constant difference of wave-numbers may disappear with a more complete and more accurate determination of the components.

TRIPLETS.

Wave-length	Wave number	Differences
7062.14	14156·17	
7014.24	14252.84	103.56
7010.84	14259.76	47.47
6990.96 d ?	14300.31	40.55
6746.65	14818.15	
6701.29	14918.45	103.66
6699.78	14921.81	48.18
6679.72	14966.63	44.82
6325.81	15803.95	
6325.4	15804.98	103.85
6284.51	15907.80	103.64
6284.19	15908.62	
6283.54	15910.27	
6269.28	15946.45	45.27
6266.36	15953.89	43.62
6177.87	16182.41	103.76
6138.51	16286.17	44.05
6121.95	16330.22	
5962.08	16768.10	
5961.7	16769.17	104.05
5925.31	16872.15	103.50
5925.13	16872.67	45.18
5909.49	16917.82	44.66
5878.88	17005.41	
5843.10	17109.54	101.13
5827.90	17154.16	44.62
5753.52	17375.92	106.41
5752.31	17379.58	102.75
5718.5	17482.33	103.42
5718.28	17483.00	
5705.18	17523.15	
5703.86	17527.20	44.20
5700.32	17538.09	
5666.95	17641.37	103.28
5652.62	17686.09	44.72
5528.64	18082.66	
5497.06	18186.57	103.88.

[1] The third line has not been observed.

It is not feasible to correct the wave-numbers so as to make the differences constant. For one does not know how to shift the weak satellites. Nevertheless, it may be satisfactorily shown that there are two series of triplets, as Plate II indicates.

The satellites are not drawn on account of the small scale. The dotted lines are not observed; they merely serve to emphasize the symmetry of the distribution.

In order to show that both series may be represented by formulæ, showing the characteristics of series, it is sufficient to take one line only of each triplet. For the triplets without satellites we take the least refrangible line; for the others the least refrangible satellite. The formulæ have been calculated from three lines only without the method of least squares. On account of the uncertainty of the satellites, it is of no use to employ this method.

First (stronger), secondary series:

$$\text{Wave-number} = 19266.54 - 108900.6n^{-2} - 94293n^{-3}$$

n	Wave-number calculated	Wave-number observed	Difference	Remarks
5	14156.17	14156.17	0.00	These three values determine the constants.
6	15804.98	15804.98	0.00	
7	16769.17	16769.17	0.00	
8	17380.80	17379.58	— 1.22	
9	17792.74	17794.91	+ 2.17	
10	18083.24	18082.69	— 0.55	
11	18295.69	18293.89	— 1.80	

The fifth wave-number, 17794.91, corresponds to the wave-length 5618.05. The other two components would be strong enough to be seen, but they are lost in one of the two green oxygen bands. It is not unlikely that there is a satellite to 5618.05. That would explain the unsystematic deviation of the formula. The last wave-number corresponds to the wave-length 5464.82. The other two components of the triplet have not been observed. Perhaps the line 5416.94 (intensity 1), wave-number 18455.57 (observed once only), belongs to the next

triplet. The calculated value is 18455.72. But the line may also be an impurity.

Second (weaker), secondary series:

Wave-number $= 19286.72 - 111959.6n^{-2} + 1227n^{-3}$

n	Wave-number calculated	Wave-number observed	Difference	Remarks
5	14818.15	14818.15	0.00	These three values determine the constants.
6	16182.41	16182.41	0.00	
7	17005.41	17005.41	0.00	
8	17539.75	17538.09	− 1.66	

The smaller values of n correspond to lines in the ultra-red, which have not been observed. The second series has also been calculated in Rydberg's form:

$$19286.76 - 111963.9\,(n + 0.00556)^{-2}$$

If we take the differences of the wave-numbers of two subsequent components to be 103.71 and 44.55, the other components are represented by the formulæ:

$$19390.47 - 111963.9\,(n + 0.00556)^{-2}$$
$$19435.02 - 111963.9\,(n + 0.00556)^{-2}$$

According to Rydberg's rules, we find for the principal series:

$$\frac{111963.9}{(2.00556)^2} - 111963.9\,(n + \sigma_a)^{-2} \quad (a = 1, 2, 3),$$

where σ_1, σ_2, σ_3 have to be determined in such a manner that $111963.9\,(n + \sigma_a)^{-2}$ assumes the values 19286.76, 19390.47, 19435.02.

We find in this way:

Strongest line, $27835.99 - 111963.9\,(n + 0.409403)$
Middle line, $27835.99 - 111963.9\,(n + 0.402951)^{-2}$
Weakest line, $27835.99 - 111963.9\,(n + 0.400195)^{-2}$

For $n = 2$ the lines are in the ultra-red, but for $n = 3$ we obtain:

		Difference
Strongest line, - - - -	18203.99	
		36.56
Middle line, - - - -	18167.32	
		15.68
Weakest line, - - - -	18151.64	

We expect these absolute values to be only rough approximations, but the differences we should expect to be in close accordance with the facts.

In the spectrum of oxygen and sulphur the observed wave-numbers of the corresponding triplet were considerably larger than the calculated values. Therefore we think it very likely that the strong triplet near 4740 is the triplet in question. We have:

	Wave-length	Wave-number	Difference
Strongest line, - - -	4731.04	21131.19	
			36.77
Middle line, - - -	4739.28	21094.42	
			14.37
Weakest line, - - -	4742.52	21080.05	

The differences are in good accordance with the expected values and so are the intensities. The strongest line and weakest line have reversed their position, as should be the case, the strongest component in the principal series now being the most refrangible, while it is the least refrangible in the secondary series. We have not observed further members of the principal series. The next triplet would be expected near 4000. The differences of the wave-lengths of the components would have to be about 2.7 and 1.2 Ångström units. If the assumption of the principal series is correct, the intensity would have to decrease very rapidly to cause this triplet to become imperceptible.

Whether the strong selenium triplet near 5370 corresponds to the oxygen line 4368 and the sulphur triplet near 5280 we cannot say. Its position makes it seem probable. But we should expect the components to be closer to one another and the relative position of the components is not similar to that of the components of the sulphur triplet near 5280.

Comparing the three spectra of oxygen, sulphur, and selenium, there are a few features that force themselves on our atten-

tion. The differences between the wave-numbers of the three components are the smallest in the spectrum of oxygen and the largest in the spectrum of selenium. They increase with increasing atomic weight, roughly proportional to the square of the atomic weight.

	Difference of wave-numbers	Atomic weight	Square of atomic weight divided by difference of wave-numbers
O, - -	3.70 and 2.08	16	69 and 123
S, - -	18.15 and 11.13	32.06	57 and 92
Se, - -	103.7 and 44.6	79.07	60 and 140

The same thing has been observed with the doublets in the spectra of the alkali metals and with the triplets in the spectra of magnesium, strontium, calcium, and of zinc, cadmium, and mercury.

On the whole the spectrum of selenium consists of lines of longer wave-lengths than that of sulphur, and this again of longer wave-lengths than that of oxygen. The greater atomic weight corresponds to slower oscillations. The same has been observed in the spectra of other groups of chemically related elements.

We tried in vain to produce the spectrum of tellurium in the vacuum tube. We introduced telluric acid, which Professor Seubert was so kind as to make for us, into the vacuum tube and treated it as we had treated the sulphuric acid and the selenious acid. But it appears that the temperature to which we could expose the evacuated glass tube was not high enough to evaporate the telluric acid and make it condense in the capillary tube. At all events we did not discover any lines belonging to tellurium.

ADDENDUM.

For the determination of the red lines in the three spectra we have used argon lines as standards. But as the red argon lines have not hitherto been determined accurately enough, they had first to be measured. We have determined their wave-

lengths by measuring their position relatively to the lines of the solar spectrum that are contained in Rowland's list of standard wave-lengths. An image of the Sun was thrown on the slit alternately with the light of the argon vacuum tube, care being taken that each time the same part of the slit was illuminated and that the whole ruled surface of the grating was filled with light. The distances of the lines were measured with an Abbe comparator, on the movable part of which we fastened a microscope of low power. In a similar way the wave-lengths of the red oxygen, sulphur and selenium lines were measured with the argon lines as standards. We also determined the wave-lengths of argon with a small, flat grating on a spectroscope of the old type by measuring the angles through which the grating is turned to bring the different lines on the cross hairs. These measurements, however, have only small weight compared with the measurements with the concave grating.

A number of weaker argon lines have occasionally been measured without having been used as standards. They are included in the following list and may be distinguished from the lines used as standards by the omission of their mean error, which we suppose to be somewhat larger than the mean error of the lines used as standards. The different determinations have been made on different days. The lines marked *r* (red) we have observed to disappear when a spark gap is interposed in the circuit. They belong to the red spectrum of argon exclusively. The lines marked *r* and *b* (red and blue) we have observed to remain when a spark gap is interposed. The lines marked *b* (blue) were seen only with a spark gap. They belong to the blue spectrum exclusively. This latter class has not been measured accurately. Nevertheless we include them in the list. The intensity is estimated by the numbers 1 to 10, the larger number signifying greater intensity. In the fifth column we give the wave-lengths as they were determined by Kayser.

LINES OF ARGON.

Wave-length	Intensity	Number of different determinations	Mean errors	Kayser	Remarks
5860.54	4			5860.6 r	Eder and Valenta : 5860.69
5864.29	<1				
5870.52	1				
5880.41	<1				
5882.88	4			5882.78 r	Eder and Valenta : 5883.03
5888.79	6	4	0.011	5888.93 r	Eder and Valenta : 5889.02
5897.75	<1				
5900.70	<1				
5904.09	<1				
5912.31	7	4	0.014	5912.22 r	Eder and Valenta : 5912.48
5916.84	3				
5920.04	<1				
5920.33	<1				
5927.34	3	3	0.015		Eder and Valenta : 5928.61
5929.06	6	3	0.015	5928.5 r	
5941.08	3				
5942.92	5	3	0.02	5943.5 r	
5949.47	3				
5960.78	<1				
5964.70	3				
5968.58	3				
5971.91	4	2	0.002		
5982.22	2				
5987.61	5	2	0.003	5987.5 r	
5994.99	2				
5999.29	4	2	0.01	5999.5 r	
6005.95	3	2	0.02		
6011.59	1				
6013.94	4	2	0.00	6013.6 r	
6015.40	<1				
6017.66	1				
6025.40	4	4	0.014	6025.8 r	
6032.39	9	4	0.017	6031.5 r	Double? Eder and Valenta: 5860.69
6035.49	<1				
6040.46	<1				
6043.48	8	4	0.02	6043.0 r	Eder and Valenta : 5928.61
6052.96	6	3	0.01	6052.7 r	
6059.62	7	4	0.02	6059.5 r	
6064.93	3	2	0.02		
6067.48	<1				
6075.20	<1				Double?
6081.50	2				
6085.90	1				
6090.97	4	3	0.06		
6093.44	1				
6096.09	1				
6099.03	6	4	0.016	6098.8 r	
6101.33	3	3	0.03		
6104.71	3				
6105.87	6	4	0.023	6106.1 r	

LINES OF ARGON (*continued*).

Wave-length	Intensity	Number of different determinations	Mean errors	Kayser	Remarks
6113.55	3	2	0.06		
6115.05	2			6114.1 b	
6119.74	3				
6121.93	2				
6123.8	<1				b
5125.96	1				
6127.57	4	4	0.03		
6129.02	3				
6134.12	<1				
6135.63	1				
6139.1	1			6140.9 b	b
6143.16	1				
6145.64	6	5	0.014	6145.6 r	
6155.46	5	3	0.014	6155.2 r	
6159.60	1				
6161.68	2				
6165.30	3				
6170.39	5	4	0.035	6170.3 r	
6172.7	5			6172.3 b	b
6173.32	6	4	0.03	6172.9 r	r and b
6179.50	2				
6183.12	<1				
6186.52	<1				
6189.5	<1				b
6194.25	<1				
6197.30	<1				
6199.44	<1				
6212.73	6	3	0.01	6212.5 r	
				6215.6 b	
6216.14	6	3	0.01	6217.5 r	
6224.85	1				
6230.96	2				
6235.99	1				
6238.58	<1				
6240.5	<1				b
6243.45	3			6243.7 b	b
6248.65	4				
6259.58	<1				
6266.70	1				
6278.80	2				
6297.15	5	4	0.012	6296.8 r	
6299.01	<1				Doubtful ?
6307.91	5	4	0.003	6307.8 r	
6309.36	1				
6334.24	<1				Uncertain.
6365.02	3				
6369.74	4			6368.0 r	
6384.89	5	4	0.04	6384.5 r	r and b
6402.21	1				

LINES OF ARGON (*continued*).

Wave-length	Intensity	Number of different determinations	Mean error	Kayser	Remarks
6416.54	8	4	0.014	6415.2 r	r and b
6431.77	3				
6466.65	3				
6481.17	2				
6483.6	3			6482.8 b	b
6494.10	2				
6513.87	1				
6538.43	3	3	0.04		r and b
6605.05	4	3	0.01		
6615.2	<1				b
6632.07	1				
6638.7	2			6638.6 b	b
6640.5	1				b
6644.3	3			6644.2 b	b
6660.92	3				
6664.27	3				r
6677.61	6	4	0.05	6676.5 r	r and b
6679.01	<1				
6682.7	2			6684.2 b	b
6684.95	<1				
6699.06	3				r
6719.33	2				
6753.15	5	4	0.02	6752.7 r	r and b
6756.58	1				
6766.97	1				r
—				6786.5 r	
6827.85	<1				
6871.56	4	5	0.02	6870.6 r	r and b (weak)
6880.26	1				
6888.83	1				
6937.99	2	3	0.04	6937.8 r	r and b (weak)
6965.81	6	4	0.05	6964.8 r	r and b (weak)
7030.54	2	2	0.03	7029.2 r	r and b (weak)
7067.54	5	4	0.03	7066.6 r	r and b
7068.83	1				
7147.30	1	3	0.08	7146.8 r	r
7273.04	2	3	0.08	7271.6 r	r and b ?
7384.22	2	2	0.05	7383.9 r	r and b
7504.5	3			7503.4 r	r and b ?
7515.4	1			7515.1 r	r and b
7636.2	1			7635.6 r	r
7725				7723.4 r	The last two wave-lengths are only rough approximations, the error being perhaps as much as 2 A. U.
7952					

TECHNISCHE HOCHSCHULE,
Hannover, May 1898.

NOTES ON THE CONCAVE GRATING.

By S. A. MITCHELL.

I. FUNDAMENTAL FORMULA.

THE general theory of the concave spherical grating has been investigated by Rowland (*Phil. Mag.*, **16**, 1883; and *Amer. Jour. Sci.* (3), **26**, 1883); by Glazebrook (*Phil. Mag.*, **15**, 1883); by Mascart (*Jour. de Phys.* (2), **2**, 1883); by Baily (*Phys. Soc. Proc.*, **5**, 1883); and by Kayser (*Winkelmann, Handbuch der Physik*, p. 408).

The following treatment is one which starts from a general condition, true for every form of grating ruled with equal spaces along a chord. By introducing the condition that the form of the grating is the section of a sphere, the general equation of the concave grating is obtained.

From the general theory of gratings (Lord Rayleigh, "Wave Theory of Light," § 14, *Ency. Brit.*, p. 437; and Kayser, *Handbuch der Physik*), we know, if we have a grating ruled with equal spaces on any surface, that

$$\lambda = \frac{\omega}{N}(\sin\gamma + \sin\mu). \tag{1}$$

where ω is the grating space, N the order of the spectrum, γ and μ the angles which the incident and diffracted light make with the normal to the surface at any point, and λ is the wave-length.

That is, in Fig. 1, if L is the radiant point, and a cone of rays from L falling on the surface of the grating QOT at P is brought to a focus at L' by any means (if the grating is plane a lens will be necessary), then λ is the wave-length of this light.

The above equation may be written:

$$(\sin\gamma + \sin\mu) = \frac{N\lambda}{\omega} \tag{2}$$

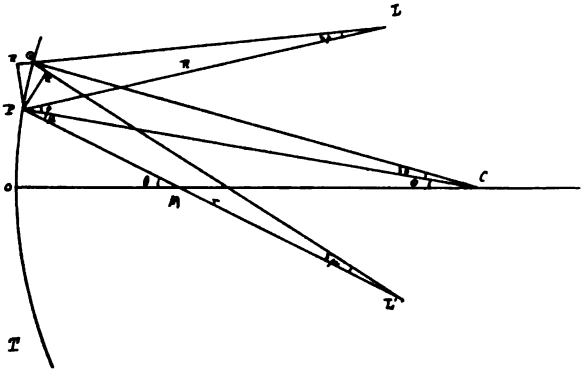


FIG. 1.

If now we allow the point P to move in the plane LoL', along the surface QOT, so that the angles which the incident and diffracted light make with the normal to the surface are now $(\gamma + \delta\gamma)$ and $(\mu + \delta\mu)$, then a new wave-length $(\lambda + \delta\lambda)$ will be brought to a focus at L', so as to satisfy the equation

$$\sin(\gamma + \delta\gamma) + \sin(\mu + \delta\mu) = \frac{N\lambda}{\omega} + \frac{\delta(N\lambda)}{\omega} \tag{3}$$

Developing each term by Maclaurin's theorem, and subtracting (2), we get

$$\cos\gamma\, d\gamma - \tfrac{1}{2}\sin\gamma\,\delta\gamma^2 + \cos\mu\,\delta\mu - \tfrac{1}{2}\sin\mu\,\delta\mu^2 = \frac{1}{\omega}\,\delta(N\lambda) \tag{4}$$

in which we neglect quantities of a higher order than the second. Hence, introducing an independent variable ϕ, we get:

$$\cos\gamma\frac{d\gamma}{d\phi} - \tfrac{1}{2}\sin\gamma\left(\frac{d\gamma}{d\phi}\right)^2 d\phi + \cos\mu\frac{d\mu}{d\phi} - \tfrac{1}{2}\sin\mu\left(\frac{d\mu}{d\phi}\right)^2 d\phi$$
$$= \frac{1}{\omega}\,\frac{d(N\lambda)}{d\phi}\,.$$

This equation gives the change in wave-length due to a change in the angles of incidence and diffraction.

If now the surface is spherical — the case of Rowland's concave grating — O is the "center of the grating," C the center of curvature, CP is normal to the surface at P, and the line CP is the radius of curvature ρ. Then the angle LPC is equal to γ,

$L'PC$ is equal to μ, and putting OCP and OMP equal respectively to ϕ and θ, we see:

$$\gamma = \theta - \phi\,; \quad \mu = \theta' - \phi\,;$$

and hence,

$$\frac{d\gamma}{d\phi} = \frac{d\theta}{d\phi} - 1\,; \quad \frac{d\mu}{d\phi} = \frac{d\theta'}{d\phi} - 1 \tag{6}$$

Calling the distances LP and $L'P$ respectively R and r, we see that by letting fall perpendiculars PR and PR' on LQ and $L'Q$ respectively that

$$PR = \rho\, d\phi \cos\gamma = R\, d\theta.$$

Hence

$$\frac{d\theta}{d\phi} = \frac{\rho\cos\gamma}{R}$$

and consequently

$$\frac{d\gamma}{d\phi} = \frac{\rho\cos\gamma}{R} - 1\,.$$

Similarly,

$$\frac{d\mu}{d\phi} = \frac{\rho\cos\mu}{r} - 1\,. \tag{7}$$

Substituting these values of $\frac{d\gamma}{d\phi}$ and $\frac{d\mu}{d\phi}$ in equation (5) we get:

$$\cos\gamma\left(\frac{\rho\cos\gamma}{R} - 1\right) + \cos\mu\left(\frac{\rho\cos\mu}{r} - 1\right) \tag{8}$$

$$= \frac{1}{\omega}\frac{d(N\lambda)}{d\phi} + \frac{1}{2}\, d\phi \left\{ \sin\gamma\left(\frac{\rho\cos\gamma}{R} - 1\right)^2 + \sin\mu\left(\frac{\rho\cos\mu}{r} - 1\right)^2 \right\}$$

which equation is true to terms of the order $d\phi^2$. To have a perfect focus for waves of length λ at L', this change in wave-length due to a change in the angle ϕ must be zero, or in other words $\frac{d\lambda}{d\phi} = 0$. Light of wave-lengths which are whole multiples of λ will also be brought to the same focus at L', since

$$\frac{d(2\lambda)}{d\phi} = 0, \quad \frac{d(3\lambda)}{d\phi} = 0 \quad . \quad . \quad \text{etc.}$$

Hence making $\frac{d(N\lambda)}{d\phi} = 0$, and neglecting infinitely small quantities of the first order $(d\phi)$, we get:

$$\cos\gamma\left(\frac{\rho\cos\gamma}{R} - 1\right) + \cos\mu\left(\frac{\rho\cos\mu}{r} - 1\right) = 0 \tag{9}$$

It may be noted that the omitted term contains as a factor:

$$\sin\gamma\left(\frac{\rho\cos\gamma}{R}-1\right)^2+\sin\mu\left(\frac{\rho\cos\mu}{r}-1\right)^2. \qquad (10)$$

Equation (9) may be put in the form

$$\frac{\cos^2\gamma}{R}+\frac{\cos^2\mu}{r}=\frac{\cos\gamma+\cos\mu}{\rho}, \qquad (11)$$

whence we get:

$$r=\frac{R\rho\cos^2\mu}{R(\cos\gamma+\cos\mu)-\rho\cos^2\gamma}. \qquad (12)$$

This is the equation of the curve on which the spectra are brought to a focus. In this, the center of the grating is the origin, the line passing through the center of curvature is the axis of reference, R and γ the coördinates of the source of light, r and μ the coördinates of the spectral line.

This is the same equation as derived by Rowland (*loc. cit.*).

Further, we see that equation (11) is satisfied by making $R=\rho\cos\gamma$, $r=\rho\cos\mu$. The same substitution makes the omitted term, viz. (10), vanish. Consequently, if the conditions $R=\rho\cos\gamma$, $r=\rho\cos\mu$ are satisfied, formula (12) is true to terms of the second order. (See Kayser, *loc. cit.*) This condition is secured automatically in "Rowland's Mounting," where $R=\rho\cos\gamma$, $\mu=0$. Therefore $r=\rho$. (See Ames, *Johns Hopkins Circulars*, May, 1889.)

This mounting consists in having slit, grating, and camera at the vertices of a right angled triangle, the camera being placed at the center of curvature of the grating.

II. ASTIGMATISM.

One of the most important properties of a concave grating is its "astigmatism," *i. e.*, the fact that a point of light as a source gives rise to a focus, not a point, but a line. The advantages arising from this fact, as pointed out by Ames, (*Johns Hopkins Circulars*, May 1889) are:

1. A narrow spark at the slit is broadened out into a wide spectrum.

2. Greater accuracy in comparing metallic and solar lines.

3. No "dust lines," as they are brought to a different focus.

4. A spectrum is obtained which is broad enough to stand enlarging.

Although this property has always been recognized, no formula giving the amount of the astigmatism has ever been published, at least to my knowledge. This quantity is therefore deduced in the following pages.

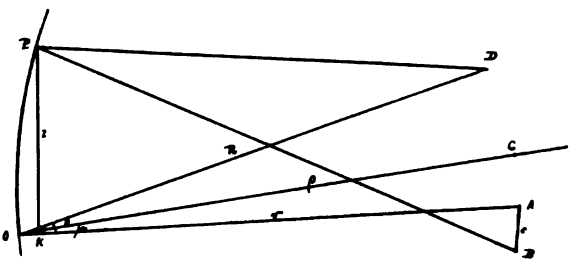


FIG. 2.

In Fig. 2 let D be the position of the point of light at the slit situated at a distance R from the center of the grating O; $A\,B$ is the spectral line under consideration, which is consequently parallel to the lines of the grating, and is situated at a distance r from the center of the grating; C is the center of curvature; $C\,O$ is the radius of curvature ρ. A, O, C, and D lie in a horizontal plane.

Let $P\,O$ be a vertical section of the grating, *i. e.*, parallel to the lines of the grating. A small pencil of rays from the slit D falling on the grating at O is brought to a focus at A; a small pencil from D falling on the grating at P is brought to a focus at B. It is our problem to find the length of $A\,B$, assuming $A\,B$ to be perpendicular to $O\,A$.

Call $A\,B = C$, $PK = Z$, where PK is the perpendicular let fall from P on the radius of curvature $O\,C$.

Since $O\,K$ is a very small quantity compared with the other quantities considered :

$$OK = \frac{Z^2}{2\rho}, \text{ and}$$

$$AK = r - \frac{Z^2}{2\rho} \cos \mu.$$

$$BP^2 = AK^2 + (KP + AB)^2.$$

$$= (r - \frac{Z^2}{2\rho} \cos \mu)^2 + (Z + C)^2.$$

If we neglect terms of a higher order than the second we shall have:

$$BP = r - \frac{Z^2}{2\rho} \cos \mu + \frac{Z^2 + 2CZ + C^2}{2r}.$$

Similarly we shall have:

$$DP = R - \frac{Z^2}{2\rho} \cos \gamma + \frac{Z^2}{2R}.$$

But since the point of light at D and the focal line AB are true foci, the differences in paths of rays falling at P and at O must be zero. Hence

$$(AO + DO) - (BP + DP) = 0$$

$$i.e. \frac{Z^2}{2}\left(\frac{\cos \mu}{\rho} + \frac{\cos \gamma}{\rho} - \frac{1}{R} - \frac{1}{r}\right) - \frac{CZ}{r} - \frac{C^2}{2r} = 0. \quad (13)$$

Solving we get:

$$C = -Z \pm Z\sqrt{r\left(\frac{\cos \mu + \cos \gamma}{\rho} - \frac{1}{R}\right)}. \quad (14)$$

Twice this value will give us the length of the line, which is parallel to the lines of the grating, when we have a point-source of light at the slit. But in practice the illuminated portion of the slit, placed parallel to the lines of the grating, is of some finite length $2b$, where b is the length of the illuminated portion of the slit above the horizontal plane. To find the length (c') of the lines of the spectrum due to this, we have merely to apply the ordinary equation for finding the magnification due to a concave mirror:

$$\frac{C'}{r} + \frac{b}{R} = 0$$

$$\text{or } C' = -\frac{br}{R}$$

Adding this to the value from equation (14) we find half the length of the spectral lines :

$$C_0 = -Z \pm Z\sqrt{r\left(\frac{\cos\mu + \cos\gamma}{\rho} - \frac{1}{R}\right) - \frac{br}{R}}.$$

Where Rowland's mounting is used the photographic plate is at the center of curvature, and hence

$$r = \rho,$$
$$\mu = 0,$$
$$R = \rho\cos\gamma.$$

Substituting these values in the above equation we get

$$C_0 = -\frac{b}{\cos\gamma} - Z \pm Z\sqrt{1 - \frac{\sin^2\gamma}{\cos\gamma}}.$$

Taking the special case when $\gamma = 0$, the image of the slit coincides with the slit itself, and hence $C_0 = -b$. Whence we see that the positive sign before the radical must be used. Hence

$$C_0 = -\frac{b}{\cos\gamma} - Z + Z\sqrt{1 - \frac{\sin^2\gamma}{\cos\gamma}}. \qquad (15)$$

From this equation it is seen that the astigmatism depends only on the length of the slit, the length of the ruled lines of the grating, and the angle which the source of light makes with the axis of the grating. Astigmatism is *independent* of the radius of curvature.

In this formula C_0 is half the length of the spectral line, b is half the length of the illuminated portion of the slit, Z is half the length of the ruled lines of the grating. The angle γ is computed from the wave-length by the formula:

$\lambda = \frac{\omega}{N}(\sin\gamma + \sin\mu)$, which for the case of Rowland's mounting, where $\mu = 0$, reduces to

$$\lambda = \frac{\omega}{N}\sin\gamma.$$

Measuring the lengths of the lines of the spectrum formed by the large concave grating spectroscope used in the Johns

Hopkins University, we get the following table, giving the lengths measured and those computed from the above formula. The width of the spectrum was measured by holding a scale in front of the camera box and observing by means of an eyepiece held in the hand.

The grating has a radius of curvature of $21^{\text{ft}}.5$, is ruled with 20,000 lines to the inch, and has a ruled surface of 2^{in} by $5^{\text{in}}.66$. The length of the slit used was 3^{mm}. In the following table the first column gives the line of the spectrum, the second the wave-length (λ), the third the order of spectrum (N), the fourth the value of γ, the fifth the computed length of the spectrum line λ, and the sixth its observed length.

Line	λ	N	γ	Computed	Observed
h	4102	1	18°50′38″	6 mm	6mm
G........................	4308	1	19 49 44	6.5	7
F........................	4861.5	1	22 30 25	8	9
b_1........................	5184	1	24 5 28	8.5	11
D	5893	1	27 38 47	10	15
C........................	6563	1	31 6 57	12.5	17
B	6767	1	32 11 50	13	19
h	4102	2	40 14 22	21	29
G........................	4308	2	42 43 14	24	32
F........................	4861.5	2	49 57 34	46	51

Special case.—When the concave grating is used directly as a star spectroscope, the source of light is at infinity, and the slit-length is zero. In this case the fundamental equation (12) reduces to

$$r = \frac{\rho \cos^2 \mu}{\cos \gamma + \cos \mu} \, .$$

Hence, making $R = \infty$, and giving r this value, equation (14) reduces to

$$C = -2Z \sin^2 \frac{\mu}{2} \, .$$

The grating is used as a star spectroscope in two ways, either the photographic plate is in the axis of the grating, when $\mu = 0$ for the center of the plate, or the photographic plate is

placed so as to make $\gamma=0$, *i. e.*, the light falls normally on the grating.

In the present work, with the direct grating spectroscope (see Poor and Mitchell, "The Concave Grating for Stellar Photography," this JOURNAL, March 1898), the large grating used had a ruled surface of 2^{in} by $5\frac{3}{4}^{\text{in}}$. The whole spectrum in the second order subtended an angle of about 6°. The grating was used in such a way that $\mu=0$, at the center of the plate; and consequently for a variation of 3° in μ, the astigmatism amounted at most to $0^{\text{in}}.003$.

If the light (in the other method of using the grating) falls normally on the grating, *i. e.*, if $\gamma=0$, then μ is about equal to 15°, and so the astigmatism amounts to about $0^{\text{in}}.07$.

Hence the grating used directly as a star spectroscope gives a spectrum which is exceedingly narrow.

III. FOCAL LINES.

By treating equation (13)

$$\frac{Z^2}{2}\left(\frac{\cos\gamma+\cos\mu}{\rho}-\frac{1}{R}-\frac{1}{r}\right)-C\frac{Z}{r}-\frac{C^2}{2r}=0$$

in a slightly different way, some interesting results are obtained. The equation gives the position of the focus conjugate to a point source; C will be zero for either of two conditions:

$$Z=0$$

$$\text{or}\quad \frac{\cos\gamma+\cos\mu}{\rho}-\frac{1}{R}-\frac{1}{r}=0 \tag{16}$$

That is, under either of these conditions, the focus is a point, not a line.

The meaning of equation (16) will be better understood if we regard the grating from another standpoint. Using the grating in the ordinary way, a cone of rays falls obliquely on its surface, and is diffracted and brought to a focus in a spectral line.

Thus we may consider the grating as some sort of a mechanism which changes a pencil of rays falling obliquely on the grat-

ing into a pencil leaving at some different angle. This action may be considered in two ways, according as we take a meridian section of the grating, *i. e.*, a section passing through the two focal points and the center of the grating; or, as we take an equatorial section, *i. e.*, a section of the grating at right angles to this. The equations of oblique pencils refracted at a spherical surface of a medium whose index of refraction is p have been fully treated and discussed by Czapski (Winkelmann, *Handbuch der Physik*, 2, 85, 86); and by Rayleigh (*Ency. Brit.*, 14, 800, art. "Optics").

These equations give the position of the *focal lines* for a meridian section and for an equatorial section. They are:

$$\frac{\cos^2\gamma}{R} - \frac{p\cos^2\mu}{r} = \frac{\cos\gamma - p\cos\mu}{\rho}, \tag{17}$$

$$\frac{1}{R'} - \frac{p}{r'} = \frac{\cos\gamma - p\cos\mu}{\rho}, \tag{18}$$

where R and R' are the distances of the radiant point from a point in the meridian and equatorial sections respectively, r and r' are the distances of the focal lines from the same point of the surface, ρ is the radius of curvature, γ and μ are the angles which the incident and refracted rays make with the normal at the surface.

Applying this to the case of the concave grating by putting $p = -1$, we get

$$\frac{\cos^2\gamma}{R} + \frac{\cos^2\mu}{r} = \frac{\cos\gamma + \cos\mu}{\rho}, \tag{19}$$

$$\frac{1}{R'} + \frac{1}{r'} = \frac{\cos\gamma + \cos\mu}{\rho}. \tag{20}$$

Equation (19) fixes the positions of the radiant and focal lines for a meridian section; equation (20), the position for an equatorial section. As the grating is generally used, the meridian section becomes a horizontal one, the equatorial section a vertical one. Thus equation (19) means that, if we use a horizontal section of the grating—a section perpendicular to the lines of the grating—we get the position of the conjugate foci;

a vertical line at the slit with coördinates R and γ is brought to a focus on the photographic plate in a vertical line whose coördinates are r and μ. This is the same equation as Rowland's. Equation (20) (which is identical with equation (16) gives the position of the radiant and focal lines using a vertical section of the grating, *i. e.*, a section parallel to the lines of the grating. This equation is satisfied by making $r' = \rho \sec \mu$, $R' = \rho \sec \gamma$; or making $\mu = 0$ (as in Rowland's mounting) by $r' = \rho$, $R' = \rho \sec \gamma$. From the theory of these focal lines (see *Ency. Brit.*, *loc. cit.*) we see that corresponding to a point-source this focal line is horizontal, and conversely. Consequently, using a horizontal slit at a distance $\rho \sec \gamma$, and the angle γ from the grating, light from it, after passing through the vertical slit, will be brought to a focus in a point in the spectrum. This fact is shown by placing the source of light at some distance in front of the slit (*i. e.*, away from the grating), and by placing an obstacle, for example a knitting needle, horizontally in front of the vertical slit and distant $\rho \sec \gamma$ from the grating. Doing this, we shall get a sharply defined horizontal line running across the spectrum. (See Sirks, *Astronomy and Astro-Physics*, 13, 1894.)

JOHNS HOPKINS UNIVERSITY.

Minor Contributions and Notes.

THE HYDROGEN ATMOSPHERE SURROUNDING THE WOLF-RAYET STAR D. M. $+30^{\circ}3639$.

WHILE examining various star spectra with the 36-inch refractor on the night of June 28, I was much interested in confirming Professor Campbell's discovery of a hydrogen envelope around the Wolf-Rayet star *DM.* $+30^{\circ}3639$.[1] With the collimator adjusted so that the spectrum was linear at $H\beta$, and with the slit wide, the $H\beta$ line appeared as a circular, fairly well defined disk, which narrowed to a fine line crossing the linear star spectrum when the slit was closed up.

That this appearance is not an illusion of any kind produced, for example, by irradiation, is proved by the following experiments. (1) When the the spectrum is made linear at the blue band λ 4652, which is nearly as bright as $H\beta$, no such appearance as that described above is observed. The band is reduced to a mere line coincident with the star spectrum. This band is not much narrowed by closing the slit. (2) With the collimator adjusted so as to make the spectrum linear at $H\beta$ and with the slit narrow, the star can be thrown off the slit by slightly displacing the telescope in declination, so that the continuous spectrum disappears. The short and narrow $H\beta$ line remains visible, however, until the slit is displaced by more than the radius of the hydrogen envelope. This latter experiment, in particular, seems to be conclusive as to the reality of the observed phenomenon.

With a large reflector I think that this hydrogen envelope could be observed visually without a spectroscope, perhaps with the aid of a piece of blue glass. With a refractor the small disk is confused with the circles of chromatic aberration when the focus is adjusted for $H\beta$.

The existence of such an extensive hydrogen envelope around a Wolf-Rayet star has an important bearing on theories of bright-line stars, as Mr. Campbell has already pointed out.

JAMES E. KEELER.

LICK OBSERVATORY,
July 6, 1898.

[1] *A. and A.*, 12, 913, 1893; *A. N.*, 3200.

NOTE ON PROFESSOR WILSING'S ARTICLE ON THE EFFECT OF PRESSURE ON WAVE-LENGTH.[1]

J. WILSING has pointed out that radiation from a vibrator has the additional effect of lowering the natural frequency of vibration. In the case of a circular disk, backed by a spring and vibrating in air, we have the equation of motion ready to hand (Lord Rayleigh's *Sound*, 2, 196).

$$\left(1+\frac{8\sigma R^3}{3M}\right)\ddot{\xi}+\frac{\sigma\pi p^2 R^4}{2aM}\dot{\xi}+p^2\xi=0.$$

Here σ is the density of air.

M is the mass of the disk.

a is the velocity of sound.

R is the radius of the disk.

This may be written, in Wilsing's notation:

$$(1+\gamma)\ddot{\xi}+2K\dot{\xi}+p^2\xi=0$$

where

$$\gamma=\frac{8\sigma R^3}{3M}$$

$$2K=\frac{\sigma\pi p^2 R^4}{2aM}$$

Now δp_1, the shift of the line due to radiation, is of the order γp.

Furthermore, there is a widening δp_2 due to the accompanying damping. The half-width of the line is of order K.

But

$$\gamma p : K = \frac{8\sigma p R^3}{3M} : \frac{\sigma\pi p^2 R^4}{2aM}$$

$$=\frac{8}{3} : \frac{\pi p R}{2a}$$

This is of order $\lambda : R$, where λ is the length of the emitted waves.

Now the diameter of a molecule is small compared with the wave-length of visible light. Therefore, if we can regard the acoustical analogy as pertinent, we should expect that the effect of radiation in shifting a spectrum line will be great compared with its effect in widening the line.

CHARLES GODFREY.

TRINITY COLLEGE, CAMBRIDGE,
July 7, 1898.

[1] This JOURNAL, May 1898.

THE NOVEMBER METEORS.[1]

On the night of November 13, 1897, ninety-one meteors were observed at the Harvard College Observatory, and forty-seven meteors at an auxiliary station twelve miles south, the Blue Hill Meteorological Observatory. A discussion of these observations by Professor W. H. Pickering will be found in the *Annals* of this Observatory, Vol. XLI, No. 5. A much greater display of meteors is expected next year, and it is very important that a continuous watch should be kept during the two or three days in which the Earth is passing through the denser portion of the meteor stream. This can only be done by establishing a series of stations in various longitudes, so that during the entire time one or more of these stations shall fulfill the conditions that the radiant point shall be above the horizon and the Sun below. Correspondence is invited with astronomers and others willing to participate in this work, especially with those who will be in the less frequented longitudes. If the weather is favorable, and the plan here proposed is carried out satisfactorily, it is expected that all the observations will be discussed here, and published in the *Annals* of this Observatory. To secure the best results a uniform plan of work is essential. Maps and forms of record will be sent to all who early signify their readiness to take part in this work. The radiant point of the meteors indicated by the cross in the accompanying map, will not rise in this latitude until $10^h\ 30^m$, and twilight will interfere at about $5^h\ 30^m$ in the morning. As the shower sometimes begins before the predicted date, a watch should be kept on November 11 and 12, from 11 to 1 o'clock, and if many meteors are seen the observations described below, for November 13, should be made on these nights, and also on the nights following the shower.

Each observer is requested to devote his attention to the region within 25° of the radiant point, and included in the map, and to send the following data regarding his observations: Name of observer, location of station, post office address, time of beginning and ending of observations, interruptions by clouds or other causes, condition of sky, as clear, hazy, passing clouds, etc.

The observations most desired are those required to determine the frequency of the meteors. They are of extreme simplicity and

[1] *Harvard College Observatory Circular* No. 31.

fourth type. The bands, however, are of shorter wave-length, and are, perhaps, identical with those of a star of the third type. The portion of the spectrum whose wave-length is less than $H\gamma$, 4341, is too faint to appear in the photographs.

$9^h\ 6^m.1$. Position for 1875, R. A. $= 9^h\ 5^m\ 50^s.3$, Dec. $= -69^\circ\ 25'\ 58''$.

$9^h\ 13^m.5$. Position for 1875, R. A. $= 9^h\ 12^m\ 59^s.8$, Dec. $= -65^\circ\ 42'\ 37''$.

$10^h\ 17^m.2$. The lines $H\beta$ and $H\gamma$ in this star have been found to be variable by Miss A. J. Cannon. On June 2, 1893, they were bright, and superposed on a broad dark band. On April 17, 1895, and March 17, 1896, these lines, like the other hydrogen lines, were dark.

$10^h\ 40^m.1$. The hydrogen lines in this star appear to be variable. On May 20, 1892, $H\beta$, $H\gamma$, and $H\delta$ were dark. On April 3, 1895, the line $H\beta$ was bright. On April 21, 1895, $H\beta$ and $H\gamma$ were bright. $H\zeta$ and $H\eta$ were dark, with the edge of greater wave-length apparently bright.

$10^h\ 42^m.6$. Position for 1875, R. A. $= 10^h\ 41^m\ 40^s.8$, Dec. $= -64^\circ\ 57'\ 20''$.

$11^h\ 55^m.3$. Position for 1875, R. A. $= 11^h\ 54^m\ 3^s.1$, Dec. $= -54^\circ\ 25'\ 24''$.

$13^h\ 26^m.8$. Position for 1875, R. A. $= 13^h\ 25^m\ 6^s.2$, Dec. $= -61^\circ\ 40'\ 30''$.

$13^h\ 29^m.1$. Position for 1875, R. A. $= 13^h\ 27^m\ 23^s.2$, Dec. $= -61^\circ\ 37'\ 4''$.

$15^h\ 26^m.7$. Position for 1875, R. A. $= 15^h\ 24^m\ 8^s.2$, Dec. $= -71^\circ\ 29'\ 26''$. This is the object whose position for 1900 is announced in *Astronomy and Astro-Physics*, 12, 546, as R. A. $= 15^h\ 27^m.0$, Dec. $= -71^\circ\ 32'$. Its spectrum is there described as of the third type having also bright hydrogen lines, and consequently it was suspected of variability. Later and better photographs show that the spectrum is continuous, with the hydrogen lines $H\beta$, $H\gamma$, $H\delta$, $H\epsilon$, and $H\zeta$ bright, resembling that of η Carinae as seen with the same dispersion. The variability is not confirmed.

$19^h\ 47^m.0$. Position for 1875, R. A. $= 19^h\ 44^m\ 37^s.0$, Dec. $= -65^\circ\ 41'\ 9''$.

$21^h\ 36^m.6$. Position for 1875, R. A. $= 21^h\ 34^m\ 37^s.9$, Dec. $= -65^\circ\ 37'\ 28''$.

STARS RESEMBLING ζ PUPPIS.

Besides ζ Puppis, the additional lines due to hydrogen are present in the spectra of the first and second classes of stars of the fifth type (*A. N.* **127**, 1), and in the spectra of the first group of stars of the Orion type according to the classification of Miss Maury (*H. C. O. Annals*, **28**, 15). The lines whose wave-lengths are there given as 4200.3, 4542.7, are probably additional hydrogen lines, and the line 4685.4 probably coincides with the bright band in ζ Puppis. Miss A. J. Cannon, from a careful study of the Draper Memorial photographs of the bright southern stars, has added the following stars to those already known to contain these lines. In *A. G. C.* 17572 the lines 3925, 4027, 4202, and 4544 are present and dark. In the fifth type stars, *A. G. C.* 8631 and 22763, the lines 4027, 4202, and 4544, and the bands 4633 and 4688 are present and bright. In the fifth type stars, *A. G. C.* 10863, 22748, and 22843, the hydrogen lines 3925, 4027, 4202, and 4544 are present and dark, and the bands 4633 and 4688 are present and bright. The band 4633 is double in the last two stars, and also in *A. G. C.* 9311, 29 Canis Majoris.

EDWARD C. PICKERING.

June 21, 1898.

A CHROMOSPHERIC LINE NEAR K.

UPON nearly all plates of the spectrum of the Sun's limb, taken for my investigation of "The Rotation Period of the Sun, and the Motions of the Sun's Atmosphere" (in course of preparation for publication), there appears, more or less faintly against the shading of the K line, a chromospheric line at wave-length 3934.954.

This line is generally stronger upon the plates showing the double reversal of the K line the most distinctly, but there does not seem to be a complete correspondence, and upon plates taken over Sun-spots where the K line is very strongly reversed, there are no traces whatever of this chromospheric line.

In the second order spectrum of the grating used, the first order ghost of the K line falls not far from the place occupied by this line, and the suspicion that the line in question might be a ghost of the reversal of K was at first entertained, being rendered more probable by the nebulous character of the line. But the wave-length is somewhat different, being 3934.954, while the first order ghost (on a

second order spectrum plate) of the violet component of the reversal is at 3935.569, and the red component at 3935.983. Besides, there is no corresponding bright line on the violet side of K, and the wave-length of the chromospheric line is the same on plates of the second and third order spectrum, which would not be the case were it a ghost (the actual distance apart of ghosts and the central line being the same on plates of the first, second, and third order spectrum, but not the difference in wave-length).

The line in question does not occur except upon plates of the spectrum of the Sun's limb. In Shackleton's flash spectrum it is not distinguishable from the K line. Professor C. A. Young writes that upon looking up some photographs of his, taken some years ago, a similar line shows, but he thinks it undoubtedly a ghost of K. Probably upon his plates the line is mixed up with a ghost of K, but with my own plates there is no coincidence and no question whatever as to the chromospheric origin of the line.

LEWIS E. JEWELL.

PHOTOGRAPH OF THE SPECTRUM OF THE "FLASH" MADE BY PROFESSOR K. D. NAEGAMVALA AT THE ECLIPSE OF JANUARY 21, 1898.

PLATE I is a negative reproduction of the spectrum of the "flash" as photographed by Professor Naegamvala at the last solar eclipse. In a recent letter Professor Naegamvala, who was stationed at Jeur, gives the following details regarding the photograph.

"The flash was visually observed with a small direct-vision combination and the signal was given to expose. The exposure was made off and on by hand and must have taken half a second, and the signal must have been taken up also half a second after it was given.

"The photograph was taken on an Edward Isochromatic plate, with a six-inch Taylor-Cooke Triplet and two objective prisms, 45°, 8 × 6 inches face.

"The ultra-violet part of the spectrum is not as extensive as might have been thought, but this is evidently due to the absorption in the lens. On the original plate the spectrum is quite conspicuous from D_3 to $H\eta$.

"The definition is unfortunately not very good. This is due to

the very few opportunities I had of adjusting the instrument, it having arrived so late as the 11th of January."

In justice to the photograph sent by Professor Naegamvala it should be stated that the half-tone cut fails to bring out all the fine lines shown on the original. Perhaps the most interesting feature of the photograph is the prominence shown in two lines between H and $H\delta$, but invisible in H and K and the hydrogen lines.

NOTE ON NEW GASES IN THE EARTH'S ATMOSPHERE.

MESSRS. RAMSAY AND TRAVERS communicated to the Royal Society on June 3 a preliminary note on *Krypton*, the new element they have discovered in the air. From $750^{\text{cu cm}}$ of liquid air they succeeded in obtaining $26^{\text{cu cm}}$ of a gas which showed, in addition to a feeble argon spectrum, two brilliant lines, one at $\lambda 5869$ (very near the D_3 line), and the other at $\lambda 5570$, with a fainter line at $\lambda 5557$. The separation of the other lines from those of argon was difficult, but those lines were assigned to the new gas which were invisible or very faint in the argon spectrum when the two spectra were simultaneously compared. The approximate wave-lengths are

$\lambda 4317$	$\lambda 4807$	$\lambda 5557$
4387	4830	5570
4461	4834	5829
4671	4909	5869 D_4
4736		6011

The density of the sample was found to be about 23, but the discoverers venture to conjecture that the density of the pure gas will turn out to be 40, with an atomic weight of 80, thus falling into the helium series. By the comparison of the length of a sound wave in it and in air, krypton is shown to be monatomic and simple.

Sir William Huggins had privately called our attention to the coincidence of the line at $\lambda 5570$ with the principal line of the aurora, as has also been noted by Schuster and by Berthelot. Thus it seems that at last the true origin of that hitherto perplexing line has been discovered.

At the meeting of the Royal Society on June 16, Professor Ramsay announced the presence in atmospheric argon of two new companion gases, to which he has assigned the names *neon* and *metargon*.

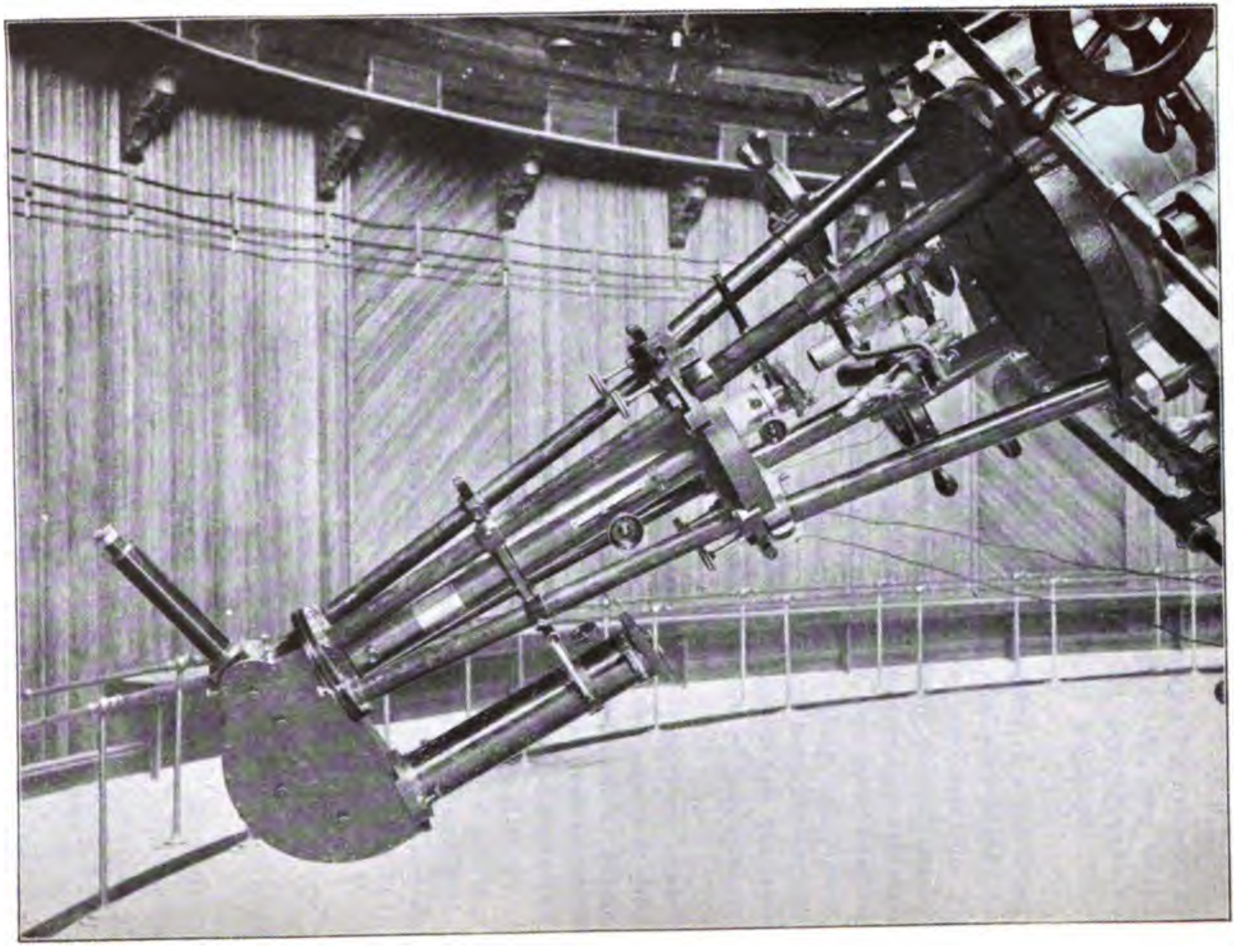
The spectrum of neon is described as containing a large number of strong lines in the red, orange, and yellow, and in the deep violet. For the line at λ5850, near the helium line D_3 (λ5876) and the krypton line D_4 (λ5869), the designation D_5 is suggested. The density of the sample was about 14.

The density of metargon is given as 19.87, that of argon being 19.94. Its spectrum showed numerous bands, the wave-lengths of which are closely coincident with those in the band spectrum of carbon, with three cyanogen bands, as was promptly pointed out by Schuster (*Nature*, **58**, 199, June 30). Thus the spectroscopic evidence is strong that metargon is not a new element, but some form of carbon. In a subsequent number of *Nature* (July 14) Messrs. Ramsay, Travers, and Baly cite several chemical tests tending to exclude the possibility of carbon from the supposed new gas, and consider that these facts will necessitate a suspension of judgment on the question.

At the session of the French Academy on June 13, a sealed package, deposited in May 1896, was opened and the paper enclosed was presented by its authors, Messrs. Moissan and Deslandres. While experimenting in 1896 on the gases obtained from the mineral *cerite* they found a residual gas which showed lines of helium and argon and lines hitherto unknown, at λ4151.7, 4143.7, 4110.0, 4108.0, 4100.5.

They expressed the opinion that these were either new nitrogen lines, peculiar to low pressure, or else were due to a new atmospheric gas resembling nitrogen in chemical characteristics—with a preference for the latter view. M. Moissan now states the belief that the lines are not due to krypton.

E. B. F.



THE MILLS SPECTROGRAPH OF THE LICK OBSERVATORY.

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THE MILLS SPECTROGRAPH OF THE LICK OBSERVATORY.

By W. W. Campbell.

It has long been planned that the great light-gathering power of the 36-inch refractor should be utilized for the spectroscopic determination of stellar velocities in the line of sight. Professor Keeler secured measures of the velocities of α Boötis, α Tauri, and α Orionis by visual methods in 1890–91 in connection with his observations of nebulæ, and a few other measures were made in 1891 by Dr. Crew and myself. The difficult character of the observations soon became forcibly evident. Even with the powerful telescope at command, it was clear that the number of stellar spectra suitable for accurate visual measurement was very small. The questionable value of previous measures secured with small telescopes was of itself sufficient warning not to expect much from the visual method. The signal success obtained at Harvard College Observatory by the photographic processes of recording spectra, and later the remarkable advance in accuracy of velocity determinations resulting from the employment of the photographic method at Potsdam, strongly discouraged further attempts to carry on the work visually.

The original star spectroscope of the Lick Observatory, though admirable for visual work, was not adapted to photo-

graphic observations on account of its great flexure even during short exposures. It was hoped to determine velocities by using a powerful Rowland plane grating, and several star spectra were thus photographed; but flexure in the framework of the instrument ruined the definition, and the attempt was abandoned.[1] Nevertheless, the instrument was used for photographing the prismatic spectra of nebulæ, new stars, Wolf-Rayet stars, etc., with more or less success. The experience thus gained was very useful in designing a new instrument suitable for efficient spectrographic work.

Thanks to the generosity of D. O. Mills, Esq., Professor Holden was able to provide the Lick Observatory with a spectrograph suitable for velocity determinations. It is now in regular use, and results of great accuracy are being obtained rapidly. It may be of value to describe the instrument and the methods of observation and reduction employed.

In designing the instrument, my constant effort was to adapt it to the determination of stellar velocities, utilizing the $H\gamma$ region of the spectrum. All other considerations were sacrificed if they in any way seemed to interfere with its efficiency or convenience for that line of work. Nevertheless, many of its features remain such that it can be converted easily and cheaply into a so-called universal spectroscope.

A large telescope has a great advantage in light-gathering power. It likewise has its own peculiar difficulties to be overcome. It might seem at first thought that a well-designed spectrograph of ordinary dimensions would be equally efficient with all telescopes whose angular apertures were equal to that of the collimator lens: in other words, that an instrument suitably designed for use on a telescope of 12 inches aperture and 19.3 feet focus, would be an efficient instrument in connection with a

[1] Judging from my experiments on the subject, the prospect is very promising for securing accurate velocity determinations of the brighter stars by means of diffraction gratings in combination with powerful telescopes. The principal difficulty to be overcome does not arise from flexure, but from the mounting of the grating. The problem of maintaining the reflection grating in position is analogous to securing the mirror of a reflecting telescope so that it cannot move, and yet be uncramped.

telescope of 36 inches aperture and 58 feet focus. But such is not necessarily the case. When our atmospheric conditions are excellent, the focal images of a star are substantially of the same size in both telescopes, and the same linear width of slit suffices for efficiency in the two cases. But the instrument must also be efficient under *average* atmospheric conditions. Now in average seeing the larger telescope forms a larger image of a star than the smaller telescope does. In order to utilize the same proportion of the light in each case, it is clear that the slit-width must be greater for the larger telescope than for the small. Therefore, to preserve the purity, the larger telescope requires the longer collimator. The length of collimator in this instrument was limited only by the size of the prisms which I thought it advisable to employ. It was decided to use a train of three dense flint prisms of such a size that they would properly receive from the collimator a circular beam of $H\gamma$ light $37^{mm}.4$ in diameter (about 1½ inches), and of such a density and angle that they would deviate it about 180°. The corresponding length of collimator would be 724^{mm} (about 28½ inches).

After the instrument arrived, I placed the slit in the focus of the collimator lens by Schuster's method, and measured the focal length of the lens in the following manner. The collimator section was separated from the rest of the instrument. A small, sensitive plate, cut to the proper shape, was pressed against the wide-open slit. The object-glass was directed to the sky immediately west of the Pleiades, and that group of stars allowed to drift across the line of sight. The trail of 20 Tauri passed centrally through the slit aperture. The distances of six other trails from that of 20 Tauri were measured with a micrometer. The focal length was computed from the formula

$$f = \frac{R - R_0}{\tan\,(\delta - \delta_0)}, \qquad (1)$$

in which δ_0 is the apparent declination of 20 Tauri, R_0 the micrometer reading on that star's trail, and δ and R the corresponding quantities for each of the other stars. The focal lengths obtained were

From 18 Tauri	$f = 722^{mm}.3$
21 "	722 .8
28 "	722 .0
"	722 .5
27 "	722 .1
23 "	722 .7
	Mean $722^{mm}.4$

This is in close agreement with the specification dimension (28.5 inches $= 723^{mm}.9$) furnished Mr. Brashear.

The diameter of the object-glass of the great refractor is $91^{cm}.44$ (36 inches) and its focal length for $H\gamma$ rays is 1768^{cm} (58 feet). Whence it follows that the effective stellar aperture of the collimator lens is $37^{mm}.4$. The lens is double, of Jena glass, cemented, with clear aperture $44^{mm}.5$ stopped down to 38^{mm}.

The camera lens first used was a double one of Jena glass, cemented, of 16 inches focus. For reasons given in the sequel, it has been replaced by a triple lens.

Mr. Wright has, at my request, determined its focal length by means of star trails. He found

$$f = 406^{mm}.0$$
$$405\ .0$$
$$\text{Mean } 405^{mm}.5$$

A second difficulty peculiar to a large refractor relates to the question of efficient guiding. A consideration of the color curve of the large object-glass will show the nature of the difficulty. The focal length is 48^{mm} greater for the $H\gamma$ rays than for the D rays, and the curve at $H\gamma$ is very steep. When the slit is placed in the $H\gamma$ focus the $H\gamma$ image is a point in the center of a large disk of light made up not only of the so-called visual rays, but also of the other blue rays. The central point cannot be clearly distinguished. It was feared that Dr. Huggins' simple method of guiding by means of reflecting-slit plates would not be efficient under these circumstances. Professor Vogel's method of viewing the slit by means of light reflected from the first prism surface would not answer: the image of a star in the

guiding telescope would be a long line with a slightly blue central portion. Guiding by means of a small diagonal telescope immediately in front of the plate would be satisfactory only in case it were inserted near the $H\gamma$ line. To insert the telescope at $H\gamma$ every few minutes would stop the exposure during the moments of guiding, and there would be more or less imperfect following between times. Likewise the danger of jarring the instrument, or of temperature changes from the proximity of the observer, would be objectionable. After numerous experiments I decided to utilize the light reflected from the first prism surface to form a guiding spectrum. The reflected light passes first through a 30° prism, and thence into the guiding telescope. The spectrum thus formed is, of course, not linear, but the region at $H\gamma$ is linear, since the $H\gamma$ rays are in focus on the slit. If the $H\gamma$ image is not in the slit there is a vacancy at $H\gamma$ in the guiding spectrum. An occulting bar in the eyepiece covers all the spectrum except the $H\gamma$ region. The light from this region alone is visible, and is easily kept at a maximum.[1]

Such, in brief, were the optical plans and dimensions adopted. The general specifications for the mounting were written by myself, though some of them were modified at Mr. Brashear's suggestion. Thus, while I desired to have the collimator enclosed in a trussed system of five inclined steel tubes, as in the Potsdam spectrograph, the number was reduced to four, for constructive reasons. Many of the parts which I desired to have of steel were constructed of brass and cast iron, on Mr. Brashear's advice, and the instrument seems to have suffered no loss thereby. The details of mechanical construction were largely left to Mr. Brashear to design.

I did not hesitate to adopt what seemed best in the applicable parts of existing spectroscopes. Many of the features are common to all instruments, and many were suggested to me by my experience of several years with imperfect spectrographs on the large telescope. A few of the features were suggested by

[1] A description of this method of guiding was published by me in 1895 — this JOURNAL, 2, 127 — but is inserted here for completeness.

the Potsdam spectrograph, and many others by Professor Keeler's Allegheny spectroscope.

Three nights a week are devoted to spectroscopy with the great refractor, and the other nights to micrometer work, etc. It is therefore necessary to mount and dismount the spectrograph once a week. This gives rise to less inconvenience than might be expected, since everything is planned with that in view. The change from micrometer to spectrograph—and *vice versa*—is made by one person with ease and safety in six minutes. When the instrument is not in use it rests on its special case. This is provided with rubber-bound casters which dampen the jarring when the case is slowly moved. When the instrument is to be mounted on the telescope, the moving floor is brought to its highest level, and the telescope is pointed to hour angle zero and declination—17½ degrees. This leaves the eyepiece about three feet above the floor, and the telescope makes an angle of 35 degrees with the horizon. The top of the spectrograph case is inclined at the same angle. The upper piece of the case is a strongly constructed framework supported on roller bearings. The framework can be moved lengthwise about three inches between guide rails by means of a lever. This movable framework carries the instrument. The lower space of the case is used for storage drawers and the Leyden jar, while the thermometer, Ruhmkorff coil and other pieces of apparatus are mounted on its sides. When the telescope has been made fast to a heavy safety weight on the floor, the micrometer weighing about 40 pounds, and a section of the telescope tailpiece weighing 85 pounds, are taken off. The spectrograph case is rolled up to the end of the telescope, and the observer, with one hand on the lever, forces the spectrograph into position, so that four lugs on the spectrograph pass over four bolts on the telescope, and, with the other hand, he screws on the four nuts. The removal of a 39-pound counter weight from the telescope completes the operation.

The end plate of the telescope is about 16 inches above the $H\gamma$ focus. It was necessary to design an extension of the tele-

scope reaching down to the point where the framework of the spectrograph must begin. This extension is left clamped to the spectrograph, but is not considered to be a part of the spectrograph. It is a part of the telescope, and remains with the spectrograph only for convenience. It consists of two circular cast iron plates, 18¾ and 12 inches in diameter, firmly joined by four inclined hollow steel tubes 1½ inches in diameter. Passing through the upper plate are four large steel screws which press against the end of the telescope and serve to collimate the instrument. The inclined rods carry the comparison apparatus for hydrogen and iron, and the lower end plate supports the electric switch board. Thus, none of the parts which require considerable handling are fixed to the spectrograph itself. The weight of this section, including the comparison apparatus, is 70 pounds, and its length is 21½ inches.

The spectrograph proper is attached to the upper section by four hinged clamps. The truss carrying the collimator consists of two brass end-plates 12 and 6¾ inches in diameter, and an intermediate web, all rigidly joined together by four inclined, hollow steel tubes 1¼ inches in diameter. The length of this truss is 23¼ inches. The tube which encases the collimator is securely fastened to the truss. The collimator as a whole is movable by rack and pinion in this tube through a range of 90^{mm}, and can be clamped at any desired reading of the millimeter scale with which it is furnished. The weight of the collimator section is forty-eight and one-half pounds.

The slit is similar to that of the original Lick Observatory spectroscope.[1] The jaws are opened and closed symmetrically with reference to the axis by means of a right and left screw whose pitch is $0^{mm}.2$. The milled head of the screw is divided to twenty parts, so that the slit-width corresponding to one division is $0^{mm}.02$. Immediately below the slit is a pair of jaws for varying the length of slit. They are moved symmetrically from or toward the center by a rack and pinion. Just behind these jaws is a diagonal eyepiece for viewing the slit. It moves

[1] Described by Professor Keeler in *Publications of the Lick Observatory*, 3, 174.

by a rack and pinion in a bearing-tube with perfect ease so that no undesirable strains are induced in the slit mechanism. As in Professor Keeler's Allegheny spectroscope, there is a thin wedge of brass mounted immediately in front of the center of the slit to protect the area on the plate occupied by the star spectrum while the comparison spectrum is photographing. An adjustable "stop" screw at the base of the wedge enables the observer to occult any desired central section of the slit. The slit apparatus as a whole is movable in the axis of the collimator by rack and pinion and can be clamped at any desired reading of a millimeter scale.

The comparison apparatus is well shown in the illustrations. The hydrogen tube is firmly mounted in the plane of the collimator axis and slit, and at an angle of 30° with the axis. An image lens of large angular aperture directs the light upon a 60° totally-reflecting prism, which sends it into the collimator. The iron electrodes and their image lens are mounted on a frame at one side of the cone of light from the object-glass. They are moved by rack and pinion into the collimator axis immediately in front of the slit when needed.

The prism box is built upon a strong brass bed-plate. One end of the plate is enlarged to the same size (six and three-fourths inches diameter) as the end-plate of the collimator truss. A turned ring on the end-plate fits neatly into a turned bearing in the prism-box plate to prevent lateral motion. The two plates are firmly held together by two strong capstan-headed clamping screws, very much as in the case of the Allegheny spectroscope. The other end of the prism-box plate receives the lower end of the camera, the camera projecting through the plate so as to bring the lens as close as possible to the third prism. The face-plates of the box are of sheet brass, held in position by brass webs, one bordering the bases of the prisms, and the other forming the curved outer face of the box.

The prisms are borne by strong mountings furnished with all necessary adjusting screws. They are pressed against their bed-plates by long, shallow springs which exert gentle pressures over

large areas of the prism ends. Each bed-plate is clamped by two strong thumbscrews to the adjacent face-plate of the prism box, and some additional support is lent by an abutting screw which passes through the opposite face-plate, and presses lightly upon the prism mounting. The 30° prism, which forms the guiding spectrum, is similarly mounted. There is no minimum deviation mechanism, as the instrument was designed for use in the $H\gamma$ region exclusively. The prisms are therefore adjusted to minimum deviation for $H\gamma$ and clamped. The weight of the prism box complete and guiding telescope is nineteen and one-half pounds.

The bearing surfaces at the lower end of the camera tube form a small section of a ball and socket joint. From exactly opposite points of it, small cylindrical rods project and rest in bearings which can be clamped. The whole camera can rotate about these rods through a small angle in the direction of the length of spectrum. The upper end of the camera is supported by two half-inch steel rods projecting out from the collimator truss. The camera can be moved to any desired position on these rods, and clamped. The outer end of the camera is provided with rack and pinion and millimeter scale for focusing, and with a shield for holding and clamping the brass plate holders. The aperture in the base of the shield, through which the light is admitted to the plate, is closed to exclude dust, when the instrument is not in use, by a thin brass slide which works in a groove. When the instrument is in use, the slide is drawn to one side and held by a spring-pin. The camera complete, two steel supporting rods, and one brass plate holder, weigh seven pounds.

Another camera, thirty-two inches long, is provided for the instrument. It is intended for use with the brighter stars which have fine lines in their spectra. It has not yet been tested, but will soon be brought into use.

The prism box and camera may be rotated 180° to the other side of the collimator truss. The collimator section is complete in itself, and may be rotated 180° on the upper section. The

upper section may be rotated through any angle along with the tail-piece of the telescope. It might occur that an instrument with a longer or shorter collimator would be desired for special purposes. Such an instrument could be attached at once to the present upper section, with the comparison apparatus ready for use.

The instrument was first used in May 1895, to determine the velocities in the system of Saturn. Excellent results were secured,[1] fully confirming the earlier results by Professor Keeler. The definition at the center of the field was excellent, but it was noticed that the plate holder required tipping through a large angle, in order to bring even a small range of spectrum into good focus. This was unexpected, as the lenses were ordered corrected for $H\gamma$. The adjustments were varied within all possible limits, without producing perceptible improvement. The difficulty seemed to lie in the lenses, and they were returned to Mr. Brashear for testing. The formulæ followed in their construction were confirmed by Professor Hastings. The lenses were examined by Mr. Brashear and tested by Professor Keeler, who found that they were properly corrected for $H\gamma$. When the lenses were returned I constructed considerable auxiliary apparatus to enable me to determine their color curves[2] by Schuster's method, using one prism.

The column "observed" in the following table contains the readings of the mm. scales resulting from several independent determinations of the foci. These were plotted with reference to the corresponding wave-lengths, and represented as well as possible by smooth curves, which furnish the numbers in the column "computed." The residuals, observed minus computed, are contained in the next column, followed by the corresponding "ordinates" to the color curve. The last two columns contain Professor Keeler's determination of the color curve of the camera lens when it was returned to Allegheny.

[1] See this JOURNAL, 2, 127–135.

[2] The original arrangement of the prism box did not permit this to be done.

	$722^{mm}.4$ collimator				$405^{mm}.5$ camera				$405^{mm}.5$ camera	
Line	Observed	Computed	O–C	Ordinates	Observed	Computed	O–C	Ordinates	O–C	Ordinates
$H\epsilon$	12.22	12.15	+.07	+0.14	14.17	14.10	+.07	+0.21	−.07	+0.13
$H\delta$	12.00	12.07	−.07	+0.06	13.94	13.98	−.04	+0.09	−.02	+0.05
λ 423 ...	12.10	12.03	+.07	+0.02	13.98	13.92	+.06	+0.03		
$H\gamma$	11.98	12.01	−.03	+0.00	13.86	13.89	−.03	0.00	.00	0.00
λ 447 ...	12.12	12.05	+.07	+0.04	13.96	13.93	+.03	+0.04		
λ 460 ...		12.13		+0.12		13.98		+0.09	+.12	+0.14
$H\beta$	12.3	12.37	−.07	+0.36	14.03	14.12	−.09	+0.23	−.03	+0.33
b_1	12.8	12.80	.00	+0.79	14.47	14.36	+.11	+0.47	+.17	+0.63
D	14.3	14.20	+.10	+2.19	15.16	15.10	+.06	+1.21	−.02	+1.35
$H\alpha$	15.3	15.45	−.15	+3.44	15.65	15.75	−.10	+1.86	.00	+2.10

Thus the color curves of the lenses were found to be correct, and the cause of the trouble remained a mystery. Could it lie in the prisms? If the surfaces were not flat, or if certain peculiarities of density prevailed, the prisms might act as under-corrected lenses and produce the observed effect. The prisms were removed from their mountings and the surfaces tested. They were found to be flat. In remounting the prisms I noticed that two of the shallow bent springs which press the prisms against their bases were too long. When the cross bars which hold them in position were forced home by the screws, the springs were unable to straighten, and therefore subjected the prisms to enormous pressure. Some improvement in the spectrum was noted when the springs were shortened, but the plates still required considerable tipping. I then tested the prisms separately in connection with the old spectroscope and found them satisfactory. I was then convinced that the difficulty lay in the camera lens. As the definition was satisfactory over the central 10^{mm} of the plate the work was allowed to proceed, and the velocities obtained were in excellent agreement.

The observed effects seemed to indicate that the color curve *at the center of the field* and the color curve *at some distance from the center* were very different. I tested the color curve with the eyepiece about 10^{mm} from the center of the field, and found that *the focal length for* λ 447 *was several mm. greater than for* λ 423.

The color curve was all right so long as the rays passed through the lens symmetrically with reference to the axis, but otherwise the curve was all wrong. The effect was practically the same with the lens reduced to one inch aperture. It had no field. Fearing that the angular aperture of the camera was too great for a double lens, I at once ordered a triple lens of Jena glass, cemented, of two inches clear aperture. That removed the difficulty. The definition is admirable over considerable range and the plate requires no tipping.

The prisms are slightly astigmatic; the focus for the dust lines is $0^{mm}.7$ longer than for the Fraunhofer lines. The astigmatism gives rise to no inconvenience, and in fact assists in distributing the light uniformly over the width of the spectrum. It is not affected by rotating the lenses and hence its cause lies in the prisms. With such dense prisms slight causes produce appreciable effects.

The average minimum deviations of the prisms, with the corresponding indices of refraction, computed from the well-known formula $n = \sin \frac{1}{2} (D+i) / \sin \frac{1}{2} i$, are

Line	D	$3D$	n
$H\epsilon$	61° 3 .3	183° 10	1.7412
$H\delta$	60 21 .7	181 5	1.7352
$H\gamma$	59 20 .5	178 2	1.7263
$H\beta$	57 47 .2	173 22	1.7124

Assuming that Cauchy's formula

$$n = a + b\lambda^{-2} + c\lambda^{-4} \qquad (2)$$

expresses the relation between index of refraction and wave-length, I substituted the values of n and λ for the three lines $H\epsilon$, $H\gamma$ and $H\beta$, and solved for the values of a, b c. The resulting equation is

$$n = 1.67284 + [7.81165]\,\lambda^{-2} + [6.83031]\,\lambda^{-4}, \qquad (3)$$

in which the quantities between brackets are logarithms and the unit of wave-length is $\mu = 0^{mm}.001$. Lord Rayleigh's formula for the theoretical resolving power is

$$R = -(t_2 - t_1)\frac{dn}{d\lambda}, \tag{4}$$

where t_2 and t_1 are the longest and shortest paths of the light in the prisms. From (3) we have

$$\frac{dn}{d\lambda} = -[8.11268]\,\lambda^{-3} - [7.43237]\,\lambda^{-5}, \tag{5}$$

whose value at $H\gamma$ is -0.3342. The effective aperture of the collimator lens is $37^{mm}.4$ and the angle of incidence is $59° 20'$. Therefore $t_2 - t_1$ is 222^{mm}. Substituting these values in (4) we have

$$R = 74.2.$$

The corresponding expression for the purity is

$$P = \frac{\lambda}{d\psi + \lambda}\,R, \tag{6}$$

in which d is the width of slit and ψ the angular aperture of the collimator lens.

The width of slit assigned by the equation

$$\frac{d}{\text{Length of collimator}} = \frac{\lambda}{\text{Effective aperture of collimator}}$$

is $d = 0^{mm}.0084$. For this case $P = \frac{1}{2}R = 37.1$, and two monochromatic lines for which $\Delta\lambda = 0.117$ tenth-meters should be resolved. The matter was tested on the solar spectrum. The lines $\lambda\lambda$ 4348.003 and 4348.130, $\Delta\lambda = 0.127$ *t.m.*, were easily separated with slit-width $0^{mm}.0084$. They were just resolvable with slit-width $0^{mm}.012$, although the theoretical limit for that width is $\Delta\lambda = 0.142$ *t.m.*

At slit-width $0^{mm}.02$, $P = 21.9$ and $\Delta\lambda = 0.198$ *t.m.* The solar lines $\lambda\lambda$ 4320.907 and 4321.119, $\Delta\lambda = 0.212$ *t.m.*, stood well apart with $d = 0^{mm}.02$, and remained separated till d was increased to $0^{mm}.029$, although the corresponding theoretical limit is 0.261 *t.m.* For practicable slit-widths the observed purity was greater than its computed value.

The photographic resolution depends largely upon the character of the plates employed. Thus, in the solar spectrum, the limit of resolution with proper exposures on Eastman's lantern

slide plates is about 0.15 *t. m.;* lines for which $\Delta\lambda = 0.20$ *t. m.* are rather widely separated. On rapid plates, with slit-width $0^{mm}.02$, a few negatives of stellar spectra show the lines $\lambda\lambda$ 4337.216 and 4337.414 to be separated. This is precisely the theoretical limit for $d = 0^{mm}.02$. With proper exposures, stellar lines for which $\Delta\lambda = 0.25$ *t. m.* are very often separated. I am unable to state the relation between diameters of the silver grains and photographic resolution. Much depends upon the exposure-time and the width of spectrum. It frequently happens that two close lines which unite at one point in a wide star spectrum are clearly separated at another.[1]

It is very unfortunate that the powerful modern spectrographs are so wasteful of light. The quantity of stellar light incident upon the object-glasses of great telescopes is sufficiently meager to start with, yet only a small fraction of the incident light succeeds in traversing the resisting media of modern spectrographs and recording itself upon the photographic plate. This is especially true for instruments photographing in the blue and violet, to which regions the line-of-sight work has practically all been confined. The question of economizing light already collected is as important as that of providing larger object-glasses, and merits the fullest consideration.

The prisms of the Mills spectrograph are slightly yellow, but may be called very clear considering their density. The average length of path which the light traverses in them is 120^{mm}. I have estimated, from the best available data, that about 50 per cent. of the $H\gamma$ light incident upon the first prism would be lost by absorption in passing through the three prisms. Again, computations show that about 40 per cent. of the light incident upon the first prism would be lost by reflection from the six prism surfaces. The combined loss due to absorption and reflection would be about 70 per cent.

[1] It would be interesting and profitable to compute the efficiency of the instrument according to Professor Wadsworth's formulae, but space is lacking, and my purpose has been simply to convey a general idea of the power of the instrument. The computations for various assumptions, and for lines of different character, can be made from the foregoing data.

In order to test the matter, Mr. Wright and I have photographed a beam of $H\gamma$ light, first with the spectrograph complete, and second with the prism box removed. A graduated series of exposures, ranging from full exposure-time down to extreme under exposure, was made in each case, on plates exactly alike and equally developed. As nearly as could be estimated, the images in the two cases were of equal intensity when the series with the prism train was exposed four times as long as the series with the prism train eliminated. That is, roughly, 75 per cent. of the light was lost in traversing the prism train. Add to this the loss in the 36-inch object-glass, in the collimator and camera-lenses, to say nothing of the great loss at the slit, and it will be seen how small a proportion of the original beam of light is able to strike the sensitive plate.

The diameter of the central disk of a star in the focus of the 36-inch refractor given by the formula $d = 1.22\lambda / R$, in which R is the radius of the object-glass, is 0.24 second of arc for the $H\gamma$ image and $0''.31$ for the visual image. Practically, these dimensions are too large, since the outer edge of the disks is faint. The double-star observers with the telescope seem to experience no special difficulty in measuring fairly equal components, on first-class nights, when the components are $0''.13$ or more apart, even with the added disadvantage of strong chromatic aberration. The linear diameter of the $0''.24$ disk is $0^{mm}.020$, and that is possibly the limiting value of the slit-width in good seeing. Thus far I have used slit-widths from $0^{mm}.01$ to $0^{mm}.03$, depending on the brightness of the star and the state of the atmosphere.

The distance of the slit from the collimator lens is left unchanged, at scale reading 12.0. The reading of the camera scale is nearly independent of the temperature; the reading at $+30°$C. is only $0^{mm}.1$ greater than for $0°$C. The reading of the collimator scale for the slit in the focus of the large object-glass was determined by making a series of exposures on the spectrum of a bright star with the scale readings differing $0^{mm}.5$ from each other. The plate having been set in focus for the

dust lines, that one of the series which is linear at $H\gamma$ corresponded to the scale reading for which the $H\gamma$ image was in focus on the slit. Nineteen determinations of the scale reading were made between June 1896 and April 1897, with the temperature varying from $+2^\circ.4$ C. to $24^\circ.8$ C. The observations were well satisfied by an equation of the form

$$\text{Scale reading} = a\,t + b,$$

in which t is the reading of the centigrade thermometer, and a and b are constants. A least squares combination of the nineteen observations gave

$$\text{Scale reading} = 0.562\,t + 49.24,$$

with a probable error of a single observation equal to $\pm 0^{mm}.4$. The instrument is occasionally used in the winter at -5°C., and in the summer at $+25^\circ$C. The scale readings in the two cases are $46^{mm}.4$ and $63^{mm}.3$, differing 17^{mm}. In addition, the length of the steel telescope tube increases 6^{mm} when the temperature rises from -5° to $+25^\circ$. The focal length of the 36-inch object-glass is therefore 23^{mm} greater at the summer temperature than at the winter. Scale readings determined in this manner are liable to some uncertainty, inasmuch as the temperature of the object-glass may be quite different from that indicated by a thermometer near the floor. I endeavored, therefore, to secure the observations when the temperature had been constant for some time.

The image lenses for the hydrogen tube and iron spark have large angular apertures. The angular aperture of the collimator lens is 3°, whereas that of the hydrogen lens is 13°, and that of the iron lens 25°. At the beginning of each night's work the observer makes sure that the beam of $H\gamma$ light fills the collimator lens. This is readily done by placing the eye at the point where the image of the $H\gamma$ line is formed on the sensitive plate. If the hydrogen tube is correctly placed, the observer will see the completely illuminated circular image (apparently) on the camera lens. The effect of the prismatic absorption is shown in this image, the edge corresponding to the vertices of the prisms

being much brighter than the edge corresponding to their bases. The $H\gamma$ line is then observed in the camera by means of an eye-piece. The hydrogen tube is gently pressed first to one side and then to the other. If it disappears in both directions under equal pressures, it is considered to be in adjustment. The iron electrodes are usually 1^{mm} or more apart, placed at right angles to the slit length, so that the image of the spark on the slit is of considerable size. The illumination of the collimator lens by the artificial light has been thoroughly tested. Among other tests, the beam of emergent light was photographed on plates held against the end of the collimator tube. The resulting circular images were of uniform intensity throughout. This test did not apply at first, owing to the absence of diaphragms to cut off the light reflected from the collimator tube. Diaphragms have been inserted in both collimator and camera.

The exposure time for the comparison spectrum is usually about 5 seconds for the $H\gamma$ line, 3 seconds for the brightest iron lines, and 60 seconds for the faint iron lines. A simple device enables these exposures to be given. The thin brass slide immediately in front of the plate, already described, was adjusted to move with accuracy and ease. I filed the edge of it away so as to leave three projections: two narrow strips in positions to cover the bright iron lines λλ 4308 and 4326, and a broad strip in position to cover the region λλ 4380–4420, which contains the bright iron lines λλ 4384, 4405, and 4415. When the iron spectrum has been photographing three seconds, the occulting strips of brass are slipped (lengthwise) over the brightest five iron lines, and the exposure on the faint iron lines goes on to the end of the minute. The device is simple and safe.

The deviation by the prisms is affected by temperature changes. Before beginning a night's work the camera is unclamped and moved so that the lines λλ 4308 and 4326 are exactly central on their occulting strips.

The value of the temperature coefficient of the deviation is not known at present. Observations for that purpose, taken by Messrs. Wright and Cottrell, yield very discordant results,

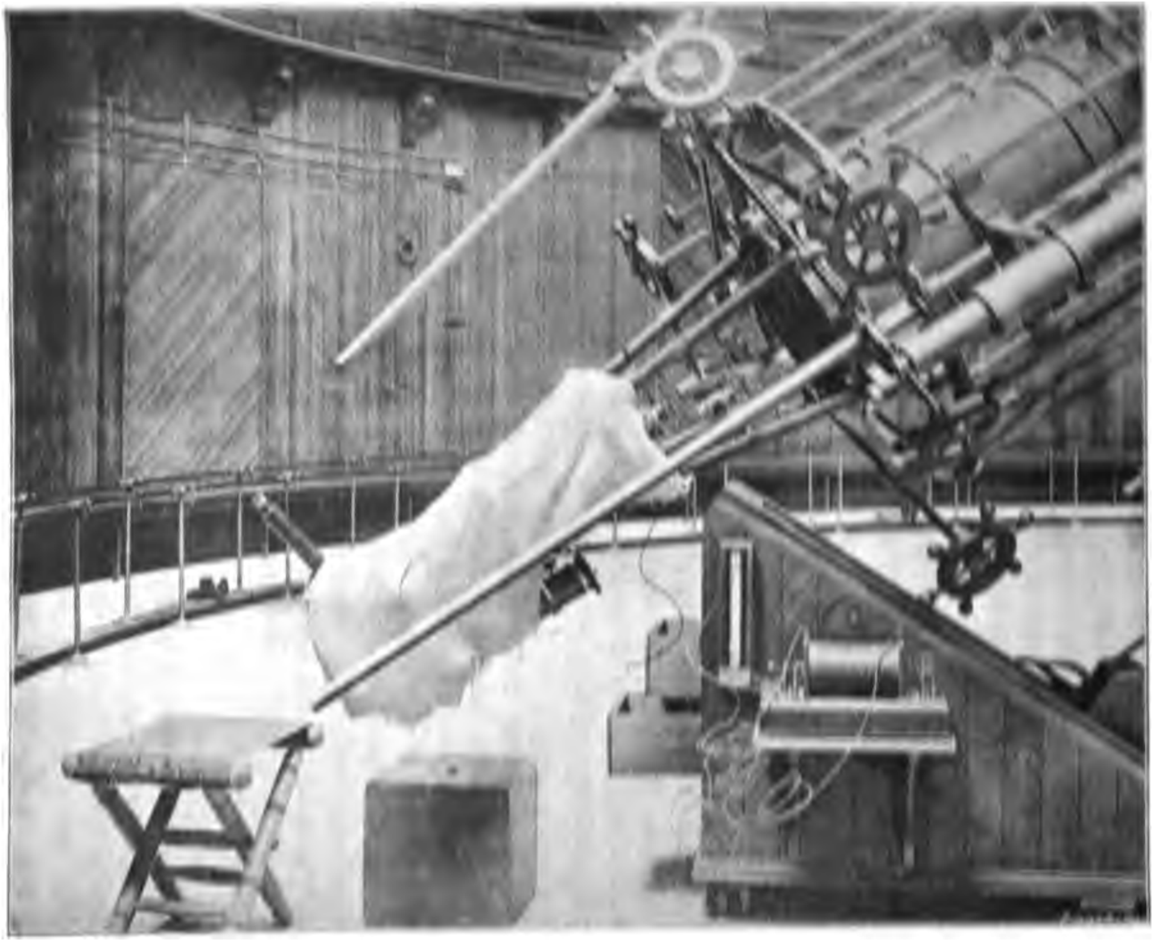
apparently from the fact that the temperature changes of the prisms lag behind those of the air in the prism box much more than was anticipated and allowed for. We shall at once mount a delicate thermometer on the spectrograph, its bulb near the center of the prism box and its tube alongside the collimator tube. It will be used both to investigate the deviation coefficient and to indicate temperature changes during stellar spectrum exposures. The temperature heretofore has been noted at the beginning and end of each exposure from the thermometer attached to the spectrograph case.

The desirability of uniform temperature in spectrographic work is very great.[1] The definition and the deviation are both affected by that factor. The Lick Observatory is fortunately situated in this respect. The temperature generally varies less than 2° Centigrade from dark to dawn. I have observed on many nights when the variation from ten o'clock to daylight has been less than 1°. It is very seldom, after nine o'clock, that the thermometer readings in the dome change a half degree during exposures of an hour.

When I first used the spectrograph, the definition of the star spectra was sometimes inexplicably poor, even when the temperature was constant. I was inclined to attribute this to the heating of the prism box by the observer's breath and body. He necessarily sits very close to it while guiding. I covered the prism box with a closely fitting hood of two thicknesses of heavy gray woolen blanket. In addition, a double hood of the same material covers the whole instrument. The prism box is therefore protected by four thicknesses. An improvement in the definition was at once noted.

The exposure-time for the star spectra is necessarily a function of the brightness, atmospheric conditions, slit-width, and plate sensitiveness. Professor Pickering has published in the *Draper Catalogue* a column of "Magnitudes," containing estimates of the photographic intensities of the spectra in the vicinity of G. These estimates are my guide in assigning the

[1] See article by M. DESLANDRES in *Bulletin Astronomique*, February 1898.



THE MILLS SPECTROGRAPH AS USED IN MAKING OBSERVATIONS.

TABLE I.

λ	Observed R	$R-32.000$	$x=\lambda-4330$	Computed R	Residuals O—C
4238.970	0.325	−31.675	− 91.030	0.326	−0.001
46.996	3.314	−28.686	− 83.004	3.313	+0.001
54.505	6.072	−25.928	− 75.495	6.073	−0.001
67.985	10.934	−21.066	− 62.015	10.943	−0.009
76.836	14.089	−17.911	− 53.164	14.083	+0.006
82.565	16.096	−15.904	− 47.435	16.092	+0.004
89.525	18.512	−13.488	− 40.475	18.508	+0.004
4303.337	23.224	− 8.776	− 26.663	23.224	0.000
13.797	26.722	− 5.278	− 16.203	26.726	−0.004
18.817	28.390	− 3.610	− 11.183	28.387	+0.003
27.274	31.157	− 0.843	− 2.726	31.156	+0.001
31.811	32.627	+ 0.627	+ 1.811	32.626	+0.001
38.084	34.645	+ 2.645	+ 8.084	34.642	+0.003
41.530	35.740	+ 3.740	+ 11.530	35.741	−0.001
59.784	41.459	+ 9.459	+ 29.784	41.467	−0.008
69.941	44.582	+12.582	+ 39.941	44.586	−0.004
79.396	47.449	+15.449	+ 49.396	47.446	+0.003
88.571	50.186	+18.186	+ 58.571	50.184	+0.002
4406.810	55.516	+23.516	+ 76.810	55.519	−0.003
17.884	58.696	+26.696	+ 87.884	58.691	+0.005
30.785	62.326	+30.326	+100.785	62.325	+0.001
4442.510	65.570	+33.570	+112.510	65.572	−0.002

Table II contains a list of 109 lines which I have had occasion to use in determining stellar velocities. Only a small portion of the table is published here, by way of illustration. The wave-lengths are Rowland's, and the corresponding micrometer readings are computed from formula (8) for the standard solar reduction plate, 433A. The quantity, rV_s, is the velocity in kilometers per second corresponding to a displacement of the lines through one revolution, r, of the micrometer screw. The velocity V_s in kilometers per second corresponding to a displacement of one tenth-meter is given by[1]

$$V_s = \frac{299860}{\lambda}. \tag{9}$$

The value of r is readily obtained from (8). Differentiating it we have

$$\frac{dx}{dR} = \frac{1}{B + 2Cx + 3Dx^2}. \tag{10}$$

[1] See my paper on "The Reduction of Spectroscopic Observations," reprinted in Frost's Scheiner's *Astronomical Spectroscopy*, p. 339.

If we assume that dR is unity, dx becomes the value of one revolution of the micrometer screw in tenth-meters; that is,

$$dx = r = \frac{1}{B + 2Cx + Dx^2}; \qquad (11)$$

the values of B, C, and D being given in (8).

TABLE II.

λ	☉ Micrometer Reading	rV_s
4238.188	0.033	188.6
38.970	0.326	188.8
..	..	..
..	..	..
..	..	..
..	..	..
4337.216	34.363	215.9
37.414	34.427	216.0
37.725	34.528	216.0
38.084	34.642	216.1
..	..	..
..	..	..
..	..	..
..	..	..
4441.881	65.399	245.5
42.510	65.572	245.6

A table computed for every five tenth-meters is very convenient in making interpolations for lines previously unused, and for many other purposes, but need not be published here. It should contain the corresponding micrometer reading, the value of one revolution in tenth-meters, its reciprocal, which is the value of one tenth-meter in terms of the micrometer revolution, and the value of rV_s.

The correction for the curvature of the spectrum lines was determined empirically from lines in the solar spectrum. Assuming that the curve is a parabola, the deviations x from the tangent at the vertex were measured for several points on the line at distances y from the axis of the parabola. The equation of the parabola was secured by a least squares solution. The equations for three lines, expressed in terms of the micrometer units, are:

$$\begin{aligned} &\text{for } \lambda\, 4238.188 && x = -\,0.00233\, y^2 \\ &\lambda\, 4338.084 && x = -\,0.00224\, y^2 \\ &\lambda\, 4342.510 && x = -\,0.00208\, y^2 \end{aligned} \qquad (12)$$

The equation for the line λ4338.084, computed from the known constants of the instrument, from Ditscheiner's formula[1] is $x = -\,0.00210y^2$. I have preferred to use the empirical formulae.

The corrections for curvature corresponding to various distances y of the comparison spectrum from the center of the star spectrum, are given in kilometers per second in Table III.

TABLE III.

y	λ4238.188	λ4338.084	λ4442.510
$0^r.0$	$-0^{km}.00$	$-0^{km}.00$	$-0^{km}.00$
0 .2	-0 .02	-0 .02	-0 .02
0 .4	-0 .07	-0 .08	-0 .08
0 .6	-0 .15	-0 .17	-0 .18
0 .7	-0 .22	-0 .24	-0 .25
0 .8	-0 .28	-0 .31	-0 .33
0 .9	-0 .36	-0 .39	-0 .41
1 .0	-0 .44	-0 .48	-0 .51
1 .1	-0 .53	-0 .59	-0 .62
1 .2	-0 .63	-0 .70	-0 .74
1 .3	-0 .74	-0 .82	-0 .86
1 .4	-0 .86	-0 .95	-1 .00
1 .5	-0 .99	-1 .09	-1 .15

The corrections for the annual and diurnal motions of the Earth are computed from my tables and formulae[2] for that purpose. I have, however, adopted the new value of the solar parallax, $8''.8$, which requires each value of log V_a to be increased by 0.0019. Likewise, the results are referred to the ecliptic and mean equinox of 1900.0.

Heretofore I have kept the violet end of the plate to the left while measuring it on the micrometer microscope. I have not detected any systematic differences between the measures made with violet to the left and violet to the right, such as those found by Professor Lord.[3] Actual trials do not show it. Further, all my measures of planetary velocities, obtained with violet to

[1] See Frost's Scheiner, p. 15.

[2] See Frost's Scheiner, pp. 338–345.

[3] See this JOURNAL, 6, 424–426.

the left, agree with the computed velocities very satisfactorily, and that is evidently the criterion in such questions. Some of Mr. Wright's measures, but not all, show a systematic difference of that character — one plate shows a change of $1^{km}.5$, or $0^{km}.75$ from the mean, by measurement in the two directions, whereas another shows no change whatever. Mr. Wright has, I believe, measured in both directions from the first. As the time formerly spent in triplicate measures in one direction may be divided between measures in two directions, I shall probably adopt that method for the sake of uniformity in our system.

In order to illustrate the methods employed, I shall insert my reduction of Plate 252 B, *α* Tauri, taken 1897 January 20, Mt. Hamilton sidereal time $3^h\ 51^m$. Column 1 contains the Rowland wave-lengths of the lines, both stellar and comparison, that were measured on this plate. The next column contains the micrometer readings for the same lines on the standard solar reduction plate, taken from Table II. It should be said that the standard solar plate was secured with the triple camera lens now in use, whereas this plate of *α* Tauri was made with the discarded double camera lens. The reduction curves for the two lenses are slightly different, but their second differences are practically identical, so that either curve will answer all requirements. Column 3 contains the micrometer readings on the star lines, each the mean of three settings, and column 4 contains the readings on the comparison lines of iron and hydrogen. The next column, "zero lines," contains the micrometer readings that comparison lines, or *lines of zero velocity*, would have if there were such lines having the wave-lengths given in column 1. They are the readings of the corresponding solar lines reduced to the curve on which the Fe and H comparison lines lie. These values are readily and accurately supplied. The micrometer readings on the comparison lines are assumed to be correct. A curve similar to the *corresponding section* of the solar curve is analytically passed through each adjacent pair of comparison lines, and the readings on this curve corresponding to the wave-lengths of the lines observed in the star spectrum are obtained

by interpolation. Sections of the solar curve are passed through the comparison lines, pair by pair, throughout the spectrum. [The iron lines $\lambda\lambda 4294.3$ and 4299.4 are usually treated as one line in their mean position $\lambda 4296.9$]. The micrometer readings supplied for the zero lines are as accurate as those for the comparison lines obtained by measurement. It would be a simple matter to plot the readings on all the comparison lines, pass a smooth curve through them as well as possible, and read off the ordinates for the zero lines. There is little to gain or lose by a choice between the two methods, and I have preferred the former.

The micrometer readings on the star lines, minus those on the comparison and zero lines, are the observed displacements. The value of rV_s varies for different plates, owing to changes of temperature, etc. The comparison lines on each plate furnish the reduction constants for that plate, and the relative linear scales of the stellar and standard solar plates permit the modified values of rV_s to be readily filled in from Table II. The measured velocities v_s are quickly found by means of Crelle's tables. The mean of the velocities from the twenty-eight star lines is $+ 79^{km}.17$. The readings on the comparison lines were made at a distance $0^r.85$ from the center of the star spectrum. Hence the correction for curvature is $-0^{km}.35$. The reduction to the Sun[1] is $v_a + v_d = - 23^{km}.90$. Therefore the velocity with reference to the solar system is $V = + 54^{km}.9$.

It will be noticed that I made little use of the iron lines in the star spectrum corresponding to the iron comparison lines. There are relatively few second-type stars in which those iron lines are suitable for measurement, owing to their breadth or to the influence of close companions. The best lines are selected irrespective of their positions, and the positions of their zero lines of reference are computed later. Thus one never knows what displacements these lines may have until column 6 on the reduction sheet is filled in.

[1] In theory, the reductions should be made with reference to the ecliptic and equinox of the observation date, but no sensible error is introduced by reducing uniformly to 1900.0.

α TAURI.— PLATE 252 B.

λ	⊙	*	Fe and H	Zero lines	Displacement	rV_s	V_s
4282.565	16.092		15.859				
87.566	17.831	17.987		17.591	$+0^r.396$	202.7	$+80^{km}.3$
94.301	20.150		19.897	19.899			
99.410	21.893		21.637	19.635			
4300.211	22.165	22.287		21.907	.380	206.1	78 .3
00.732	22.342	22.469		22.084	.385	206.3	79 .4
02.692	23.006	23.136		22.746	.390	206.9	80 .7
08.081	24.819		24.553				82 .1
15.262	27.213		26.942				
16.962	27.775	27.892		27.502	.390	210.5	
25.939	30.721		30.441				
26.520	30.911	31.016		30.631	.385	213.3	82 .1
27.274	31.156	31.225		30.876	.349	213.5	74 .5
28.080	31.418	31.501		31.137	.364	213.7	77 .8
33.925	33.308	33.394		33.026	.368	215.3	79 .2
37.216	34.363	34.443	34.080		.363	216.3	78 .5
37.725	34.528	34.603		34.244	.359	216.4	77 .7
38.084	34.642	34.708		34.358	.350	216.5	75 .8
38.430	34.753	34.840		34.469	.371	216.6	80 .3
38.854	34.888	34.973		34.603	.370	216.7	80 .2
40.634	35.456	35.535	35.169		.366	217.1	79 .5
41.167	35.626	35.711		35.339	.362	217.1	78 .6
41.530	35.741	35.811		35.454	.357	217.2	77 .6
43.861	36.481	36.564		36.194	.370	217.9	80 .6
44.670	36.738	36.811		36.451	.360	218.1	78 .5
47.403	37.601	37.678		37.313	.365	218.9	79 .9
49.107	38.137	38.212		37.849	.363	219.4	79 .7
55.257	40.062	40.141		39.773	.368	221.1	81 .4
59.784	41.467	41.543		41.178	.365	222.4	81 .2
69.941	44.586	44.645		44.295	.350	225.3	78 .9
76.107	46.456	46.515		46.165	.350	227.1	79 .5
79.396	47.446	47.495		47.155	.340	228.0	77 .5
83.720	48.741		48.449				
89.413	50.433	50.487		50.141	.346	230.9	79 .9
4404.927	54.975		54.683				
06.810	55.519	55.554		55.227	+0.327	235.8	+77 .1

Mt. Ham. Sid. T. 1897 Jan. 20 $3^h\ 51^m$ Mean $= +79\ .17$

α 1900.0 $=$ 4 30 Corr. for Curvature $= -\ 0\ .35$

$\tau = -\ 0\ 39$ Reduction to ⊙ $= -23\ .90$

δ 1900.0 $= +16°\ 18'$ V $= +54^{km}.92$

Greenwich M. T. 1897 Jan. 20 $15^h\ 55^m$

β 1900.0	$= -\ 5°\ 28'.5$	$\log v_a$	$= 1.4805$
λ 1900.0	$=\ 68\ 23\ .5$	$\sin(\lambda - ⊙ + i)$	$= 9.9009$
⊙ 1900.0	$=\ 301\ 28\ .6$	$\cos\beta$	$= 9.9980$
i	$= +\ 20\ .0$	$\log v_a$	$= 1.3794n$
$\lambda - ⊙ + i$	$=\ 127\ 14\ .9$	v_a	$= -\ 23^{km}.96$
		v_d	$= +\ 0\ .06$

Several hundred plates of star spectra have been obtained, and about 150 have been measured and reduced. In order to show the nature of the results, I append a list of fifty velocity determinations for eleven stars, the only stars for which I have reduced four or more plates each. The results obtained by other observers are printed for comparison in the adjoining columns. Those by Professors Vogel and Scheiner found in the same horizontal line are the results obtained by the two observers from one and the same plate. The results are all expressed in kilometers per second.

DETERMINATIONS.

Star	Mt. Hamilton Sid. time	Campbell	Vogel	Scheiner	Other observers	
α Cassiopeiae	1896 Nov. 12, $0^h\ 54^m$	— 4.1	—14.8	—17.0	Lord, photo.	— 2.8
	Dec. 8, 1 00	— 4.1	—15.0	—14.1	" "	+ 1.6
	" 17, 1 42	— 4.9				
	" 24, 1 45	— 4.2				
	Means	— 4.3	—14.9	—15.6		— 0.6
β Andromedae	1896 Dec. 8, 2 58	+ 0.8	+ 3.9	+11.7		
	" 17, 2 36	— 0.8	+14.0	+14.9		
	" 24, 2 40	— 1.0				
	1897 July 8, 22 56	+ 0.6				
	" 21, 22 27	+ 2.1				
	Means	+ 0.3	+ 9.0	+13.3		
α Ursae Minoris	1896 Sept. 8, 2 00	—20.1	—21.1	—25.9		
	" 15, 2 23	—19.1	—29.9	—26.5		
	" 23, 1 34	—18.9				
	Oct. 5, 1 52	—19.0				
	Nov. 11, 2 40	—20.1				
	Dec. 8, 1 50	—20.3				
	Means	—19.6	—25.5	—26.2		
γ Andromedae	1896 Nov. 12, 2 32	—12.0	— 5.6	—16.1		
	1897 Jan. 5, 2 35	—12.8		—21.6		
	July 22, 23 36	—10.8	—10.3	—15.7		
†	1898 July 12, 22 45	—10.6				
*	" 12, 22 45	— 9.5				
†	" 13, 23 09	—11.2				
	Means	—11.2	— 8.0	—17.8		

* Independent measures and reductions by Mr. Wright.
† Photographs taken by Mr. Wright.

DETERMINATIONS—*continued.*

Star	Mt. Hamilton Sid. time	Campbell	Vogel	Scheiner	Other observers	
α Arietis	1896 Aug. 19, 0 30	−13.8	−12.8	−21.0	Lord, photo.	−12.6
	" 25, 1 11	−14.0	−13.2	−10.5	" "	−15.3
	Dec. 9, 0 36	−14.2	−17.4	−13.2		
	" 9, 1 07	−14.4				
	Means	−14.1	−14.5	−14.9		−14.0
α Persei	1896 Nov. 11, 3 35	− 2.0	−10.8	− 9.5		
	" 12, 3 24	− 1.8	−10.8	−10.1		
	1897 Jan. 19, 3 57	− 3.5				
†	1898 July 12, 23 40	− 2.1				
	Means	− 2.4	−10.8	− 9.8		
α Tauri	1896 Aug. 23, 3 04	+55.7	+44.4	+49.0	Keeler, visual	+57.3
	Sept. 16, 3 15	+54.6	+49.0	+49.0	" "	+60.1
	Dec. 8, 4 03	+54.5	+46.3	+46.7	" "	+48.3
	" 17, 3 19	+54.3	+50.6	+52.8	Campb'll, visual	+49.1
	1897 Jan. 20, 3 51	+54.9			Newall, photo.	+49.2
‡	1898 Jan. 19, 4 55	+55.2				
	Means	+54.8	+47.6	+49.4	Keeler	+55.2
α Canis Minoris	1897 Jan. 20, 6 33	− 5.0	− 8.5	−13.7	Newall, photo.	− 4.9
	" 21, 6 38	− 4.3	− 9.3	−13.4		
	Feb. 21, 8 01	− 4.8	− 5.9	− 4.4		
	" 22, 7 42	− 5.0				
	Means	− 4.8	− 7.9	−10.5		− 4.9
α Ursae Majoris	1897 Jan 21, 11 20	−10.3	− 8.1	−15.3		
	Feb. 24, 12 10	− 9.5	− 9.9	−14.7		
	April 15, 11 42	−10.0	−10.6	− 8.8		
†	Dec. 23, 11 33	− 9.5	−12.5	−11.9		
*	" 23, 11 33	− 9.5				
	Means	− 9.8	−10.3	−12.7		
γ Draconis	1897 May 4, 19 00	−27.4				
	" 27, 19 09	−28.4				
	July 20, 18 58	−26.3				
	Aug. 3, 19 05	−27.4				
	Mean	−27.4				
ε Pegasi	1896 Aug. 27, 21 06	+ 5.5	+ 5.5	+ 2.6	Lord, photo.	+ 6.8
	1897 July 8, 21 03	+ 5.7	+ 9.1	+14.7	" "	+ 8.3
†	1898 June 15, 21 10	+ 6.4			" "	+14.7
†	" 26, 21 25	+ 5.2				
	Means	+ 5.7	+ 7.3	+ 8.6		+ 9.9

‡ Photograph, measures and reductions by Mr. Wright.

The extreme accuracy required and attained in this class of work is evident from the following statement. The linear value of 0.01 second of arc in the focus of the 36-inch refractor is $0^{mm}.000857$. On the spectrum plates, $0^{mm}.000857$ is 0.0034 revolution of the screw, corresponding to $0^{km}.74$ per second displacement. It is not surprising, therefore, that great care and considerable experience are absolutely necessary for suitable measurement of the plates. The lines to be measured require good judgment in their selection. Some of the best lines in the solar spectrum are practically useless in many stars, owing to the changed intensities of close companion lines. To mention only one case, the line at λ4318.817 is excellent in the solar spectrum, but a close companion seriously affects it in nearly all the second type stars thus far measured.

The necessity for guarding against systematic errors has been constantly held in mind. No difficulties of that sort have yet been encountered. The velocities of Mars and Venus have been observed on several occasions, with the telescope on both sides of the pier, and at both large and small hour angles. The accompanying tables will show the agreement between observed and computed velocities.

It is desirable to have a simple formula for computing the velocity of a planet at any instant with reference to the Earth, and with reference to the Sun.

Let D be the distance between the two bodies whose relative velocity is required. The American Ephemeris tabulates the function

$$f = \log D$$

at regular intervals and stated times, for each planet and the Earth, and for each planet and the Sun. Let T be the date in the ephemeris nearest the instant for which the velocity is required, and let ω be the tabular interval of time. Then the adjacent dates in the ephemeris may be represented as in column 1 of the table below, and the corresponding values of the function $f = \log D$ as in column 2. The remaining columns

$T-2\omega$	$f(T-2\omega)$				
		$a_{,,}$			
$T-\ \omega$	$f(T-\ \omega)$		b'		
		$a_{,}$		$c_{,}$	
T	$f(T)$	a	b	c	d
		a'		c'	
$T+\ \omega$	$f(T+\ \omega)$		$b_{,}$		
		a''			
$T+2\omega$	$f(T+2\omega)$				

contain the first, second, third, and fourth "differences" of the function f, formed in the usual manner. Lastly, the quantities $a = \frac{1}{2}(a_{,} + a')$ and $c = \frac{1}{2}(c_{,} + c')$ are inserted in the positions indicated.

Let the instant for which the velocity is wanted be represented by $T+t$; and let n be the ratio of t and the tabular interval ω; *i. e.*, $n = t/\omega$, or $t = n\,\omega$. The formula for computing log D at the time $T+t$ from the above data is[1]

$$\log D = f(T+t) = f(T) + \left(a - \frac{c}{6}\right)n + \left(\frac{b}{2} - \frac{d}{24}\right)n^2 + \frac{c}{6}n^3 + \frac{d}{24}n^4 + \dots \qquad (13)$$

The rate of change of log D at that instant is

$$\frac{\delta}{\delta t}\log D = \frac{\delta}{\delta t} f(T+t);$$

whence, letting m be the modulus of the logarithmic system used,

$$\frac{\delta D}{\delta t} = \frac{D}{m}\left[a - \frac{c}{6} + \left(b - \frac{d}{12}\right)n + \frac{c}{2}n^2 + \frac{d}{6}n^3 + \dots\right]. \qquad (14)$$

Now $\frac{\delta D}{\delta t}$ is the desired velocity at the time $T+t$, but expressed in terms of the astronomical unit of distance employed in the ephemeris, and of the tabular unit of time ω. The astronomical unit of distance corresponding to the solar parallax 8″.80 is 149,500,000 kilometers. If ω is expressed in

[1] An adaptation of formula (71), Vol. I, Chauvenet's *Spher. and Prac. Astronomy*, 5th edition.

MARS.

Mt. Hamilton M. T.	No. of lines measured	Measured velocity v_s	Corr. for curv.	Observed velocity	Relative velocity ♂ − ⊕	Relative velocity ♂ − ⊙	Diurnal velocity, $-v_d$	Computed velocity	Residuals O − C
1896, Sept. 15ᵈ 16ʰ 13ᵐ	33	− 8.66	−0.39	− 9.05	−10.38	+1.94	−0.11	− 8.55	−0.50
Oct. 3 16 43	30	− 7.86	−0.39	− 8.25	−10.35	+2.12	+0.01	− 8.22	−0.03
1897, Jan. 14 08 05	18	+14.79	−0.39	+14.40	+12.71	+1.90	−0.08	+14.53	−0.13

VENUS.

Mt. Hamilton M. T.	No. of lines measured	Measured velocity v_s	Corr. for curv.	Observed velocity	Relative velocity ♀ − ⊕	Relative velocity ♀ − ⊙	Diurnal velocity, $-v_d$	Computed velocity	Residuals O − C
1897, July 22ᵈ 16ʰ 09ᵐ	37	+13.92	−0.82	+13.10	+13.56	−0.17	−0.33	+13.06	+0.04
* July 22 16 09	32	+14.80	−0.81	+13.99				+13.06	+0.93
† Aug. 26 16 52	31	+12.14	−0.62	+11.52	+12.06	−0.24	−0.31	+11.51	+0.01
‡ Aug. 26 16 52	27	+12.36	−0.70	+11.66				+11.51	+0.15
1898, June 8 07 58	31	− 9.04	−0.62	− 9.66	− 9.56	+0.08	+0.34	− 9.14	−0.52
* June 8 07 58	22	− 8.71	−0.90	− 9.61				− 9.14	−0.47

* Independent measures and reductions of the same plate by Mr. Wright.

† Photograph, measures, and reductions by Mr. Wright.

‡ Independent measures and reductions of the same plate by Mr. F. C. Cottrell.

seconds of mean solar time, the desired velocity V will be given in kilometers per second by

$$\frac{\delta D}{\delta t} = V =$$

$$\frac{149{,}500{,}000}{\omega m} D\left[a - \frac{c}{6} + \left(b - \frac{d}{12}\right)n + \frac{c}{2}n^2 + \frac{d}{6}n^3 + \ldots\right]; \quad (15)$$

or, for logarithmic computation,

$$\log V = \log \frac{149{,}500{,}000}{\omega m} + \log D + \log [\ldots.].^* \quad (16)$$

The following table contains the values of $\log \frac{149{,}500{,}000}{\omega m}$ in the cases which may arise.

If $\omega = 1$ mean solar day,	$\log \frac{149{,}500{,}000}{\omega m}$	$= 3.6003$
" $\omega = 2$ " " days,	"	$= 3.2993$
" $\omega = 4$ " " "	"	$= 2.9983$
" $\omega = 8$ " " "	"	$= 2.6973$

Example. Required the velocity of Venus with reference to the Earth at Mt. Hamilton mean time, 1898, June $8^d\ 7^h\ 58^m$.

The Greenwich mean time is June $8^d\ 16^h\ 05^m$. The American Ephemeris gives for

Greenwich M. T.	log D	a	b	c	d
1898, June 5.0......	0.1591081				
		—32651			
" 7.0......	0.1558430		—818		
		—33469		—12	
" 9.0......	0.1524961	—33884	—830	—14	—3
		—34299		—15	
" 11.0......	0.1490662		—845		
		—35144			
" 13.0......	0.1455518				

In this case the assigned instant precedes $T =$ June 9.0 by $7^h\ 55^m$, and $\omega = 2$ days. Therefore

$$n = -\frac{7^h\ 55^m}{2 \text{ days}} = -0.165.$$

*The form [. . . .] is used to express the quantity within the brackets in (15).

Solving (13) and (16) we have

$$
\begin{array}{rl}
f(T) = & +\ 0.1525 \\
\left(a - \frac{c}{6}\right) n = & +\qquad\quad 6 \\
\hline
\log D = & +\ 0.1531
\end{array}
\qquad\qquad
\begin{array}{rl}
a - \frac{c}{6} = & -\ 0.003388 \\
\left(b - \frac{d}{12}\right) n = & +\qquad\quad 14 \\
\hline
[\,.\ .\ .\ .\ .\,] = & -\ 0.003374
\end{array}
$$

$$
\begin{array}{rl}
\log \dfrac{149{,}500{,}000}{\omega\, m} = & 3.2993 \\
\log D = & 0.1531 \\
\log [\,.\ .\ .\ .\,] = & 7.5281_n \\
\hline
\log V = & 0.9805_n \\
V = & -9^{\mathrm{km}}.56
\end{array}
$$

Example. Required the velocity of Venus with reference to the Sun at Mt. Hamilton mean time, 1898, June $8^d\ 7^h\ 58^m$.

As in the preceding example, the Greenwich mean time is 1898, June, $8^d\ 16^h\ 05^m$, and we have from the Ephemeris for

Greenwich M. T.	log R	a	b	c	d
1898, June 1.0........	9.8563710				
		+ 568			
" 5.0........	9.8564278		+371		
		+ 939		−14	
" 9.0........	9.8565217	+1118	+357	−16	−4
		+1296		−18	
" 13.0........	9.8566513		+339		
		+1635			
" 17.0........	9.8568148				

In this case $T =$ June 9.0 and $n = -0.082$.

The solutions of (13) and (16) are

$$
\begin{array}{rl}
f(T) = & 9.8565 \\
\left(a - \frac{c}{6}\right) n = & \qquad\quad 0 \\
\hline
\log R = & 9.8565
\end{array}
\qquad\qquad
\begin{array}{rl}
a - \frac{c}{6} = & +\ 0.000112 \\
\left(b - \frac{d}{12}\right) n = & -\qquad\quad 3 \\
\hline
[\,.\ .\ .\ .\ .\,] = & +\ 0.000109
\end{array}
$$

$$
\begin{array}{rl}
\log \dfrac{149{,}500{,}000}{\omega m} & = 2.9983 \\
\log R & = 9.8565 \\
\log [\,.\ .\ .\ .\ .\,] & = 6.0374 \\
\hline
\log V & = 8.8922 \\
V & = +0^{\mathrm{km}}.08
\end{array}
$$

I have endeavored throughout to employ only those methods of reduction which will readily enable revisions to be made in case it is desired to use slightly different values of the wave-lengths: whether due to changes in the tables of standard wave-lengths, or to effects of pressure, or to any other causes.

The results contained in this paper were practically ready for publication one year ago, but the publication was delayed by my preparations for observing the total solar eclipse in India, and by an absence of seven and a half months from the Observatory. The work has been carried on during my absence by Mr. W. H. Wright, Assistant Astronomer.

LICK OBSERVATORY,
UNIVERSITY OF CALIFORNIA.
July 20, 1898.

SOME STARS WITH GREAT VELOCITIES IN THE LINE OF SIGHT.

By W. W. CAMPBELL.

IN the course of our work on the spectrographic determinations of stellar motions, I have detected a number of cases of great velocities in the line of sight.

Mr. Wright and I have secured four plates of the spectrum of η Cephei, which furnish the following velocities with reference to the solar system:

Date			No. of lines measured	Velocity
1897	September	29	34	$-87^{km}.6$
	"	*	18	$-87\ .2$
1898	July	20	39	$-86\ .2$
	August	21	34	$-86\ .9$
	August	25	31	$-86\ .2$
			Mean	$-86^{km}.8$

The proper motion of η Cephei is about $0''.8$.

We have confirmed M. Belopolsky's results for the brighter component of ζ Herculis [*A. N.*, **133**, 257–262]. Our results from four plates, together with M. Belopolsky's results from seven plates, are as follows:

Date			No. of lines measured	Velocity	Belopolsky
1897	April	29	22	$-69^{km}.1$	-68^{km}
1898	May	11	42	$-70\ .4$	-84
	May	23	36	$-70\ .0$	-75
	May	*	32	$-71\ .1$	-67
	August	19	34	$-70\ .9$	-66
					-64
					-69
			Means	$-70^{km}.3$	-70^{km}

* Independent measures and reductions of the same plate by Mr. Wright.

We may refer, in this connection, to the large velocity obtained by Professor Keeler for the planetary nebula *G. C.* 4373. His result from measures on six nights is $-64^{km}.7$ per second.

It should be noted that the results for all the above objects will be numerically decreased when we apply the correction for solar motion. Thus, if we assume that the solar system is moving toward the point $\alpha = 267°$, $\delta = +31°$, with a velocity of 17^{km} per second, the corrections for the solar motion are as given in column two below, and the velocities with reference to the sidereal system become as in column three.

Object	Correction	Velocity
η Cephei	+12.7	$-74^{km}.1$
ζ Herculis	+16.4	$-53\ .9$
G.C. 4373	+13.8	$-50\ .9$

LICK OBSERVATORY,
August 29, 1898.

THE VARIABLE VELOCITY OF η PEGASI IN THE LINE OF SIGHT.

By W. W. CAMPBELL.

MY first measures of the velocity of η Pegasi from three spectrum plates gave $+7.1$, $+5.1$, and -2.2 kilometers, respectively. Inasmuch as an extreme range of four kilometers is never expected and would lead to a careful reëxamination of the plates and of the results obtained for other stars on the same night, I felt sure that the velocity of η Pegasi varied. Additional plates were secured and measured and another earlier plate was reduced. All confirm the variation. The velocities obtained up to date are as follows:

1896	Aug. 27	$+7^{km}.1$
	Sept. 23	$+5\ .1$
1897	July 8	$-6\ .4$
	Sept. 28	$-2\ .2$
1898	Aug. 29	$+16\ .5$
	Aug. 30	$+15\ .6$
	Sept. 4	$+16\ .5$

The extreme range observed is 23^{km}. It is pretty certain that the period is a long one, possibly in the neighborhood of two years. Since it may be several years before the character of the motion can be investigated, it seems proper to announce the fact of the variation at once. If other observers have secured observations of this star, I should be glad to receive their results to assist in determining the period.

LICK OBSERVATORY,
September 5, 1898.

A SPECIMEN CHART FROM THE ATLAS STELLARUM VARIABILIUM.

By J. G. HAGEN, S. J.

THE following lines are written in response to the kind invitation of one of the editors, as an explanatory text to the specimen chart from the forthcoming *Atlas of Variable Stars.*

1. It will not be without interest to know the general plan of the atlas; because the plan of a work is its most essential part, it might be called its soul; and if this atlas ever succeeds in being useful to astronomy, it will be owing to its well defined and limited plan. This remark is called forth by an inspection of the late Pogson's manuscripts, which I had the privilege of making lately at the Harvard College Observatory. While a fuller report on these manuscripts is reserved for another occasion, it may be well to state here in a general way that Pogson had been preparing charts for all the known variable stars, including the southern sky, with all the surrounding stars down to the 13th magnitude within 80′ square. Unfortunately his labors of more than a quarter century have not become available to his fellow astronomers because his work had the germ of death in its very plan. It was too extended for him to finish, and put in a shape that makes it inconvenient for practical use.

It will be seen from the accompanying chart, that the field containing the faintest comparison stars has been confined to 30′ square, or less than one seventh of Pogson's area.

Excluded from the atlas are, for the present, all those variable stars, whose range of variability is not yet established with certainty, also the so-called Novae, and finally the variable star clusters. The Novae are not variable stars in the ordinary sense of the word, and ought not to be embodied in the catalogues of variable stars, lest novices in this branch of astronomy be misled to waste time on these historical objects.

As regards the few star clusters that are known to contain

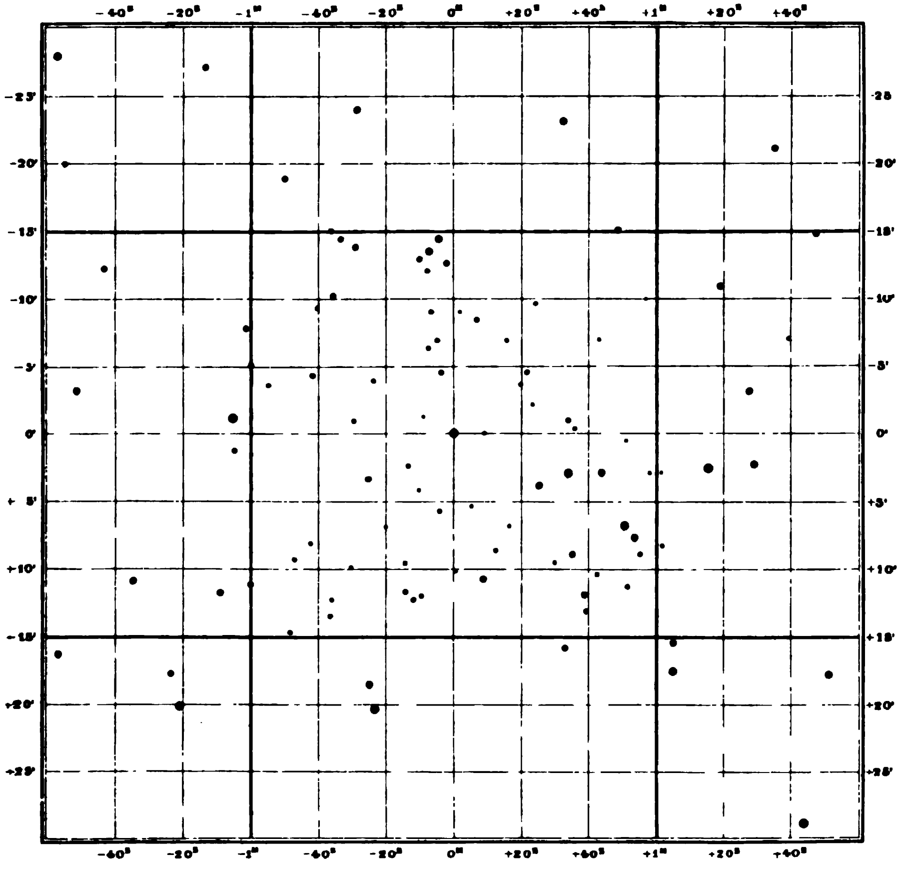
PLATE V.

2857

U Puppis

(1900.0) $7^h\,56^m\,8^s$ ($+2^s.81$) $-\,12^\circ\,33'.8$ ($-0'.16$)

Color: 3.2; III. **Magnitudo:** $8\frac{1}{2}$ – < 14.



7 8 9 10 11 12 13

Series I.

variable stars, observers will obtain at their request photographic prints from the Harvard College Observatory, as was announced by the Director at the astronomical congress held there this summer.

The first three series of this atlas, of which the accompanying chart is a specimen, will contain charts for all variable stars (not excluded by the above rule), within 0° and 115° polar distance, whose minimum brightness is beyond the reach of a three-inch telescope.

The fourth and fifth series will be devoted to those variable stars whose entire range of variation can be followed respectively by a three-inch telescope and by the naked eye.

Observers of variable stars will thus be able to procure for themselves that one of the five series which best suits their instrumental means and their geographical latitude. They will find the programme of their work prepared, the comparison stars mapped, and, what is the greatest advantage of the atlas, the variable star in the center of each chart identified beyond doubt.

2. The methods by which the stars on the charts were determined both as to position and magnitude, would require more space than is intended for this explanatory note. We can refer the reader who wishes to inform himself on this point, to the first announcement of the Atlas in the *Vierteljahrschrift* (XXXI, p. 278), or to a fuller description of the observations and reductions in the *Publications of the A. S. P.* (X, p. 100).

3. An inspection of the accompanying chart shows that it is divided into two parts, one interior and the other exterior, separated from each other by heavy lines. The interior square represents the chart proper, with the variable in the center and all the stars around it that can be easily seen and measured with our twelve-inch equatorial, while the outer part, surrounding this chart and forming itself a square of 1° in both coördinates, contains only the *BD* stars, corrected and supplemented where it was necessary. The circle in the middle of the chart indicates, by its diameter, the maximum light of the variable star. The minimum light, whenever visible in our twelve-inch refractor, is denoted by a smaller disk in the center of this circle. The

identification of the variable star, which is the essential purpose of these charts, seems to be put beyond doubt, because no chart was sent to the engraver before the star in the center was seen to vary by myself, and all the proof sheets have been examined by two observers of high authority, Dr. Hartwig and Mr. H. Parkhurst.

The projection of the charts is, for the first four series, based on Mercator's principle, except that on each chart the lines representing the parallels are placed at equal distances. In this way the engraving, which is done directly from the catalogue by means of a micrometer screw, is greatly facilitated, whilst the distortion of the small field is almost insensible. The red color of the net will completely disappear to the eye when a red lantern is used during the observation. The chart will then resemble the sky as nearly as possible.

The inscription of the chart furnishes what is useful at the telescope, and no more. The "color" is the "redness" of the variable, on Chandler's scale of ten units from white to red, and the Roman figure denotes the spectrum of Secchi's types.

The readers of this JOURNAL may remember, from a previous communication (Vol. VI, pp. 441–443, of this JOURNAL), that the atlas will be accompanied with a catalogue, which gives the positions of all the stars, differentially from the variable, to full seconds of time and to tenths of minutes of arc, and besides the estimates of brightness in steps and the computed magnitudes. The meaning of these magnitudes has been defined in the paper just referred to.

It may be of interest to know that the engraving of the first series, which comprises the variables between 0° and -25° declination, is finished, and that the whole atlas is published by Mr. Felix L. Dames, Berlin W. 62, Landgrafenstrasse 12.[1]

Finally I may be allowed to repeat in this place what has been stated elsewhere, that astronomers are indebted to Miss Catherine W. Bruce for aiding the expensive publication of this atlas.

GEORGETOWN COLLEGE OBSERVATORY,
September 8, 1898.

[1] We are informed that the subscription price for all the five series is 1 mark per chart, and that of a single series 1.20 marks per chart, including the catalogue.

THE VARIABLE STAR U PEGASI.[1]

By G. W. Myers.

The excuse for my infringement on your time and patience lies in the fact that Professor Pickering saw fit to put me here, and some of us have come to think that when he makes up his mind on any subject relating to variable star astronomy, the best thing we can do is to concur in his judgment. In this instance, however, I shall allow my opinion that your time and thought might be more profitably employed in practical matters than in listening to me, to assert itself to the extent of enforcing brevity on my part, perhaps even at the expense of clearness To conform to this purpose I shall be able to give but a very meager outline of my recent study of U Pegasi's light changes. The discussion will be published in detail as a *Bulletin* of our Observatory, and copies of it may be had by applying to the library of the University of Illinois.

Professor Pickering has shown, in *H. C. O. Circular* No. 23, that U Pegasi is to be considered a member of the β Lyrae class of variables. In that circular he gives the observations, in the form of a light curve, on which he bases his conclusion, and in the Astrophysical Journal for March of this year he shows from the internal evidence of the observations that the reality of the difference of 0 15 of a stellar magnitude in the brightness at the minima cannot be denied. The writer wishes to add, moreover, that a recent observational study of β Lyrae's light change with one of Professor Pickering's polarizing photometers removes from his mind the last vestige of doubt as to whether the attainable accuracy of photometric measures with this instrument is sufficiently high to detect and confirm so small a difference of brightness as this. He is convinced that, with a little experience and care, the probable

[1] Read at the Harvard Observatory Astronomical Conference, August 19, 1898.

error of a single observation in a set of ten could hardly exceed one-fifth of this difference.

For example, with photometer *V*, I triangulated the three stars β, ν, and 8 Lyrae on August 5 inst., with the following results, which I think may be regarded as fairly typical of the character of the work of a beginner with this instrument:

$$\nu-\beta=1.96 \quad 8-\beta=2.48 \text{ and } 8-\nu=0.46.$$

From the last two we have

$$\nu-\beta=2.02,$$

giving a discrepancy of 0.06 of a magnitude. The result of each of six other evenings' work are also well represented by this. I wish, moreover, to emphasize that this is the work of a *beginner.*

Granting the reality of this difference in the minima, the light curve appears to be susceptible of treatment by essentially the same method as that adapted and used by the writer in his recent discussion of β Lyrae's light curve entitled: *Untersuchungen ueber den Lichtwechsel des Sternes β Lyrae*, Muenchen, 1896.

Assuming provisionally, that both bodies are spheres, a lower limit for the eclipse-duration at Min. I can be easily obtained from the observational curve given in *Circular* 23. A little reflection will make it clear that the shorter the eclipse-duration be taken, the larger will be the corresponding distance between centers of the components. If, then, we assume that the eclipse has not begun until the light curve has fallen quite appreciably and that it has ended shortly before the curve ceases to rise, we shall obtain a value for the duration of the eclipse, at all events short enough—perhaps too short—and the corresponding value of the distance of centers must be at all events great enough—perhaps too great. Proceeding thus, I obtain $3^h.3$ for the interval shorter than which the eclipse-duration at Min. I cannot be. The corresponding value of the distance between centers may then be regarded as fixing a superior limit for this orbital element.

ECCENTRICITY.

The fundamental hypothesis underlying the whole discussion is that the light curve of U Pegasi is capable of being explained on the satellite theory.

The uncertainty in the instants of maximum brightness as indicated by the light curve obviously precludes the possibility of deriving an approximate value of the orbital eccentricity of the component from the chief epochs of light variation, as was done with β Lyrae. One may readily convince himself by considerations adduced below, however, that this eccentricity must be quite small.

Taking account of the relative positions of the components at the beginning and end of the eclipse at Min. I, the most elementary considerations give for the distance of centers, r, the expression:

$$r \leqq (x + \kappa) \csc 66^\circ$$
$$\leqq 1.0946\,(x + \kappa)$$

Where x and κ denote magnitudes of such nature that $x \leqq 1$ and $\kappa \leqq 1$. We shall have then $r \leqq 2.189$ times the radius of the larger companion. So small a distance of centers relative to the dimensions of the primary would be physically impossible unless the radius of the secondary were quite inconsiderable compared with that of the primary, which is shown later not to be the case.

It was therefore assumed as a first approximation that $e = 0$, and we then proceed to determine the value of the ratio of the brightness of the companions, and to fix the limits within which the ratio of the radii must be comprised. We then undertake to find the most probable value of this latter ratio by direct reference to the light curve of the star.

CIRCULAR ORBITAL ELEMENTS AND LIGHT RATIO OF THE COMPONENTS OF U PEGASI.

The chief epochs of the light curve shall be designated in order of time as Min. I, Max. I, Min. II and Max. II. From

the curve Max. I is seen to have a brightness of 9.32 magnitude and Max. II of 9.34 magnitude, so that the mean value $9^m.33$ has been used throughout the discussion for the brightness at both the maxima. For the brightness at Min. I, the value 9.90 magnitude has been used, and for Min. II, 9.75 magnitude. Reducing these differences in stellar magnitudes at the chief epochs of variability to their equivalent light ratios, by the aid of Pogson's scale, we obtain:

$$\frac{\text{Brightness at Min. II}}{\text{Brightness at Min. I}} = c = 1.1480$$

$$\frac{\text{Brightness at Mean Max.}}{\text{Brightness at Min. I}} = m = 1.6904$$

A STUDY OF THE LIGHT CURVE.

Calling the light ratio of the components λ and the portion of the disks common to both bodies at the middle of the eclipses a, the preceding equations give the following:

$$\frac{1 + \kappa^2\lambda - a\kappa^2\lambda}{1 - a\kappa^2 + \kappa^2\lambda} = c \tag{1}$$

$$\frac{1 + \kappa^2\lambda}{1 - a\kappa^2 + \kappa^2\lambda} = m, \tag{2}$$

where κ is the ratio of radii of the components.

If it be thought desirable to include the possibility of a flattening of the disks, we may assume, as a means of making a first approximation to the general effect of such deformation, that the bodies are similar ellipsoids of revolution and designate by q the common ratio of the semi-major to the semi-minor axis, whereupon equation (2) must be replaced by

$$q\,\frac{1 + \kappa^2\lambda}{1 - a\kappa^2 + \kappa^2\lambda} = m. \tag{2a}$$

(Cf. this JOURNAL, 7, 13, where $a\ \kappa^2$ should be stricken from the numerator of (e).)

From (1) and (2a) we find readily

$$\frac{a\kappa^2\lambda}{1 + \kappa^2\lambda} = (m - c\,q)/m, \tag{3}$$

and
$$\frac{a\kappa^2}{1+\kappa^2\lambda}=(m-q)/m, \tag{4}$$
whence, dividing, we get
$$\lambda=(m-cq)/(m-q). \tag{5}$$

Neglecting the flattening provisionally, *i. e.* putting $q=1$, (5) gives, when the foregoing values of c and m are substituted,
$$\lambda=0.7865.$$

By making some easy transformations and reductions, we find:
$$\frac{m-q}{cq}\leqq\kappa^2\leqq\frac{q}{m-cq}. \tag{6}$$
From this, we have also,
$$\frac{m-q}{cq}\leqq\frac{q}{m-cq},$$
and finally
$$q\geqq\frac{m}{c+1}.$$

Substituting now the former values of m and c, we obtain
$$q\geqq 0.787.$$

It does not therefore appear to be necessary to assume the existence of a flattening for U Pegasi, such as was shown to be necessary in my "Dissertation on β Lyrae," p. 30, for the latter star.

Taking again the value of q as unity, and substituting in (6) we find:
$$0.6014\leqq\kappa^2\leqq 1.845, \text{ or } 0.7755\leqq\kappa\leqq 1.358.$$

The following test values distributed linearly over this interval were, therefore, selected for criteria to an approximation to κ:
$$0.80,\quad 0.85,\quad 1.00,\quad 1.15,\quad \text{and}\quad 1.35,$$
and for each of these values a light curve was computed by the method and with the results given below.

Using the portion of the light curve lying within 1.5 hours before and after Min. I., and the notation (see Fig. 1) and equations developed in my dissertation and published in the ASTROPHYSICAL JOURNAL for January 1898, I have to compute the

values of M and H from the data furnished by the light curve, and then for $\kappa < 1$, to solve the transcendental equations:

$$\begin{cases} M = \phi + \kappa^2 \phi'' - \kappa \sin (\phi'' + \phi) \\ H = \kappa^2 \phi' - \phi + \kappa \sin (\phi' - \phi), \end{cases} \tag{7}$$

and for $\kappa > 1$,

$$\begin{cases} M = \phi + \kappa^2 \phi'' - \kappa \sin (\phi' + \phi) \\ H = \phi_1 - \kappa^2 \phi'' + \kappa \sin (\phi_1 - \phi'') \end{cases} \tag{8}$$

for ϕ and ϕ'', and when $\kappa = 1$,

$$M = 2\,\phi - \kappa \sin 2\,\phi = 2\,\phi - \sin 2\,\phi \ (H \text{ being here zero}). \tag{8a}$$

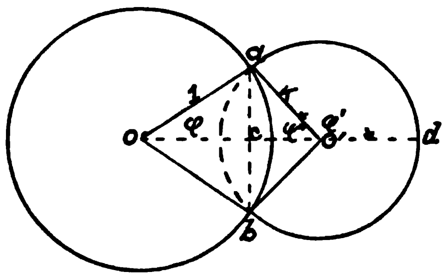


FIG. 1.

These solutions may be made most conveniently by means of tables giving the values of M and H for suitably chosen values of ϕ and ϕ', from which approximate values of ϕ and ϕ' may be interpolated, which may then be corrected by suitable differential formulæ.

Having thus obtained the value of ϕ for a number of points on the light curve (15 were used), the corresponding radii vectors ρ of the apparent orbit were computed by means of

$$\rho = \frac{\kappa \sin (\phi' + \phi)}{\sin \phi}, \ \sin \phi' = \frac{\sin \phi}{\kappa}$$

for each test value of κ.

We then readily obtain the relation

$$\rho^2 = r^2 \sin^2 \beta + r^2 \cos^2 i \cos^2 \beta$$

Where β is the longitude in the apparent orbit reckoned from Min. I, r is the radius vector in the true orbit and i the inclination.

Putting:

$$x = r^2 \text{ and } y = r^2 \cos^2 i,$$

we have from the above relation:

$$\rho^2 = x \sin^2 \beta + y \cos^2 \beta. \quad (\beta = \mu t = 40^\circ \times t, \ t \text{ in hrs. from Min. I}).$$

In this we shall have the values of ρ^2 and β for the chosen points, and, solved by least squares, the equations furnish the most probable values of x and y, and, through these, of r and i.

The mean values and probable errors for each of the assumptions for κ are: For $\kappa = 0.80$, $r = 1.6636 \pm 0.0485$; for $\kappa = 0.85$, $r = 1.7512 \pm 0.0494$, and for $\kappa = 1.00$, $r = 1.9341 \pm 0.0535$. The individual determinations of $\cos^2 i$ are not given here, but the corresponding means and probable errors are, for the respective cases:

$$\cos^2 i = +\ 0.0275 \pm 0.0069; \ = +\ 0.0482 \pm 0.0072;$$
$$= +\ 0.0547 \pm 0.0074.$$

The difference of the probable errors is not great in any case, but both r and $\cos^2 i$ agree in their testimony favoring the smallest value of κ as being the most probable. Assuming this value of κ however, a physical peculiarity, though not an impossibility, is met in the circumstance that the most probable distance of centers (1.6634) is considerably less than the sum of the radii ($= 1.8$), *i. e.*, the masses must interpenetrate, and consequently form a single body (Poincaré's apiod).

The probable errors not differing by enough to enable them to pronounce with sufficient emphasis for any one of the hypotheses, it seemed desirable to approach the problem also indirectly to see whether the conclusions will be the same as those given by this direct solution. That the foregoing discussion, however, indicates conclusively that the correct value of κ is smaller than 0.85, there can be no doubt.

INDIRECT SOLUTION.

The mode of procedure here is to read from the light curve, for suitably chosen epochs, the instantaneous brightnesses in stellar magnitudes, to form the differences between these bright-

nesses and the maximum brightness, to convert these differences, by means of the Pogson scale, into their equivalent light ratios, to compare these ratios with the corresponding ratios, computed from certain assumed elements, and finally, after finding sufficiently close approximations to the correct values of the elements, to adjust these differences in the sense computation minus observation, by the method of least squares.

Letting J' and J'' denote the instantaneous brightnesses in the neighborhood of Min. I. and Min. II., respectively, and M', H', M'' and H'', the corresponding values of the M and H defined by equations (12), it will be seen by referring to my article on β Lyrae, in the January ASTROPHYSICAL JOURNAL, that

$$1-J'=\frac{M'}{\pi(1+\lambda\kappa^2)}; \quad 1-J''=\frac{\lambda M''}{\pi(1+\lambda\kappa^2)}; \tag{10}$$

and hence there is an obvious advantage in adjusting $1-J'$ and $1-J''$ instead of J' and J''. The former quantities were therefore used throughout the reductions.

The equations for computing M', or M'' are:

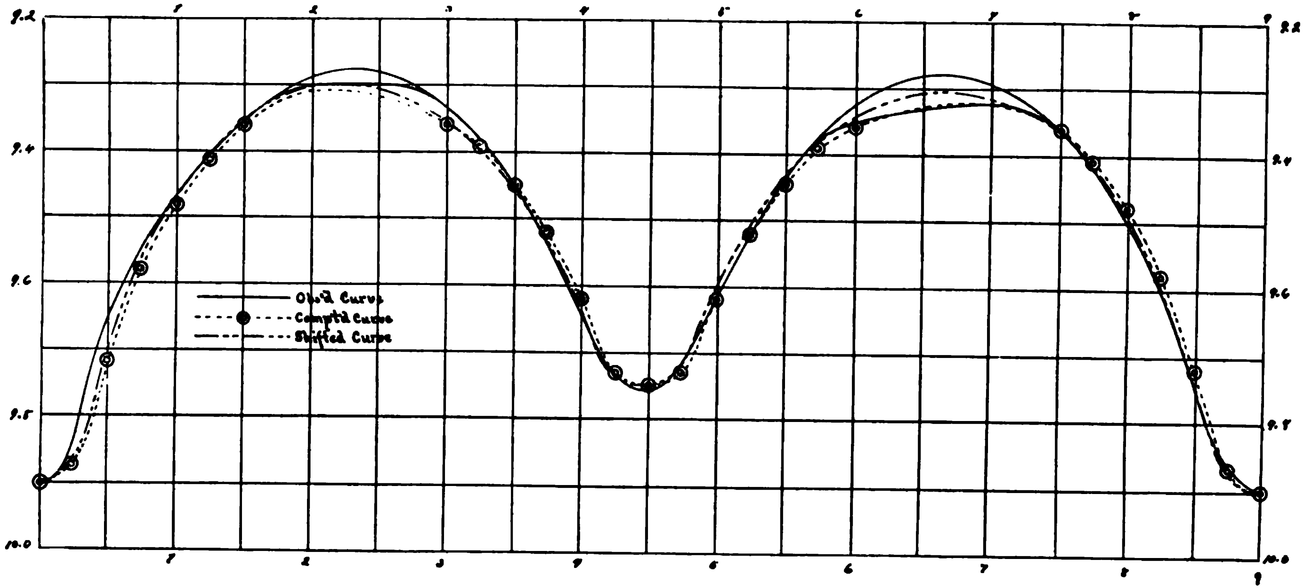
$$\left\{\begin{array}{l}
(a)\ \beta=40^\circ\times t. \\
(b)\ \rho=r\sqrt{\sin^2\beta+\cos^2 i\cos^2\beta}.\ \text{If } i=\frac{\pi}{2}-i' \text{ is near } \frac{\pi}{2}, i' \text{ is small and} \\
(c)\ \rho=r\sqrt{\sin^2\beta+i'^2\cos\beta}.\ \text{If } i'=o,\ \rho=r\sin\beta. \\
(d)\ \cos\phi=\dfrac{1+\rho^2-\kappa^2}{2\rho}. \\
(e)\ \sin\phi'=\dfrac{1}{\kappa}\sin\phi. \\
(f)^*\ M',\text{ or } M''=\phi+\kappa^2\phi''-\kappa\sin(\phi+\phi'')=\phi+\kappa^2\phi''-\rho\sin\phi.
\end{array}\right. \tag{11}$$

Guided by the results of the direct solution, certain values were assumed for r, κ, q, λ and i', and a light curve was computed. Arbitrary but small changes were then made in each of the elements, each time being guided, of course, by the results of the foregoing assumptions, until, in all, about twenty light curves were computed. The values $r=1.7816$, $\kappa=0.7785$,

*N. B.—If it be desired to introduce the effect of flattening, we have:

$$M=\phi+\kappa^2\phi''-\sin\rho'\phi, \text{ where } \rho'=\frac{1}{f}\rho \text{ and } \frac{1}{f}=\sqrt{\sin^2\beta+q^2\cos^2\beta}.$$

PLATE VI.


Obs'd Curve
Comp'd Curve
Shifted Curve
9.2
9.4
9.6
9.8
10.0
1
2
3
4
5
6
7
8
9

$\lambda = 0.7748$, $q = 1.02$ and $i' = 0$, gave quite a satisfactory curve, so satisfactory as to seem to warrant the use of a least square adjustment. The resulting corrections contained discordances among themselves and contraventions of fundamental physical laws to so great an extent as to cast entire discredit on the value of the least square adjustment. Several attempts were made to adjust the residuals furnished by other sets of elements with no better outcome.

Making a slight shift in the epoch of Min. I, which Professor Pickering has informed me personally to be allowable in consideration of the rather scanty observational data available for its determination, the close conformity of the computed with the *mean* observed curve may be seen from this table:

COMPARISON OF COMPUTED WITH OBSERVED CURVE.

t	J_o	J_c	ΔJ_{o-c}	J_o	J_c	ΔJ_{o-c}
	m.	m.	m.	m.	m.	m.
1.50	9.35	9.36	− 0.01	9.34	9.36	− 0.02
1.25	9.41	9.41	0.00	9.37	9.39	− 0.02
1.00	9.48	9.48	0.00	9.43	9.45	− 0.02
.75	9.58	9.58	0.00	9.52	9.52	0 00
.50	9.70	9.72	0.02	9.62	9.62	0.00
.25	9.84	9.87	0.03	9.72	9.73	− 0.01
0	9.90	9.90	0.00	9.75	9.75	0.00

Average deviations = 0.009 = 0.01

The closeness of the correspondence of the computed with the *actual*, not *mean*, observed curve may be judged by reference to the detailed discussion contained in the *Bulletin* referred to at the outset, and is seen by comparing the shifted with the observed curve on the accompanying plate. I do not believe, after a protracted and rather tedious study of the star, that a better theoretical representation of U Pegasi's light curve is possible in the present *status* of the observational material. It seems to me, moreover, that the following results are pretty clearly indicated:

1. The light curve of U Pegasi given in *Harvard College Observatory Circular* No. 23, is satisfactorily represented by the satellite theory.

2. The distancc of centers does not materially differ from the sum of the radii of the components, suggesting the probable existence of the "apiodal" form of Poincaré.

3. The smaller companion is about 0.77 as bright as the larger, and the ratio of radii is approximately 1:0.78.

4. The inclination of the orbit is very nearly 90°, and the disk of one or both bodies, if separate, is slightly flattened.

5. The accuracy of present observations does not suffice to determine the elements of the "system" completely, since the foregoing discussion shows the residuals to be incapable of adjustment by least squares.

6. The manner of rise and fall of the observed curve after and before the minima, which portions of the curve were determined with especial care, fails to confirm one's first impression on examining the curve, viz., that the components are separated enough to remain apart for an appreciable time at the maxima. The difference between the durations of uniform brightness at the maxima, as shown by the curve, would seem to indicate a considerable orbital eccentricity, whereas the small distance of centers nullifies the possibility of its existence. It, therefore, seems desirable to direct attention to the importance of a careful photometric study of U Pegasi's light curve near the maxima, with a view to ascertaining whether or not the form of this curve near these epochs is real.

It may be objected that the inclusion of points in the vicinity of the maxima might exercise a modifying influence on the conclusions of this paper. A very little reflection will make it clear, when we compare the computed with the observed curve, that including such points can have no such influence whatever. The points selected were taken from those parts of the curve where the observations were most accordant, and where the accompanying description indicated that most care had been taken to eliminate errors of various sorts. To have included more points would only have rendered the computations more laborious and less readily comprehensible.

THE K LINES OF β AURIGAE.[1]

By Antonia C. Maury.

A series of two hundred photographs of the spectroscopic binary β Aurigae, obtained at Harvard Observatory in the work of the Henry Draper Memorial, shows a periodic change in the intensity of the K lines of the combined spectra.

The components of the binary revolve in a period of $3^d\ 23^h\ 37^m$ approximately, and with a combined velocity in the line of sight of 240^{km} a second. The distance between their centers, supposing the line of sight to lie in the plane of the orbit, is only about eight million miles. When the revolving stars are traveling at right angles to the line of sight, the lines of the combined spectrum appear single, but when one star is approaching and the other receding, the spectral lines are seen double, the components belonging to the approaching star being shifted toward the violet and those of the star receding, toward the red. The lines are accordingly double on every alternate night, and at intervals of two nights the relative position of the stars is reversed.

The photographs were taken with the 11-inch Draper telescope, two objective prisms being used in the case of 120 photographs, and three or four prisms for the eighty remaining, The series extends over a period of nine years, from 1889–90 to 1897–8, the photographs having been taken during from two to five of the winter months of each year, excepting 1896–7. These observations sometimes began as early as October and sometimes ended as late as April.

A detailed examination of the plates shows that the relative intensity of the K lines of the component stars is reversed each year as compared with the year preceding. The following table shows the results obtained. The first column gives the year of the observations, the second the number of plates taken, the third

[1] Read at the Harvard Conference, August 20, 1898.

the percentage of these in which the K line of one star, which we may call A, appeared the more intense; the fourth the percentage in which the K line of star B appeared more intense, and the fifth the percentage showing the K lines equal.

ALTERNATION IN INTENSITY OF K LINES.

Year	Number of plates	Stronger K line		K lines equal
		A	B	
1889–90......	23	70	17	13
1890–91.....	72	10	68	22
1891–92......	29	86	7	7
1892–93......	13	8	77	15
1893–94......	11	46	18	36
1894–95......	24	13	58	29
1895–96......	9	89	11	0
1897–98......	23	83	0	17

A few photographs showed a slight change in the intensity of the components of the double $H\gamma$ and $H\delta$, and the line of wave-length 4481.4, which is the most conspicuous of the fainter lines. This appeared to correspond with the variation in the K lines, but was too slight to be satisfactorily verified. It does not, however, seem unlikely that the spectra as a whole may vary, since the K line is the line of first type spectra best adapted to show slight changes, the lines of hydrogen being too wide and hazy, and the remaining lines too faint.

In the case of the spectroscopic binary μ' Scorpii it was observed by Mr. Bailey that the difference in the intensity of the lines of the component spectra appeared to change as if due to a variation in the light of one of the components.[1]

It does not seem probable that the apparent change is due to difference in the quality of the plates. Here both spectra are of the Orion type and the variation appeared in the hydrogen lines and lines peculiar to the Orion type.

Photographs of ζ Ursae Majoris, like β Aurigae, of the first type, give also interesting results. Twenty-two plates, taken

[1] *Harvard College Observatory Circular* No. 11, August 31, 1896.

between March 1887 and June 1890 show the line toward violet to be in most cases the more intense; while in nearly all the seventy-seven plates taken between May 1891 and July 1896 the line toward red is more intense. As the period of ζ Ursae Majoris is as yet undetermined, it is not known whether this indicates an actual reversal of intensity, as in β Aurigae, or whether the component stars have reversed their relative position.

The components of β Aurigae are probably nearly or quite equal in mass, and their spectra are closely similar. The most probable theory would seem to be that the revolving stars induce in one another reciprocal variability. If the period of this variability be two years it would represent about 180 revolutions. The components are conjectured to have each a probable mass of 1.25 times that of the Sun, and the probable distance between their centers being only about eight million miles, their influence on one another, electromagnetic or tidal, might be very great.

It is to be hoped that more definite results from the other spectroscopic binaries may show whether or not their components are in general variable.

NOTE.

It was stated at the Conference that in the double spectrum of β Aurigae the K of the line star approaching in the line of sight appeared wider than that of the star receding. From a later examination of the plates it seems probable that the greater width observed in the K line of the spectrum lying toward the violet may be due to a faint line of the superposed spectrum combining with this K line. It was previously supposed that the line referred to was too faint to alter the appearance of the K line, but it has since been found that the observed effect can be in part imitated by superposing two plates in which the spectra are single. It seems probable, though not quite certain, that this may explain the observed appearance.

OBSERVATIONS ON THE ABSORPTION AND EMISSION OF AQUEOUS VAPOR AND CARBON DIOXIDE IN THE INFRA-RED SPECTRUM.

By H. RUBENS and E. ASCHKINASS.[1]

THE researches of Langley[2] show that the energy in the infra-red spectrum decreases very rapidly from wave-length $2^{\mu}.7$ onward, and that measurable amounts of energy occur only at a few places beyond the wave-length $\lambda=5^{\mu}$. The cause of this falling off in energy, which is very marked as compared with that in terrestrial sources of light, was recognized by Langley in the absorption by the Earth's atmosphere. In so far as it was not already established by Langley's own experiments, this view has been fully confirmed by the researches of Ångström[3] and Paschen.[4] Accordingly, there can be no doubt that aqueous vapor and carbon dioxide possess a very strong absorptive power in the spectral region in question, and that our atmosphere contains a sufficient amount of these gases to reduce the intensity of the solar rays practically to zero in most of these parts of the spectrum.

One of us, in conjunction with E. F. Nichols,[5] has recently shown that aqueous vapor and carbon dioxide exert only a very slight absorption upon the rather homogeneous bundle of rays of a mean wave-length of $24^{\mu}.4$, which can be separated from the total emission from a source of radiation by repeated reflection from surfaces of fluor-spar. It therefore did not seem impossible that the presence of these rays of wave-length 24^{μ} in the solar spectrum beyond the above-mentioned region of intense absorption by these gases might be demonstrated.

[1] Translated from *Wiedemann's Annalen*, **64**, 1898.

[2] LANGLEY, *Phil. Mag.* (5), **26**, 505, 1888.

[3] ÅNGSTRÖM, *Bihang till K. Svenska Vet. Akad. Handlingar*, **15**, Afd. 1, Nr. 9, 1889.

[4] PASCHEN, *Wied. Ann.*, **53**, 341, 1894.

[5] H. RUBENS and E. F. NICHOLS, *Wied. Ann.*, **60**, 418, 1897.

That the earlier observers did not succeed in detecting these rays is sufficiently explained by the fact that they worked with prisms of fluorite and rock salt—substances through which these rays do not pass.

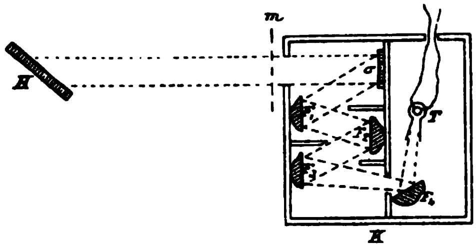


FIG. 1.

In answering the questioh as to the presence of these rays in the solar rays that reach us, we made the following experiment: We caused the solar rays reflected from the front silvered surface H of a heliostat mirror (Fig. 1) to undergo four reflections on the fluor-spar surfaces F_1 to F_4, after they had been rendered slightly convergent by the concave mirror σ. A linear thermopile, constructed by one of us,[1] served as the measuring instrument, in connection with a sensitive galvanometer protected against magnetic disturbances. The sensitiveness of the thermopile, which in these experiments was provided with a cone of 2 square centimeters' aperture, is approximately indicated by the fact that the radiation of a candle at 2 meters distance produced a throw of about 400^{mm} on the scale, on which fractions of a millimeter could be readily perceived. To avoid disturbances by currents of air and diffuse rays the mirror σ as well as the four fluor-spar surfaces and the thermopile were placed in a box K, in which screens were set in suitable places. The observer at the galvanometer could cause the heat-rays to enter the box by drawing up with a cord the glass screen m which was in the path of the rays.

The result of the experiment was a negative one. We did

[1] A full description of this thermopile may be found in *Z. f. Instrum.*, March 1898, p. 65.

not succeed in obtaining measurable deflections on drawing up the glass screen. But when we sent into the box the rays from a zirconium burner, rendered parallel by a concave mirror, instead of the solar rays, we observed a galvanometer throw of more than 200^{mm}, which wholly subsided when a rock salt plate of 5^{mm} thickness was inserted in the path of the rays, thus proving the accurate adjustment of the fluor-spar surfaces, and the considerably high sensitiveness of the apparatus.

We therefore held it to be not improbable that the residual rays of the fluor-spar, as well as the greatest part of the infra-red spectrum thus far investigated, were absorbed in the atmosphere, and it seemed desirable to repeat the earlier experiments on the absorption of carbon dioxide and aqueous vapor.

For this purpose we conducted a stream of dry carbon dioxide into the box *K* through a lateral aperture. The extinction of a burning match indicated that the air had been entirely displaced. In this way the rays from the zirconium burner were made to traverse a layer of carbon dioxide 60^{cm} long. In this experiment an air-tight chloride of silver plate was cemented in front of the aperture of the cone so that the carbon dioxide could not penetrate within. This precaution is necessary because otherwise the observed deflections would be altered in consequence of the slight thermal conductivity of the carbon dioxide, even without the effect of absorption.

The intensity indicated by the thermopile was, however, just the same as when the box was filled with air, so that no absorption of the heat rays by the carbon dioxide could be detected.

An appreciable absorptive effect of aqueous vapor was equally impossible to obtain when a stream of aqueous vapor was introduced in the path of the rays as above described.

Since, however, with this arrangement only a rather slight layer of the vapor is traversed by the rays, we varied the experiment as follows. A cast-iron tube, *E*, 40^{cm} long and 5^{cm} wide, was so arranged (Fig. 2) that the rays from the concave mirror *S* had to pass through its entire length. Steam could be conducted at pleasure into the tube *E* by heating the water in the

flask *F*, while the tube was maintained by three Bunsen burners at a temperature above 100° in order to avoid the condensation of the water vapor inside the tube. The drop-screen *m* was placed directly in front of the zirconium burner *Z* in order to

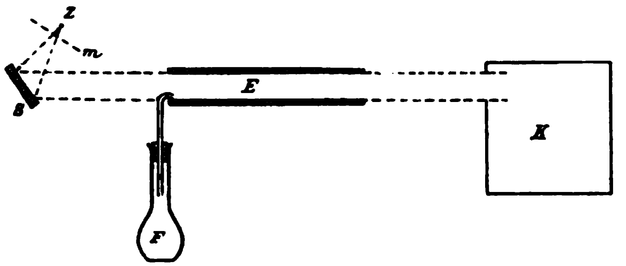


FIG. 2.

avoid the errors which would result from the heat emission of the hot tube. The distance of the opening of the tube *E* from the point of entrance of the rays into the box was made sufficient (about 25^{cm}) to make it impossible for the steam to enter the box. This provision is important, as otherwise the water vapor might condense on the reflecting surfaces. We have convinced ourselves by special tests that this did not occur.

It now appeared that the intensity of the residual rays from the fluor-spar was reduced to 31 per cent. of its original value as soon as the tube *E* was filled with water vapor. Hence, with the use of the longer layer, water vapor exhibited a clearly perceptible absorptive power for these rays sufficient to prevent their transmission by the Earth's atmosphere.

After this result had been gained, the next investigation was to see whether an emission of these rays from the heated water vapor could be proven. For this purpose the zirconium lamp was removed and the screen *m* placed again directly in front of the box *K*, without other change of the apparatus.

We then observed that when the tube *E* was filled with air a deflection of 20^{mm} occurred on drawing the screen *m*, chiefly due to the radiation from the tube itself; when the steam was conducted through the tube the deflection increased to 25^{mm}; hence

we obtained a distinct emission from the vapor in amount pretty nearly represented by the difference of the deflections.

It was to be expected after this experiment that the spectrum of the Bunsen burner would contain the residual rays of the fluor-spar. In fact, an ordinary triple burner placed in front of the opening of the box *K* gave deflections of 25^{mm}. A proof of the purity of the radiation was furnished by the insertion of a rock salt plate 5^{mm} thick in their path, with the result that the rays were wholly absorbed.

After we had further experimentally shown that the residual rays of quartz are present in considerable quantity in the radiation of the Bunsen burner, we resolved to undertake a systematic investigation of the spectral distribution of its radiation, as well as of the emission and absorption of aqueous vapor and carbon dioxide.

Paschen's[1] careful measurements of radiation and absorption of these gases include the region from immediately beyond the visible spectrum to about wave-length, $\lambda = 9^{\mu}$, a limit which could not be passed on account of the absorption by the fluorite prism which produced the spectrum. Our observations begin at this point and extend approximately to wave-length $\lambda = 20^{\mu}$.

EMISSION SPECTRA.

We began with the investigation of the emission of the Bunsen burner. The spectrum was produced by the large mirror spectrometer already often employed by one of us,[2] carrying a sylvin prism of 6^{cm} height, and $43° 57' 50''$ refracting angle, which almost entirely filled the objective of the reflecting telescope of $5.^{cm}5$ aperture and 56^{cm} focal length. The prism was maintained at minimum deviation by an automatic attachment. The dispersion was calculated according to the observations recently published by one of us in conjunction with Trowbridge.[3] According to these the spectrum extended over $1° 33'$ from the

[1] F. PASCHEN, *Wied. Ann.*, 51, 1; 52, 209; 53, 335, 1894.

[2] A short description of the instrument may be found in *Wied. Ann.*, 54, 270, 1894.

[3] H. RUBENS und A. TROWBRIDGE, *Wied. Ann.*, 60, 724, 1897.

D lines to 9^{μ}, and over $3^{\circ}\ 21'$ from 9^{μ} to 20^{μ}. Thus the conditions are distinctly more favorable in the latter region. Moreover, there are here no sharp absorption bands of sylvin like those observed at $\lambda\ 3.^{\mu}20$ and $\lambda\ 7^{\mu}.08$, but the absorption begins at 13^{μ} and increases very slowly and steadily. According to a rough estimate, at $\lambda\ 18^{\mu}$ some 70 per cent., and at $\lambda\ 20^{\mu}$ some 30 per cent. of the incident radiation is transmitted by the prism.

As in the previous experiments, the observations of energy were made with a linear thermopile constructed for measurements in the spectrum. The fifteen odd junctions, which were in a vertical line, were cut out by a number of rectangular diaphragms, the smallest of which was $0^{mm}.6$ wide and close to the thermopile. A bright sheet of copper was attached directly behind the thermopile, in order to reflect back upon the elements a part of the energy that had escaped between them. In the spectral region concerned the reflecting power of copper is sufficiently constant to prevent errors in the distribution of the energy from the use of the copper mirror.[1] Three slits were made in the copper to permit optical settings. The theoretical sensitiveness of the thermopile, made from wires of iron and "constantan" of $0.^{mm}1$ thickness, is calculated to be $15 \times 53 = 800 \times 10^{-6}$ volts per centigrade degree. Its internal resistance amounted to about seven ohms, the external resistance of the circuit (galvanometer and connecting wires) to little over five ohms. The current sensitiveness usually employed was about 3.3×10^{-10} amperes per millimeter of deflection. It follows from these data that a rise of temperature of the odd junctions of 5.9×10^{-6} centigrade degrees corresponds to 1^{mm} of galvanometer deflection. With this sensitiveness the constancy of the zero point, particularly in the evening hours, was very satisfactory, and the consequent high accuracy of the measurements made it possible to greatly reduce the number of single observations and the duration of the series of experiments. This was of special importance in measurements of absorption in which the constancy of the source of heat during the series of measurements had to be

[1] This is known from unpublished observations in this physical laboratory.

assumed. In consequence of the small capacity for heat of the thermopile, the warming and cooling of the junctions was effected in a few seconds, so that the instrument behaved like a bolometer on raising and dropping the drop-screen.

The radiation was emitted by four Bunsen triple burners set up in a row in the prolongation of the axis of the collimator. A plane mirror, silvered on the front surface, placed behind the burners, practically doubled the number of radiating flames and produced a considerable increase in the energy.

One of the most important sources of error, which greatly increases the difficulty of measuring in the region of long wave-lengths, is the impurity of the spectrum, due to stray radiations. All except a minute fraction of these stray radiations belong to the region of short waves ($1-7^{\mu}$) of great energy, and can therefore be suppressed, by the use of a fluor-spar screen instead of a glass screen, which transmits these rays, down to a very small part, some 6 or 7 per cent. due to reflection of the fluor-spar screen. It is true that this procedure is theoretically not quite so correct as that of double spectral dispersion, but it has the practical advantage of greater simplicity, and avoids the considerable losses of energy in the latter process. We have convinced ourselves by investigating the selective absorption of rock salt that both methods furnish like results within the limits of errors of observation. We have, therefore, chiefly employed a fluor-spar screen in the region beyond 12^{μ}.

Beyond the limit $\lambda = 18^{\mu}$ the use of a rock salt screen is permissible if of sufficient thickness. In this part of the spectrum we repeatedly used as screen a rock salt plate of 23^{mm} thickness, which transmitted 4.9 per cent. at 18^{μ}, and 0.1 per cent. at 19^{μ}.[1] The results when this screen was used showed no appreciable differences from those obtained with a fluor-spar screen.

For the sake of completeness and to test our arrangement of experiments, we also measured the region of short-waves in the emission spectrum of the Bunsen burner already investigated by

[1] *Cf.* H. Rubens u. A. Trowbridge, *loc. cit.*, p. 736.

Paschen, but we would distinctly state that, on account of the slight dispersion and the selective absorption of sylvin, the

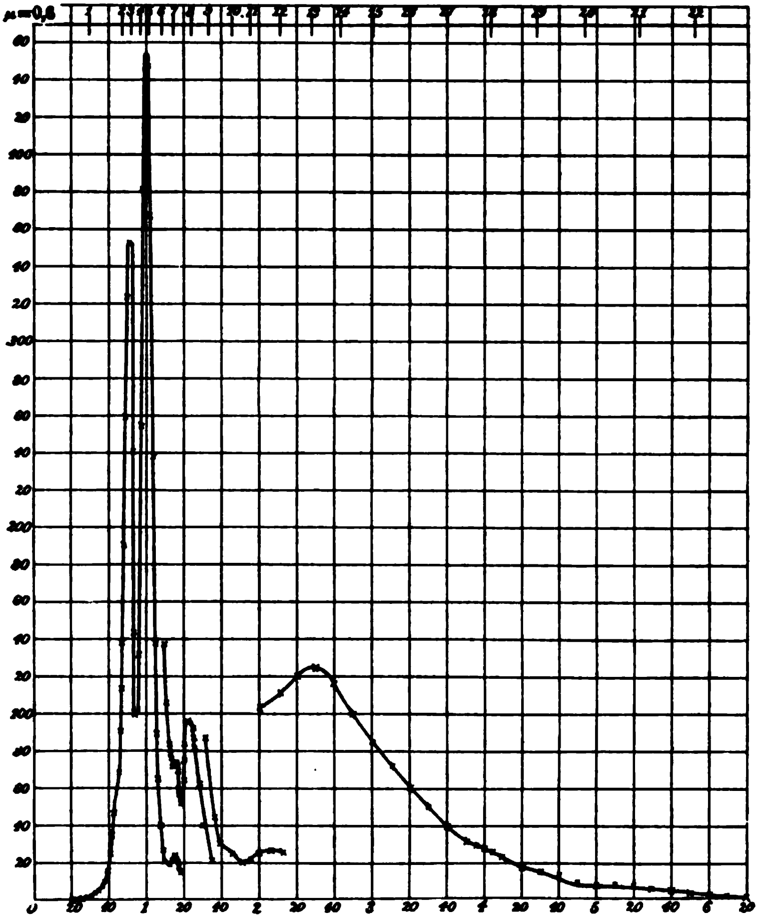


FIG. 3.

measures could not be such as to be comparable in accuracy with those of Paschen. The slit width was 0.^{mm}2, the width of the thermopile 0.^{mm}6 (see above). The results are shown graphically in Fig. 3. The ordinates represent the galvanometer deflec-

tions, the abscissas the differences of minimum deviations from those of the D line, and of wave-lengths. A closer examination of the energy curve shows the presence of all the stronger maxima of emission of water vapor and of carbon dioxide at the same positions where they were observed by Paschen in the dispersion spectrum of fluorite, with the exception of the elevation at $\lambda = 5^{\mu}.4$ which belongs to the emission spectrum of water vapor, and which is here wholly concealed by the strong emission band of carbon dioxide at $\lambda = 4^{\mu}.40$. With the width of slit employed the energy of the spectrum was sufficient for observation as far as $\lambda = 7.4$. At this point the bilateral slit was opened to $0.^{mm}4$, and then to $1^{mm}.0$, and with this width the measurements were continued respectively to $\lambda = 9.1$ and $\lambda = 12^{\mu}$, where a further opening of the slit to $5^{mm}.5$ was necessary. Accordingly the curve in Fig. 3 is in separate sections.

In the part of the curve of energy beyond $\lambda = 9^{\mu}$, here published for the first time, the emission apparently assumes a more continuous character. The curve shows at $\lambda = 10^{\mu}.7$ a minimum, at $\lambda = 13^{\mu}.1$ a faint maximum, and then approaches the axis of abscissas as an asymptote. We have not succeeded in resolving this broad band, reaching from $\lambda = 11^{\mu}$ to beyond $\lambda = 20^{\mu}$, into separate lines by narrowing the slit. The experiments with absorption communicated below teach, on the other hand, that we are here dealing with a large number of neighboring bands.

In order to determine what part the heated water vapor has in the region of long wave-lengths in the radiation of the Bunsen burner, we now placed a hydrogen flame before the slit of our spectroscope. In order to bring as thick a layer of water vapor as possible into radiation we had inclined the hydrogen flame at a very acute angle to the axis of the collimator. The radiation entering the slit was also intensified by use of a concave mirror. Fig. 4 shows the results of the observations which were again made with different widths of slit ($0^{mm}.5$ and $3^{mm}.5$), as is apparent from the jump in the curve. The first part of the curve contains the maximum of water vapor, lacking in Fig. 3, which is covered up by the emission band of carbon dioxide, at

$\lambda = 5^{\mu}.4$, and which was given by Paschen. Beyond $\lambda = 9^{\mu}$ the curve runs about as in Fig. 3.

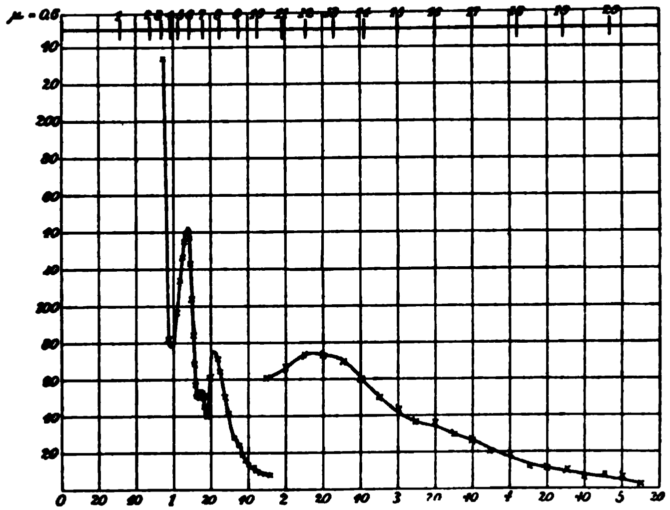


FIG. 4.

We investigated the emission spectrum of the heated carbon dioxide in the following manner. We placed (Fig. 5) before

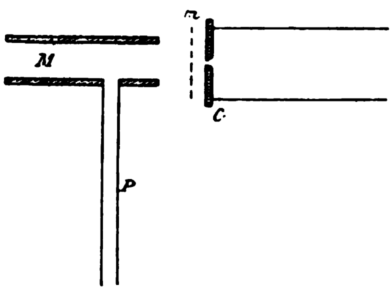


FIG. 5.

the spectrometer slit C a brass tube M, 25^{cm} long, and 4^{cm} wide, which was maintained at a high temperature by a triple burner. Into the side of this entered a platinum tube P, 14^{cm} long, and 8^{mm} in diameter, which was held at a bright red heat by a

second triple burner. A steady stream of dry carbon dioxide was conducted into this from a gasometer, and was thus brought to radiating. We satisfied ourselves by trials that no radiation from the burners or from the walls of the hot tube could enter into the spectrometer.

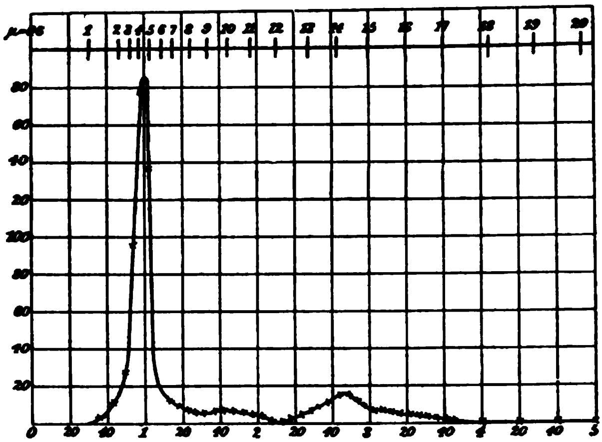


FIG. 6.

The curve in Fig. 6, which represents the results of our observations of the emission of hot carbon dioxide, shows that beside the previously known emission bands at $\lambda = 4^{\mu}.4$ and $\lambda = 2^{\mu}.7$, there is a third maximum with its greatest elevation at about $\lambda = 14^{\mu}.1$. The probable reason for its non-appearance in the emission curve of the Bunsen burner is because it is covered up by the considerably stronger maximum of water vapor lying near it.

ABSORPTION SPECTRA.

The above experiments on emission can for several reasons furnish only an incomplete picture of the spectral properties of the gases investigated. For one thing the observed energy of the radiation is so slight that beyond 9^{μ} in part a very wide slit (up to $5^{mm}.5$) had to be employed. The spectrum is, therefore, very impure, and the possibility is excluded of observing details,

which extend over only a few minutes of arc. Further, the position of the maxima and minima are affected by three minor circumstances which can be only partially eliminated. These are: first, the temperature of the source of radiation, by which, as is well known, the short waves are relatively the more favored, as the temperature is elevated; secondly, the dispersion, and thirdly, the absorption of the prism. We have, therefore, refrained from drawing extensive conclusions from the above observations of emission, and only briefly communicate them as furnishing a good control on the measurements of absorption now to follow.

In these measurements the source was a zirconium burner whose radiations were concentrated on the slit of the spectrometer by one or more concave mirrors. A layer of water vapor or carbon dioxide could be suitably inserted in the path of the rays. Quite different arrangements were necessary in the two cases on account of the different characteristics of the two gases.

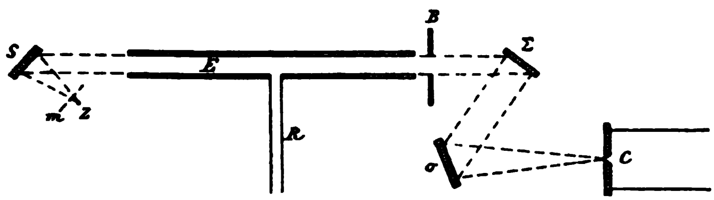


FIG. 7.

Fig. 7 shows the arrangement employed in investigating the absorption of aqueous vapor. The rays from the zirconium mantle Z are rendered parallel by the concave mirror S, and then traverse the cast iron tube E, 75^{cm} long, which is heated above 100° by four Bunsen burners beneath it, and can be fed with a permanent stream of vapor through the brass tube R entering at the side. Directly behind the iron tube is a circular diaphragm B which excludes the radiation of the wall of the tube, and a plane mirror Σ which reflects the rays emerging from the tube to the concave mirror and by which they are concentrated upon the slit of the spectrometer. This arrangement,

resembling in many points that used by Paschen, permitted us to make observations in quick succession, if alternately air and water vapor occupied the tube *E*, so that we could obtain the absorption of a layer of vapor 75^{cm} thick independently of the distribution of energy in the spectrum, and without putting too high demands upon the constancy of the source of light.

The arrangement employed for observing the absorption spectrum of carbon dioxide was as follows. (Fig. 8.) A zirconium burner *Z* was placed horizontally a few centimeters above a

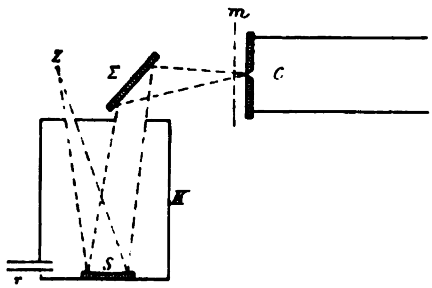


FIG. 8.

wooden box *K*, 30^{cm} deep, 12^{cm} wide and 30^{cm} long. The rays passed through a circular aperture in the top of the box to a concave mirror *S* on the bottom of the box, which reflected them through another hole in the top to a plane mirror Σ. The focal length of the mirror *S* was such that the rays reflected from Σ gave a sharp image of the zirconium mantle in the plane of the slit. A glass tube *r* entered the box near the bottom, through which a stream of dry carbon dioxide could be continuously led into the box. All of the joints and cracks in the box were carefully filled air-tight with wax so that after displacing the air the carbon dioxide could escape only through the two apertures in the top. The mean path traversed within the box by the rays was about 65^{cm}. As the preliminary experiments showed that the absorption of carbon dioxide beyond 9^{μ} was limited to a very small region, it was possible to observe three energy curves in succession, the first when the box was filled with air, the second

when filled with dry carbon dioxide, and the third, which served as a control, under the same conditions as the first. The carbon dioxide in the box was replaced by the air in the room by attaching a water air-pump to the glass tube which exhausted the carbon dioxide and the air entered the apertures in the top of the box.

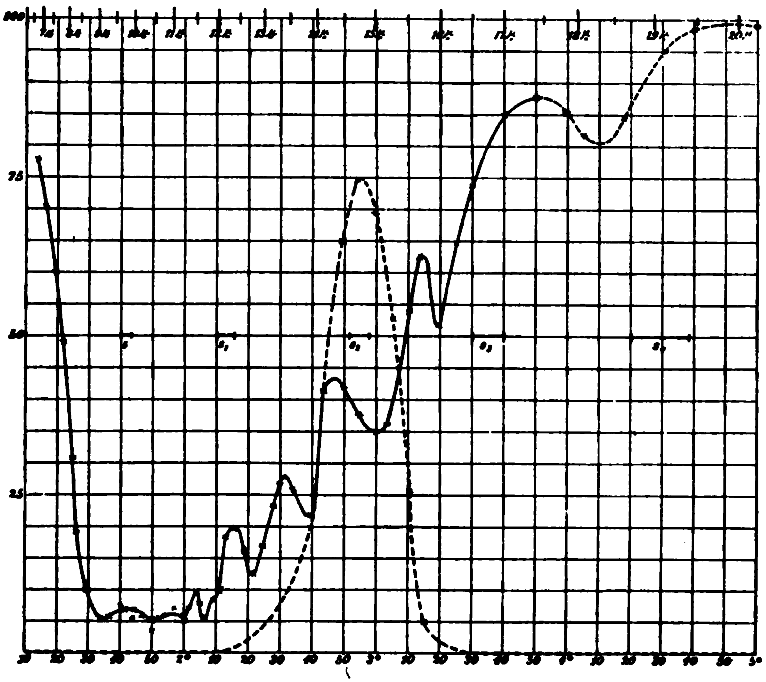


FIG. 9.

The results of our observations of absorption are graphically shown in Fig. 9. On account of the uncommonly strong energy of the source of light, the observations could be made with quite a narrow slit. All the measures on carbon dioxide were made with a slit width of 1^{mm}; in those on water vapor the width varied from $0^{mm}.5$ to $3^{mm}.0$, being $0^{mm}.5$ between 7^{μ} and 11^{μ}, $0^{mm}.8$ between 11^{μ} and 14^{μ}, $1^{mm}.1$ between 14^{μ} and 16^{μ}, $1^{mm}.7$

between 16^{μ} and $17^{\mu}.6$, and $3^{mm}.0$ between $17^{\mu}.5$ and 20^{μ}, while the thermopile was without change $0^{mm}.6$ wide. The apparent width of the slit in minutes of arc is represented in Fig. 9 by the length of the lines $s, s_1 \ldots\ldots s_4$. At $\lambda = 17^{\mu}.5$ the deflections were respectively 22^{mm} and $2^{mm}.6$ with and without the introduction of the layer of water vapor. On account of the lesser accuracy we have therefore indicated the absorption from here on by a dotted curve.

In the curves of Fig. 9, as also previously in the curves of emission, the angles of deviation α, or more correctly their differences from the minimum deviation of the D line, $\alpha_D - \alpha_\lambda$, are entered as abscissas. The ordinates represent the amount of the absorbed energy, the incident energy being called 100. The scale at the upper edge of the figure gives the wave-lengths corresponding to the different angles of deviation.

Water vapor shows only faint absorption in the spectral region between $\lambda = 9^{\mu}$ and $\lambda = 11^{\mu}$, as compared with shorter and longer waved parts of the infra-red. From this follows the minimum observed in the emission at $\lambda = 10^{\mu}.7$. Beyond 11^{μ} the absorption begins to increase and becomes almost total at $\lambda = 20^{\mu}$, whereby the maximum observed in the emission at $\lambda = 13^{\mu}.1$ is explained. In the region between 11^{μ} and 18^{μ} water vapor possesses six conspicuous maxima of absorption, which have according to our observations the wave-lengths $\lambda = 11^{\mu}.6$, $12^{\mu}.4$, $13^{\mu}.4$, $14^{\mu}.3$, $15^{\mu}.7$, and $17^{\mu}.5$.

It is highly probable that in the region of shorter wave-lengths between 9^{μ} and 11^{μ} a number of such absorption bands are also present, of which two are suggested by the curve, but here the dispersion of our sylvin prism is plainly no longer sufficient for their complete resolution. The same is true for the portion of spectrum between 7^{μ} and 9^{μ} which we added for the purpose of connection with Paschen's observations. His observations give for a layer of water vapor 7^{cm} thick a continuous falling off in the absorption from 82.3 per cent. down to 8.0 per cent. between wave-lengths $6^{\mu}.53$ and $8^{\mu}.26$, thus qualitatively according well with our measures. A quantitative comparison of our

results with those of Paschen seems however to be impossible, since the law of absorption cannot be applied to spectra which consist of small separate absorption bands whose separation cannot be accomplished on account of the impurity of the spectrum. This follows from Paschen's own observations, for he finds for example at λ $7^{\mu}.87$ 12.5 per cent. of absorption for layer of water vapor 7^{cm} thick,[1] and 13 per cent. for a layer 33^{cm} thick.[2] Ångström has also made similar observations in the study of the absorption spectrum of other gases.

Our experiments carried out as described above on the absorption spectrum carbon dioxide very soon showed that we were dealing with a single absorption band whose maximum lies near $\lambda=14^{\mu}.7$. However, the thickness of the absorbing stratum of carbon dioxide proved to be too great to give a distinct picture of the way the absorption runs in the spectrum, since between 14^{μ} and $15^{\mu}.5$ an almost complete extinction of the energy occurred. We therefore filled the box only to about one-third of its depth with carbon dioxide and then obtained the absorption spectrum represented in the dotted curve of Fig. 9. The whole region of absorption is limited to the interval from $12^{\mu}.5$ to 16^{μ}, with the maximum at $14^{\mu}.7$. Aside from this region not the slightest absorption could be detected between 8^{μ} and 20^{μ} even when the box was completely filled with carbon dioxide. The observed emission spectrum of carbon dioxide is thus easily explained. The maximum of the radiated energy is displaced toward the side of shorter waves in consequence of the three circumstances mentioned above and appears at $14^{\mu}.1$ (see Fig. 6).

The absorption band at $14^{\mu}.7$ is so sharp that it comes out distinctly in every energy curve in consequence of the carbon dioxide in the air of the room, while the absorption bands of water vapor cannot be observed in this way under an average humidity.

The observations now communicated show that the Earth's

[1] PASCHEN, *Wied. Ann.*, 52, 214, 1894.

[2] *Wied Ann.*, 51, 19, 1894.

atmosphere must be wholly opaque for the rays of wave-length 12^{μ} to 20^{μ} as well as for those of wave-length $24^{\mu}.4$. In fact Langley's observations on the spectrum of the Sun and Moon only extend to minimum deviations of his rock salt prism of about 36°, corresponding to an extreme wave-length of from 10^{μ} to 11^{μ}.

For practical meteorology the fact that the solar rays beyond $\lambda = 12^{\mu}$ are absorbed in the Earth's atmosphere is after all of slight importance, as the energy of these rays is very small in comparison with the total emission of the Sun.

Physical Laboratory of the Technical Hochschule.
Charlottenburg, December 1897.

Minor Contributions and Notes

THE HARVARD CONFERENCE.

The selection of the Harvard College Observatory as the place of meeting for the Second Annual Conference of astronomers and astrophysicists doubtless had much to do with the highly successful outcome of the gathering. The numerous instruments, many of them operated by automatic devices, together with the unrivaled collection of celestial photographs, afforded the visitors opportunities for study not to be found elsewhere. The meetings were held in the drawing-room of the Director's residence, which was most hospitably thrown open for the occasion. At the conclusion of the first morning's session, the members of the conference were entertained at luncheon by Professor and Mrs. Pickering, and on the second day similar entertainment was provided at Memorial Hall by the President and Fellows of Harvard University. On the third day, after the adjournment of the regular sessions, a large party visited the Blue Hill Meteorological Observatory at the invitation of the proprietor, Mr. A. Lawrence Rotch. Ample opportunity was afforded between the sessions for the inspection of the laboratories and museums of Harvard University, while the members of the Observatory staff were always ready to assist visitors in their examination of the instruments and photographs so bountifully displayed.

The following persons were registered as attending the conference:

Mr. C. G. Abbott, Smithsonian Astrophysical Observatory, Washington, D. C.
Mr. W. H. Atwill, Harvard College Observatory, Cambridge, Mass.
Professor S. I. Bailey, Harvard College Observatory, Arequipa, Peru.
Professor E. E. Barnard, Yerkes Observatory, Williams Bay, Wis.
Mr. N. E. Bennett, Wilmington, O.
Mr. S. H. Brackett, St. Johnsbury, Vt.
Professor H. S. Carhart, University of Michigan, Ann Arbor, Mich.
Dr. F. J. Chase, Yale University, New Haven, Conn.
Mr. H. Helm Clayton, Blue Hill Meteorological Observatory, Hyde Park, Mass.
Professor W. H. Collins, Haverford College, Haverford, Pa.

Mr. H. R. Colson, Harvard College Observatory, Cambridge, Mass.
Professor George C. Comstock, Washburn Observatory, Madison, Wis.
Professor Charles R. Cross, Massachusetts Institute of Technology, Boston, Mass.
Professor A. E. Dolbear, Tufts College, Somerville, Mass.
Miss H. R. Donaghe, Morristown, N. J.
Professor C. L. Doolittle, Flower Observatory, Upper Darby, Pa.
Professor H. W. Du Bois, Central High School Observatory, Philadelphia, Pa.
Mr. J. A. Dunne, Harvard College Observatory, Cambridge, Mass.
Professor J. R. Eastman, U. S. Naval Observatory, Washington, D. C.
Mrs. I. W. Eddy, Harvard College Observatory, Cambridge, Mass.
Dr. W. S. Eichelberger, U. S. Naval Observatory, Washington, D. C.
Professor W. L. Elkin, Yale University Observatory, New Haven, Conn.
Mr. S. P. Fergusson, Blue Hill Meteorological Observatory, Hyde Park, Mass.
Professor R. A. Fessenden, Western University of Pennsylvania, Allegheny, Pa.
Mr. Edward P. Fleming, Harvard College Observatory, Cambridge, Mass.
Mrs. M. Fleming, Harvard College Observatory, Cambridge, Mass.
Professor A. S. Flint, Washburn Observatory, Madison, Wis.
Professor Edgar Frisby, U. S. Naval Observatory, Washington, D. C.
Mr. R. H. Frost, Harvard College Observatory, Cambridge, Mass.
Miss Caroline E. Furness, Vassar College Observatory, Poughkeepsie, N. Y.
Miss E. F. Gill, Harvard College Observatory, Cambridge, Mass.
Professor H. M. Goodwin, Massachusetts Institute of Technology, Boston, Mass.
Miss Ida Griffiths, Poughkeepsie, N. Y.
Rev. J. G. Hagen, Georgetown College Observatory, Georgetown, D. C.
Professor George E. Hale, Yerkes Observatory, Williams Bay, Wis.
Mr. J. F. Hayford, U. S. Coast and Geodetic Survey, Washington, D. C.
Miss Lillian Hodgdon, Harvard College Observatory, Cambridge, Mass.
Professor G. W. Hough, Dearborn Observatory, Evanston, Ill.
Professor Harold Jacoby, Columbia University, New York City.
Mr. E. S. King, Harvard College Observatory, Cambridge, Mass.
Mr. Lawrence La Forge, Cambridge, Mass.
Miss E. F. Leland, Harvard College Observatory, Cambridge, Mass.
Professor F. H. Loud, Colorado College, Colorado Springs, Colo.
Mr. Carl Lundin, Cambridgeport, Mass.
Dr. Alex. Macfarlane, Lehigh University, South Bethlehem, Pa.
Miss A. C. Maury, Harvard College Observatory, Cambridge, Mass.
Professor C. H. McLeod, McGill University, Montreal, Canada.
Professor Dayton C. Miller, Case School of Applied Science, Cleveland, O.
Professor E. W. Morley, Adelbert College, Cleveland, O.
Professor G. W. Myers, University of Illinois, Champaign, Ill.

Professor Simon Newcomb, Washington, D. C.
Mr. H. M. Parkhurst, Brooklyn, N. Y.
Professor H. M. Paul, U. S. Naval Observatory, Washington, D. C.
Professor B. O. Peirce, Cambridge, Mass.
Professor E. C. Pickering, Harvard College Observatory, Cambridge, Mass.
Mrs. E. C. Pickering, Harvard College Observatory, Cambridge, Mass.
Professor W. H. Pickering, Harvard College Observatory, Cambridge, Mass.
Professor Charles Lane Poor, Johns Hopkins University, Baltimore, Md.
Miss Mary Proctor, New York City, N. Y.
Rev. Alden W. Quimby, Berwyn, Pa.
Mr. F. G. Radelfinger, Nautical Almanac Office, Washington, D. C.
Mr. W. Maxwell Reed, Andover, Mass.
Mr. Charles H. Rockwell, The Observatory, Tarrytown, N. Y.
Mr. Jonathan T. Rorer, Central High School Observatory, Philadelphia, Pa.
Mr. A. Lawrence Rotch, Blue Hill Meteorological Observatory, Hyde Park, Mass.
Professor W. C. Sabine, Harvard University, Cambridge, Mass.
Mr. F. E. Seagrave, Private Observatory, Providence, R. I.
Professor Arthur Searle, Harvard College Observatory, Cambridge, Mass.
Professor A. N. Skinner, U. S. Naval Observatory, Washington, D. C.
Mr. Frederick Slocum, Ladd Observatory, Providence, R. I.
Professor M. B. Snyder, Central High School Observatory, Philadelphia, Pa.
Rev. John Stein, Leyden, Netherlands.
Miss M. C. Stevens. Harvard College Observatory, Cambridge, Mass.
Mr. Charles E. St. John, Oberlin, O,
Mr. A. E. Sweetland, Blue Hill Meteorological Observatory, Hyde Park, Mass.
Professor D. P. Todd, Amherst College Observatory, Amherst, Mass.
Professor Winslow Upton, Ladd Observatory, Providence, R. I.
Professor J. M. Van Vleck, Wesleyan University, Middletown, Conn.
Professor F. W. Very, Providence, R. I.
Mr. Robert De C. Ward, Harvard University, Cambridge, Mass.
Mr. Charles F. Warner, Cambridge Manual Training School, Cambridge, Mass.
Mr. W. R. Warner, Cleveland, O.
Professor A. G. Webster, Clark University, Worcester, Mass.
Professor O. C. Wendell, Harvard College Observatory, Cambridge, Mass.
Miss Sarah F. Whiting, Wellesley College, Wellesley, Mass.
Professor F. P. Whitman, Adelbert College, Cleveland, O.
Miss Mary W. Whitney, Vassar College Observatory, Poughkeepsie, N. Y.
Miss A. Winlock, Harvard College Observatory, Cambridge, Mass.
Miss L. Winlock, Harvard College Observatory, Cambridge, Mass.
Miss E. G. Wolffe, Harvard College Observatory, Cambridge, Mass.

Miss I. E. Woods, Harvard College Observatory, Cambridge, Mass.
Professor R. S. Woodward, Columbia University, New York City.
Mr. Paul S. Yendell, Dorchester, Mass.

As it is intended to publish in the November number of this JOURNAL abstracts of all the papers presented to the conference, no account of this part of the proceedings will be given here. Mention should be made, however, of certain actions of the conference which will not find a place in these proceedings.

In the Friday morning session, which was devoted to a discussion of various matters of general interest, the question of forming an astronomical and astrophysical society was considered, and referred to a committee, consisting of Professors Pickering, Newcomb, Comstock, Morley, and Hale. Before the committee was appointed it was resolved, by unanimous vote, that the annual conferences should be continued, either in their present form, or under the auspices of an organized society. The committee, in offering its report at the next session of the conference, recommended that a society be formed, and presented the first draft of a constitution. It also recommended that on the following Tuesday a meeting for the purpose of effecting a preliminary organization should be held by those who had previously signed a statement signifying their wish to become charter members of the society. The meeting was duly held at the Massachusetts Institute of Technology, sixty-one persons having signed the statement. After a brief discussion, the same committee of five, with power to add four to its number, was appointed to act as the first council of the society. The duties of the committee include the drafting of a constitution, the election of members to the society, arrangements for the next meeting, and other business of a similar nature.

At the Friday morning session of the conference some time was devoted to a discussion of the United States Naval Observatory. No action was taken at this time, but on Saturday the following resolution, offered by Professor Flint, was unanimously adopted:

Resolved, That a committee of three be appointed by the conference to consider the question of the proper organization and function of the United States Naval Observatory; to draw up resolutions expressing different representative views; to obtain signatures of astronomers and astrophysicists of the country; and to present the same in person to the Secretary of the Navy by January 1899, and to other authorities at their discretion.

Further, **That said committee be instructed to coöperate with any other committees that may be appointed in this country to consider questions connected with the scientific functions of the national government; and to take such further action as may seem to said committee expedient.**

The committee, appointed by ballot, consists of Professors Pickering, Comstock, and Hale. The American Association for the Advancement of Science at its meeting in Boston, subsequently appointed a committee for a similar purpose, consisting of Professors Pickering, Mendenhall, and Woodward.

The total solar eclipse of May 28, 1900, was also discussed by the conference, and the chairman was instructed to appoint a committee to coöperate with observers and to take such action as might be deemed necessary to secure the best results. Professors Pickering, Barnard, and Comstock were named. At a subsequent session of the conference, the committee recommended that another committee, consisting of Professors Newcomb, Barnard, Campbell, and Hale, be appointed to act in its place. This recommendation was adopted, and the new committee was given power to add to its number should this seem desirable.

The committee on the *Northern Durchmusterung*, appointed at the Yerkes Observatory conference, reported that satisfactory progress was being made in printing the new edition. Largely through the efforts of the committee, the new edition has been freely subscribed for by American astronomers.

The best method of filling vacant positions in astronomy and physics was discussed by Professor Pickering. It was arranged that he should act as a committee of the new society on this subject. Those desiring to obtain positions, or having vacant positions which they wish to fill, may therefore advantageously communicate with him.

The meeting of the American Association in Boston during the week following the adjournment of the conference practically amounted to an extension of the Harvard meetings, so far as the astronomical work of Section A was concerned. Professor Barnard's important address on Astronomical Photography, and the numerous interesting papers presented to the Section, added greatly to the value of the previous week's proceedings. The conference had already enjoyed the hospitality of the Institute of Technology, as Professor Cross had provided a stereopticon for an evening meeting held there at his invi-

tation. The continuity of the Cambridge and Boston meetings was thus almost unbroken, over a week being devoted to the discussion of astronomical and astrophysical topics. G. E. H.

PHOTOGRAPH OF "FLASH" SPECTRUM.

To the Editors of the ASTROPHYSICAL JOURNAL:

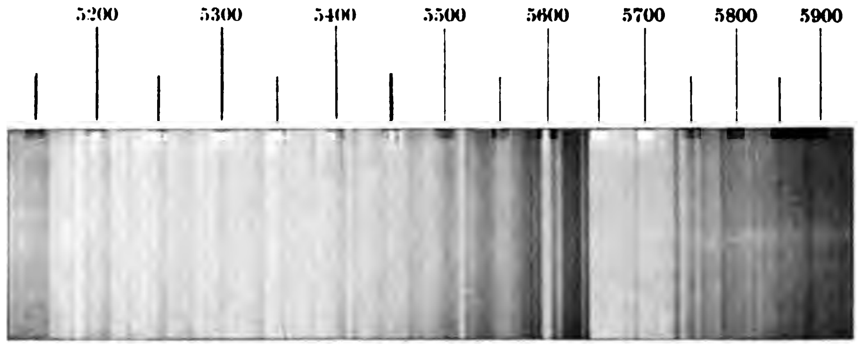
AS PROBABLY you have found out before now, there appears to be a mistake in the last sentence of the article on above, on p. 121 in your August issue. A comparison with Evershed's photograph in *Knowledge*, June 1898, shows that the two images of the prominence really belong to the lines H and K; the prominence being lofty enough to appear projecting beyond a part of the Moon's disk where nothing else is shown in the photograph. There appears also an image of the prominence formed by the $H\gamma$ line, in contact with the outer edge of the next conspicuous line toward the red. This latter line also has an image of the prominence as a very faint spot; and there may be others; but it is rather singular $H\beta$ has no image, seeing it is conspicuous in Evershed's spectrum.[1]

T. W. BACKHOUSE.

WEST HENDON HOUSE, Sunderland,
September 7, 1898.

[1] An examination of another photograph, recently received from Professor Naegamvala, shows that Mr. Backhouse is undoubtedly right. In this photograph the $H\beta$ image is also clearly shown.— EDS.

PLATE VII.



SPECTRUM OF 152 SCHJELLERUP.

PHOTOGRAPHED WITH A THREE PRISM SPECTROGRAPH ATTACHED TO THE FORTY-INCH YERKES TELESCOPE.

THE

ASTROPHYSICAL JOURNAL

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AND ASTRONOMICAL PHYSICS

VOLUME VIII NOVEMBER 1898 NUMBER 4

THE PROBABLE RANGE OF TEMPERATURE ON THE MOON. I.[1]

By FRANK W. VERY.

THE subject may be approached from the experimental side in two ways:

The first supposes the radiation from different lunar regions to have been observed at various phases, and the proper radiation of the heated lunar surface to have been distinguished from solar radiation immediately reflected. The observations require further correction for the changes produced by the absorbing action of the Earth's atmosphere upon different rays.

The radiant values thus given will be stated in terms of some arbitrary scale of units. Before the results can be interpreted, the values must be transformed and given in terms of absolute measurement. This involves two steps:

a. The determination of the law connecting radiation and temperature for the measuring instrument and comparison body used in observing the lunar radiation, and the transformation of arbitrary into absolute values for the standard substance.

b. A comparison of the radiant emissivity of the standard substance with that of such materials as may reasonably be assumed to make up the lunar surface, and a fixing of the prob-

[1] Read at the Harvard Conference, Aug. 25, 1898.

able variation of temperature to be anticipated with departure from standard conditions.

The second method of attacking the problem of lunar temperature starts with the solar radiant constant, as determined by observations corrected for atmospheric absorption, and assuming such laws of reflection and absorption of solar rays, and such retention of heat and powers of emission in the lunar surface as are indicated by observation, proceeds to trace the consequences as far as these affect the diurnal range of temperature on the Moon.

The law connecting radiation and temperature will form the subject of a special paper, but the general results are given in Table B (Part II).

The first part of the present article describes the author's measurements of emissivity for several substances at moderate temperatures.

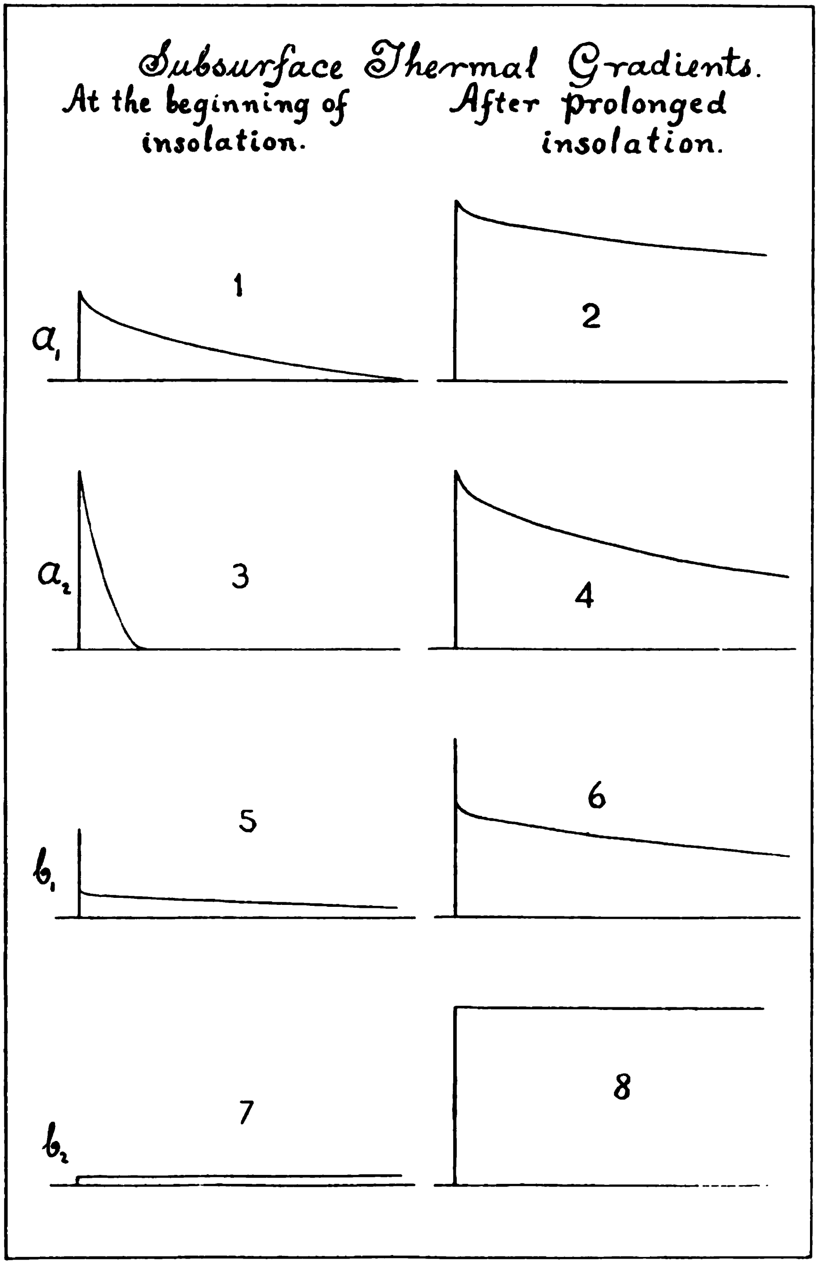
Radiators may be divided somewhat arbitrarily into the following classes :

(*a*) Good surface radiators which are
  (a_1) good conductors of heat,
  (a_2) bad conductors of heat.
(*b*) Bad surface radiators which are
  (b_1) good conductors of heat,
  (b_2) bad conductors of heat.

Classes a_1, a_2, and b_1 absorb the greater part of the infra-red rays in layers of slight thickness ; but class b_2 is made up of a few substances which are eminently transmissive of these low temperature rays, and we find that radiation from the interior of a body cannot be excluded in even the roughest attempt at a classification. Moreover, under certain circumstances, the same body may have affinities with several classes.

A point which needs to be emphasized is the compensation involved in the radiation of interior layers, together with the thermal replenishing of surface layers by conduction, and the consequent narrowing of the range of temperature and radiant emissivity in different materials, after prolonged insolation.

PLATE VIII.



Subsurface Thermal Gradients.
At the beginning of insolation.
After prolonged insolation.
1
2
a_1
3
4
a_2
5
6
b_1
7
8
b_2

Considerable duration of exposure to the heating rays may be needed to affect compensation, but the great length of the lunar day gives ample time for bringing substances having the widest diversity of conductive and absorptive properties to approximately the same temperature. Given unlimited depth of absorbent material, the thermal subsurface gradients of the different classes, when first exposed to radiation, and after prolonged insolation, are indicated diagrammatically in Figs. 1 to 8, Plate VIII, abscissæ being depths, and ordinates temperatures.

Internal absorption of radiation by the transmissive substances of class b_2 gives the effect of almost perfect conductivity. The initial rise of temperature is small, but the internal distribution of heat approaches uniformity, and reaches great depths. In the end, a high temperature is maintained to a great distance from the surface, and the general effect of the combined rays from all depths of this large mass is similar to that of a nearly perfect radiator. Prolonged exposure largely evens up differences of absorbent or of conductive power. Good absorbents of slight conductivity (class a_2) attain a high surface temperature quickly, and further exposure chiefly affects the internal distribution of heat. Class b_1 is composed of metals, in which reflected radiation, represented symbolically in Figs. 5 and 6 as an excess of apparent surface temperature, is large.

The chief point to be noted is that if exposure to solar radiation is sufficiently prolonged to give a stable subsurface temperature-gradient, a large part of the absorbed radiation is re-radiated with but little delay, and the combined emission and reflection from both good and bad radiators is very nearly equal to the radiation simultaneously received. This is a fact of importance in the present case, since it greatly narrows the uncertainty which attaches to the interpretation of observations.

The term emissive power, or emissivity, is sometimes used in a loose way for loss of heat from the unit of surface of an isolated body by radiation and convection combined, and, as so used, is dependent on such a wide range of variable conditions as to be ill adapted to purposes of scientific definition; but, even

when so restricted as to mean loss of heat from the unit of surface by radiation in a vacuum, emissivity, or absolute radiative power, is still a quantity depending on a large variety of circumstances.

A common method of determining relative emissivities of solids at a particular temperature is to coat the faces of a Leslie's cube with thin layers of the various substances, and to compare the radiation from these faces, either in totality by thermopile or bolometer, or analyzed as spectral energy-curves by the spectrobolometer. This involves the assumption that radiation is a surface phenomenon; and the endeavor is to measure the radiation from a surface of a given material at a definite temperature. It is needless to say that the assumption is incorrect. No substance is absolutely opaque. It is not merely a surface layer of molecules which radiates, but deeper and deeper layers contribute to the effect, of course in relatively smaller and smaller proportion. If the coating be thin, it transmits rays proceeding from the underlying material. If the coating be thick enough to prevent the transmission of rays from a second substance, it is in general no longer permissible to assume that the outer surface is at the same temperature as the well-stirred liquid within the cube. Indeed, there must, in any case, be a temperature-gradient from the outer to the inner surface of the combined solids, and one which, as Leslie's cubes are commonly constructed, is by no means negligible. The best cube is one with walls of very thin and conductive sheet-metal, such as copper, with a reciprocating brush-stirrer, by which the layers of liquid adherent to the inner faces are continually removed and mixed more effectually than by unaided convection. The ideal aimed at is a knowledge of the amount and quality of the radiation from a given material at a definite temperature. The result attained is only approximate, even for highly absorbent substances which can be used in very thin layers, and does not give us information as to the radiation from a considerable mass of transmissive and slightly conductive material gradually heating in the sunshine, for here internal conductivity comes into

play, establishing a temperature-gradient beneath the surface, the radiation proceeding from layers at different temperatures whose range, constituting the gradient, is determined by the conductivity and the coefficient of transmission. Since transparent bodies are poor conductors of heat, while the highly conductive metals are very opaque, it follows that there is a certain degree of compensation in the radiation of various cooling bodies, those bodies in which a rapid flow of heat maintains a high surface temperature during cooling, radiating by means of a very thin layer, but the poor conductors, in which the surface layers are at a lower temperature, radiating from greater depths.

A plate of rock-salt, 5^{cm} thick, heated to 95° C., and compared with a black screen at 20°, the excess being 75°, gave a radiation of 150 arbitrary units (see Table 1). I estimate, graphically, that a block of salt 100^{cm} thick might give 350 arbitrary units. In studying the radiation from such a thick block of highly transmissive substance, after long heating, it is not necessary to consider the distribution of temperature near the surface, since very little of the radiation comes from the outer layers affected by the internal temperature-gradient produced by convective cooling, at least not until the cooling has progressed considerably.

A layer of lampblack not over $0^{cm}.05$ thick (perhaps $0^{cm}.03$ to $0^{cm}.5$) deposited on copper (temperature 99° C., black comparison screen 26°, excess 73°) gave with the same instruments a radiation of 507, which is very near the maximum obtainable from this combination. Reduced to the same excess of 75° C., the radiation of sooted copper is 521. This is 10 to 15 per cent. greater than the radiation of lampblack would be without a metal backing. Compare initial radiation of lampblack on thick glass (Table 2), and radiation of lampblack on thin glass containing mercury (Table 4), allowing for the difference in temperature, and noting that the difference of radiation and surface temperature would be very much greater, if the cooling were prolonged, and that a large amount of heat is removed by

the evaporation of water in experiment IV. For a short exposure, the conductivity of lampblack being very small (about 0.0002), the radiation is that of a thin layer of carbon, let us say $0^{cm}.05$ thick.

The radiation of glass is 93 per cent. of that of lampblack (see Table 5). Glass has about ten times the conductivity of finely powdered carbon, but is more absorbent for long waves, and, owing to this compensation, the depth of its efficient radiant layer may be taken the same as for subdivided carbon with similar backing, or $0^{cm}.05$. By direct experiment, I have found scarcely any difference between the radiation from $\frac{1}{2}^{mm}$ and from 2^{cm} of glass, the thicker layer giving a barely perceptible increase, showing that the adopted thickness cannot be far from the truth.

For the efficient radiating layer of copper, a thickness of $0^{cm}.000001$ may be adopted provisionally. In spite of the high conductivity of copper, the small thickness of the radiating layer causes its radiation to fall far below that of the other substances. Freshly scoured, but not burnished, copper gave a radiation of 64, reduced to the same excess of 75° C. (see Table 1, *d*). Collating these data, we have:

TABLE A.

Substance	Thickness of radiating layer	Conductivity	Radiation	Volumetric radiation	Transparency for self-radiation
	cm.				
Carbon	0.05	0.0002	521	10420.	0.000096
Glass	0.05	0.0020	485	9700.	0.000103
Rock-salt	100.	0.0110	350	3.5	0.29
Copper	0.000001	1.0	64	64000000.	0.000000016

The reciprocals of the volumetric radiations, or of equivalent radiations from efficient layers of equal thickness, which are given in the last column, are approximately proportional to the transparency of the materials for the complex of self radiation, although reversal of position may occur in closely succeeding members of the series, as already suggested above in the

case of glass and lampblack with varied backing, owing to differences of conductivity. It is hardly necessary for me to say that no experiments have been made on the radiation of so great a thickness of rock-salt as one meter. I have never seen a block of clear rock-salt approaching that size, and it is quite possible that with pure material the effective radiant depth would be *many* meters, and that in this case this feebly radiant substance might stand at the head of the list when radiating in mass of unlimited depth. But while the results given in Table A are somewhat rude, it seems to me that, as thus stated, they have a meaning, and a practical applicability, which is lacking in the tabular statements of relative emissivities which do service in most of our text books. A single example of the fallacies which underlie many of these numerical statements must suffice. Tyndall, in his *Contributions to Molecular Physics* (p. 248) states that "MM. de la Provostaye and Desains find the relative emissive powers of the two substances [glass and rock-salt] to be as 17 to 6;" and in a later part of the book he gives some results of his own on the relative radiative power of a number of substances, including rock-salt. To show how little real meaning attaches to some of these comparisons, let it be noted that the powdered salt was applied as a thin layer to a metallic cube. Although it is probable that with less transmissive substances the discontinuity of the particles will diminish the rays from the underlying metal by successive reflections, even to the point of extinction, nevertheless it is certain that with powdered salt a large part of the radiation was that of the metal under it.

Two experiments are described:

a. When rock-salt powder was held by sulphur cement, its radiation was 35.3, that of lampblack being 84.

b. When the rock-salt powder was held by electric attraction, its radiation was 24.5, that of black oxide of iron being 65.8 (*loc. cit.*, p. 320 and 321).

Testing further the transmission of these alleged rock-salt rays, Tyndall found that a rock-salt plate, 0.8 inch ($2^{cm}.03$) thick, transmitted—

67.2 per cent. of supposed rock-salt radiation (condition *a*)
62.8 " " " " " " (condition *b*)
(*loc. cit.*, p. 322).

Now I found that using radiation from a block of clear rock-salt, 5^{cm} thick, and with polished faces, instead of the powdered salt on a metal ground (see Table 1), a much thinner plane-surfaced and polished plate of rock-salt, perfectly clear (thickness, $1^{cm}.14 = 0.^{in}.45$), transmitted only 50.8 per cent. of the genuine rock-salt radiations; and since a plate of cold rock-salt $5^{cm}.38$ thick, transmitted 35.1 per cent. of the radiation from a similar hot plate, it may be concluded that a plate of the thickness used by Tyndall will allow 45 per cent. of rock-salt radiation to pass, and by no means so large a quantity as 65 per cent. In all of these measures, the relatively insignificant loss by surface reflection is included with loss by absorption in the body of the medium. As indicated by the dispersion researches of Rubens and Nichols (*Phys. Rev.* **5**, 162, Sept. 1897), the proper radiations of rock-salt presumably approach a wave-length of 50μ. For ordinary sources whose spectral energy-curves have maxima at wave-lengths not so extreme, the absorption is very much less. Thus the rock-salt plate, $5^{cm}.38$ thick, transmits 74.1 per. cent. of the radiation from sooted copper, and 73 per cent. of the radiation from bright copper; from all of which it is evident that Tyndall's value for rock-salt transmission is intermediate between the transmission of this substance for its own and for metallic radiations; and granting that the polish of his surfaces was as good as mine (which owed their perfection to Brashear's unrivaled skill), something over half the radiation from the powdered metal was metallic, while if Tyndall's transmitting surfaces were less perfect, a still larger percentage of the combined radiation must have been due to the metal, and but little to the salt.

Owing to the great conductivity of copper, a considerable mass of this substance continues to radiate for a long time with only slight change, while glass, being a poor conductor, falls off rapidly in its radiation, as is evident from successive values in

Table 2; and the same would be true for lampblack if it were not backed by metal. After some time spent in cooling, glass in mass will radiate less than copper in mass. Rock-salt, owing to the great depths from which the rays proceed, in spite of the feebleness of its volumetric radiation, must eventually surpass all other substances as a radiator. Thus the combination of diverse properties tends to equalize radiant effects in large masses, and with lapse of time, which is not a little remarkable when it is remembered that the volumetric radiating power of these various substances is so different. To be told that the radiating powers of equal efficient volumes of copper and of rock-salt are something like 20,000,000 to 1, and that nevertheless the rock-salt is the better radiator in large masses, is rather startling, but it illustrates the power of compensation.

The substances used in my measures are fairly typical of the following classes:

1. Lampblacked copper is a good radiator of slight transmissive power, but with tolerably rapid transference of heat to the surface. Water, which covers so large a part of the globe, is in nearly the same case. A block of ice at zero Centigrade, radiates almost identically with lampblacked copper of the same temperature. Two sets of measurements gave me:

For ice, 13.6 ± 0.23, and
For sooted copper, 14.0 ± 0.15.

Liquid water radiates more powerfully than lampblack, according to Péclet, who found the radiation of water 32 per cent. greater than that of lampblack on metal. I do not know what precautions were taken to eliminate the influence of water vapor from the heated liquid, nor whether the lampblack had a thickness of maximum efficiency, but admitting the reliability of the measurement, it seems that liquid convection is more efficient in maintaining surface temperature, than metallic conduction, hampered by the retardation of so bad a conductor as lampblack.

2. Glass will serve as a representative of a very large class, including most of the surface rocks. Measurements by W.

Hopkins (*Proc. R. Soc.*, 10, 514, 1860) made in a unit of 1000 grain-degrees Centigrade per square foot per minute, equivalent to 1.16×10^{-3} C. G. S. units, gave the following values of radiation in a vacuum at moderate temperatures per degree Centigrade per square centimeter per second:

Glass	0.000 0856 (C. G. S.)
Chalk	0.000 0871
Sandstone	0.000 0795
Polished limestone	0.000 0815
Unpolished "	0.000 1146 (same block).

These values are not absolute. They were obtained by a calorimetric method involving cooling through a wide range of temperature and the formation of an internal heat-gradient, since the substances are poor conductors. Hence the results are smaller than would be obtained by direct measures of initial radiation at the same temperature.

The initial radiation of glass, at temperatures not much below the boiling point of water, and in nearly normal direction, is 93 per cent. of that of carbon (Table 5), and allowing for the smaller radiation of glass at large angles of emission, its radiation to a hemisphere is about 85.7 per cent. that of carbon; but taking the absolute radiation of carbon at 50°C., or the radiation of carbon to absolute zero for an excess of 323° absolute, as a transferring of 0.0126 small calories per square cm. per second, and the rate per degree Centigrade at this temperature as 0.000128, which is given by Bottomley's measures for sooted copper, the absolute radiation of glass at the same temperature is $0.857 \times 0.0126 = 0.0108$, and its rate per degree 0.000110; and this also represents fairly well the unimpeded radiation from a surface of bare rock in full sunshine, where the renewal of heat from the outside maintains the initial temperature. Hopkins' values suit better for cooling rocks soon after removal of the Sun's rays.

3. Salt covers the surface in some parts of the Earth, but generally in so impure a condition that the relatively large radiation indicated by experiments for deep layers of pure salt, can-

not be expected. A mere incrustation will give a value not far removed from that of the underlying rock. On this Earth, saline crusts cover only a small part of the surface; but it is possible that in such an arid world as the Moon, salt deposits are very extensive. Nevertheless, although salt may be a large ingredient in lunar surface formations, it is certain that few areas on the Moon are white enough to be composed of anything like pure salt; and it is also probable that the original surface lies buried under deposits of cosmic dust.

4. It becomes increasingly probable that cosmic dust is largely metallic, but since its particles are apt to be incrusted by oxidation, such radiating surfaces if composed, for example, of magnetic oxide of iron, or black oxide of manganese, are more nearly represented by sooted than by bright copper, and I am inclined to think that sooted copper is not a bad choice for a comparison body in trying to arrive at a knowledge of the Moon's surface-temperature. Somewhere between this radiative standard and sunlit rock-surface which, as we have seen, does not fall very far below it in efficiency, the emissivity of a considerable part of the lunar surface is likely to be found.

It has been with the design of furnishing a legitimate means of interpreting measures of lunar radiation, and arriving at a better knowledge of the lunar temperature, that I have undertaken these measurements which now follow in detail:

TABLE I.

EXPERIMENTS WITH ROCK-SALT.

a. After being heated for two hours in a hot-air chamber, a rock-salt plate, $5^{cm}.03$ thick, was placed in front of a black copper screen containing water at the temperature of the room, and exposed to the bolometer by removing a similar screen.

a_1. Salt at 85°.6 C. Screen 24°.0. Excess 61°.6 C.

a_2. After heating one-half hour longer: Salt at 86°.8 C. Screen 24°.4. Excess 62°.4 C.

a_3. An absorbent plate of clear rock-salt, $1^{cm}.14$ thick, covered the aperture of the bolometer case, reducing the aperture

in the ratio 23 : 17, whence the deflections must be multipled by 1.35 to be comparable with the others. Same temperature.

a_4. Absorbent plate away. Comparable with a_2 and a_3 Relative humidity 39 per cent. Dew point 11°.1 C., 10.0 grams of water per cubic meter of air.

Radiation:	a_1	a_2	a_3	a_4
	div.	div.	div.	div.
	108.5	110.6	41.5	109.6
	108.5	112.1	43.4	110.8
	109.1	107.5	42.0	111.2
	106.3	108.3	39.7	111.3
	107.3	107.3	40.3	110.5
Mean	107.9	109.2	41.4	110.7

Transmission of rock-salt radiation by $1^{cm}.14$ of rock-salt $= \frac{41.4 \times 1.35}{110.0} = 0.508$. Absorption $= 49.2$ per cent.

b. Rock-salt plate, $5^{cm}.03$ thick, at 95° C. Comparison screens 20°. Excess 75° C.

b_1. Hot salt radiating freely.

b_2. Radiation absorbed by a plate of clear rock-salt, $5^{cm}.38$ thick.

b_3. Hot salt radiating freely. The temperature of the salt has evidently fallen slightly. Relative humidity 31 per cent. Dew point 6°.1 C., 7.27 grams of water per cubic meter of air.

Radiation:	b_1	b_2		b_3
	div.		div.	div.
	149.5	47.5	48.2	132
	147	47.9	48.6	126
	145	51.0	47.9	126.5
	148.5	48.4	46.5	126
	145	47.7	47.6	125
Mean	147	48.1		127.1

Transmission of rock-salt radiation by $5^{cm}.38$ of rock-salt $= \frac{48.1}{137.} = 0.351$. Absorption $= 64.9$ per cent.

c. A bright copper screen was freshly sooted to a thickness, as nearly as could be estimated, of $0^{cm}.03$ to $0^{cm}.05$, until a maximum radiation was obtained.

c_1. Sooted copper screen with boiling water at 99° C.; blackened copper comparison screen, 25°.4. Excess of temperature, 73°.6 C.

c_2. Sooted copper at 99° C., radiating through a plate of clear rock-salt $5^{cm}.38$ thick, at the temperature of the room. Relative humidity, 22 per cent.; dew point, 6°.7 C.; 7.56 grams of water per cubic meter of air.

Radiation:	c_1	c_2
	div.	div.
	504	378.5
	506	375
	503	373.5
	507	373.5
	507.5	378.5
	509	379
	505.5	373.5
	504.5	374.5
	511	378
	514	375.5
Mean	507.2	376.0

Transmission of radiation from thickly sooted copper by $5^{cm}.38$ of rock salt.

$$\frac{376.0}{507.2} = 0.741$$

Absorption 25.9 per cent.

d. A screen of bright copper was freshly scoured with turpentine and pumice, and wiped dry, but was not burnished.

d_1. Bright copper screen, filled with boiling water at 99° C.; blackened copper comparison screen at 24°.6. Excess of temperature, 74°.4 C.

d_2. Bright copper at 99° C., radiating through $5^{cm}.38$ of clear rock salt at the temperature of the room.

Humidity as in experiment *c*.

Radiation:	d_1	d_2
	div.	div.
	60.6	47.1
	66.1	49.1
	64.0	44.7
	63.9	47.9
	62.6	42.7
Mean	63.4	46.3

Transmission of radiation from bright copper by $5^{cm}.38$ of rock salt.

$$\frac{46.3}{63.4} = 0.730$$

Absorption 27.0 per cent.

TABLE II.

EXPERIMENTS WITH GLASS.

a. Two pieces of glass, one $0^{cm}.18$ thick, the other $1^{cm}.97$ thick, were heated in an air-bath for four hours, and upon removal were placed in front of the bolometer. The rapid diminution of successive deflections may be contrasted with the comparative constancy of rock-salt radiation and temperature. Conduction of heat from the interior enables the larger mass to preserve its radiating power better, but allowing for the greater rapidity of cooling of the thin piece, there does not seem to have been much difference in the initial radiation and surface temperature of the thick and thin glass, and this preliminary experiment indicates that the radiation cannot proceed from any great depth.

Glass $0^{cm}.18$ thick.

	div.		div.	
Radiation:	339.5	first diff.	57.0	= cooling in 20 sec.
	282.5		34.5	" " "
	248.0		37.0	" " "
	211.0		29.0	" " "
	182.0			

Glass $1^{cm}.97$ thick.

	div.		div.	
Radiation:	379.0	first diff.	17.5	= cooling in 20 sec,
	361.5		10.5	" " "
	351.0		4.0	" " "
	347.0		6.0	" " "
	341.0			

b. Comparison of radiation from glass and lampblack. The glass plate, $1^{cm}.97$ thick, was sooted on one side until quite opaque, and heated for five hours in an air-bath at 99° C. Exposures were made by withdrawing a blackened cold screen at 24°.5 C., the black and clear faces of the glass being alternately turned towards the bolometer. It will be seen by comparing the first deflection, taken immediately after the removal of the glass from the hot air-bath, with Table I, *c*, that the radiation of lampblack on glass is considerably less than that of lampblack on copper. No instrumental correction is needed in this comparison, and temperatures are similar.

Relative humidity, 37 per cent.; dew point, 9°.7 C.; 9.16 grams of water per cubic meter of air.

Face of glass exposed: Black	Face of glass exposed: Clear	Ratio of radiation	Glass / Lampblack
415			
	365.5	$\frac{366.5}{397.5} = 0.922$	
380			
	344	$\frac{344.0}{366.5} = 0.939$	
353			
	319.5	$\frac{319.5}{340.3} = 0.939$	0.9322
327.5			
	294.5	$\frac{294.5}{317.0} = 0.929$	
306.5			
	276	$\frac{276.0}{296.0} = 0.932$	
285.5			
	257.5	$\frac{257.5}{273.8} = 0.940$	
262			
	242	$\frac{242.0}{253.0} = 0.957$	
244			
	219	$\frac{219.0}{236.0} = 0.928$	0.9396
228			
	204.5	$\frac{204.5}{218.8} = 0.935$	
209.5			
	190	$\frac{190.0}{202.5} = 0.938$	
195.5			

TABLE III.

EXPERIMENTS WITH THIN GLASS BACKED BY WATER.

One-half of the cylindrical surface of a large glass beaker was coated with lampblack until opaque for the Sun's light. The beaker, filled with boiling water (temperature 98°.5 C.), was taken from the fire for each exposure, and placed at a marked point in line with the axis of the bolometer case and in front of a blackened comparison screen at 20° C. After the deflection was recorded, the glass vessel was restored to the fire, and as soon as it was again in vigorous ebullition, the experiment was repeated with the opposite face of the beaker turned toward the bolometer. The radiation from the clear side is that of glass, ½^{mm} thick, together with a feeble trace of radiation from water.

	Face of glass exposed: Black	Face of glass exposed: Clear	Ratio of radiation		Glass / Lampblack
First series	485.5				0.9236
		445.0	$\frac{445.0}{47.88}$	= 0.929	
	472.0				
		446.0	$\frac{446.0}{475.8}$	= 0.937	
	479.5				
		445.0	$\frac{445.0}{486.5}$	= 0.915	
	493.5				
		445.0	$\frac{445.0}{488.5}$	= 0.911	
	483.5				
		450.5	$\frac{450.5}{486.5}$	= 0.926	
	489.5				
Second series	505.0				0.9232
		466.0	$\frac{466.0}{505.0}$	= 0.923	
	505.0				
		472.5	$\frac{472.5}{506.8}$	= 0.932	
	508.5				
		465.0	$\frac{465.0}{509.0}$	= 0.914	
	509.5				
		474.0	$\frac{474.0}{512.3}$	= 0.925	
	515.0				
		470.5	$\frac{470.5}{510.5}$	= 0.922	
	506.0				

Relative humidity 34 per cent.; dew point 6°.9 C.; 7.65 grams of water per cubic meter.

TABLE IV.

EXPERIMENTS WITH THIN GLASS BACKED BY MERCURY.

A large glass beaker, with walls ½ mm thick, was prepared with an inner smaller beaker held concentrically by cork plugs, the space between the two being filled with mercury, and the inner one with water. One-half of the outer surface was blackened by camphor smoke until completely opaque. The vessel was heated until the water began to boil, and was then taken from the fire and allowed to cool for about two minutes, in order that the mercury might fall to approximately the same temperature as the water. In the course of a series the temperature fell from about 93° C. to 73° C. Exposures were made, as in experiment III, by replacing the black comparison screen by the hot vessel.

First and second series: comparison screen 28° C.; relative humidity 41 per cent.; dew point 14°.4 C.; 12.26 grams of water per cubic meter of air. Third series: comparison screen 25°.3 C.; relative humidity 51 per cent.; dew point 14°.4 C.; 12.26 grams of water per cubic meter of air.

	Face of glass exposed: Black	Face of glass exposed: Clear	Ratio of radiation	Glass / Lampblack
First series	421.5			
		375	$\frac{375}{403.3}$	= 0.930
	385			
		341	$\frac{341}{368.5}$	= 0.925
	352			
		311	$\frac{311}{336.5}$	= 0.924
	321			
		290.5	$\frac{290.5}{306}$	= 0.949
	291			
		267	$\frac{267}{281.0}$	= 0.950
	271			
			Mean	0.9356

Second series	408			
		357	$\frac{357}{387.5}$	= 0.921
	367			
		324.5	$\frac{324.5}{352.3}$	= 0.921
	337.5			
		298	$\frac{298}{321.0}$	= 0.928
	304.5			
		275	$\frac{275}{293.8}$	= 0.936
	283			
		248	$\frac{248}{271.8}$	= 0.912
	260.5			
			Mean	0.9236
Third series	391.5			
		354.5	$\frac{354.5}{377.5}$	= 0.939
	363.5			
		331.5	$\frac{331.5}{352.0}$	= 0.942
	340.5			
		307	$\frac{307}{329.0}$	= 0.933
	317.5			
		283.5	$\frac{283.5}{307.3}$	= 0.923
	297			
		267.5	$\frac{267.5}{287.0}$	= 0.932
	277			
			Mean	0.9338

No change in the ratio of radiations from glass and lampblack was produced by the interposition of a thick plate of rock-salt, the series giving the ratio 0.9274.

TABLE V.

SUMMARY OF EXPERIMENTS ON THE RADIATION OF GLASS AND LAMPBLACK.

Comparisons by several methods furnish nearly the same value for this important ratio.

Radiator	Radiation of glass / Radiation of lampblack		
Thin glass backed by water,	0.9236	±	0.0035
" " " " "	0.9232	±	0.0018
" " (absorbed by rock-salt)	0.9274	±	0.0032
Thick glass, - - -	0.9322	±	0.0023
" " - - -	0.9396	±	0.0030
Thin glass backed by mercury,	0.9356	±	0.0047
" " " " "	0.9236	±	0.0029
" " " " "	0.9338	±	0.0023
Radiation of glass =	0.9299		
" " lampblack =	1.0000		

A glass vessel with thin walls, nearly normal to the direction of emission, and containing a boiling liquid of nearly constant temperature, with screens arranged to shut out radiation from hot vapors, constitutes a good standard of radiation, perhaps better than either lampblack on metal, or polished metal; for unless special precautions are taken to get a thick coat of fresh lampblack whenever the comparison screen is used, the maximum efficiency of a blackened screen is seldom obtained, and the emission of bright metal varies with its polish, or its tarnish.

A SIMPLE INTERPOLATION FORMULA FOR THE PRISMATIC SPECTRUM.

By J. HARTMANN.

THE Appendix to the twelfth volume of the Publications of the Astrophysical Observatory at Potsdam, contains a new dispersion formula proposed by myself, which is worthy of attention on account of its great simplicity and accuracy. I beg to present here an abstract of that article.

When λ denotes the wave-length and n the index of refraction the new formula reads

$$n = n_0 + \frac{c}{(\lambda - \lambda_0)^a},$$

where n_0, λ, c, and a are constants depending upon the substance of which the prism is made. For the kinds of crown and flint glass in ordinary use, $a = 1.2$, so that for a glass prism it only remains to calculate the three constants n_0, λ_0, and c from the measures. In the above reference I have given a very simple method for making this computation, and here I shall only show by an example how accurately the indices of refraction can be represented by the formula. The measurements of indices by Professor Müller given in the fourth volume of the Potsdam Publications are, on account of their high accuracy, especially adapted for testing a dispersion formula. The following table contains in the second column the wave-lengths (Potsdam system) of the observed lines, in the third column the indices reduced to 0° C, as measured by Professor Müller on the flint prism designated as No. 3, and in the fourth column the probable errors in units of the sixth decimal of n. The sixth column gives the values of n resulting from the employment of Cauchy's formula heretofore most often used, viz.,

$$n = a + \frac{b}{\lambda^2} + \frac{c}{\lambda^4}.$$

The constants of this formula, calculated by the method of least

squares from the whole series of measures, are $a = 1.5604998$, $b = 0.006216900$, $c = 0.000168871$. The residuals O — C in units of the sixth decimal place follow. The last two columns contain the indices and their residuals as calculated from the new formula. The values of the constants of this formula for the flint glass used are $n_0 = 1.555166$, $\lambda_0 = 0.17060$, $\lg c = 7.938724$.

Line	λ	n	P. E.	Cauchy		Hartmann	
				n	O–C	n	O–C
B	$0^{\mu}.68685$	1.574359	± 5.0	1.574436	−77	1.574365	− 6
C	0 .65631	1.575828	6.2	1.575843	−15	1.575823	+ 5
a	0 .62780	1.577377	6.3	1.577361	+16	1.577378	− 1
D	0 .58932	1.579855	6.8	1.579800	+55	1.579850	+ 5
$0^{\mu}.55$	0 .55287	1.582703	7.2	1.582646	+57	1.582703	0
b_1	0 .51839	1.585999	3.7	1.585972	+27	1.586008	− 9
F	0 .48616	1.589828	3.9	1.589826	+ 2	1.589826	+ 2
$0^{\mu}.47$	0 .46678	1.592565	4.6	1.592590	−25	1.592565	0
Hγ	0 .43407	1.598204	5.0	1.598252	−48	1.598203	+ 1
g	0 .42271	1.600543	5.2	1.600582	−39	1.600541	+ 2
h	0 .41020	1.603398	2.7	1.603412	− 14	1.603399	− 1
H_1	0 .39688	1.606838	12.4	1.606775	+63	1.606826	+12

The great advantage of the new formula is clearly apparent from a comparison of the two columns O — C, for while the residuals of Cauchy's formula show a systematic progress, and exceed the probable error by almost tenfold, the deviations of the new formula from the observations lie within the limits of error of the observations and their signs are quite irregularly distributed.

But in addition to its good representation of the dispersion curve the formula has the second very important property of being readily solved for the wave-length λ, which is not the case for any of the previous dispersion formulæ. The formula thus solved for λ,

$$\lambda = \lambda_0 + \frac{c}{(n - n_0)^{\frac{1}{a}}},$$

in which $\frac{1}{a}$ is about $\frac{5}{6}$, would serve for the calculation of the wave-length corresponding to a given index of refraction; and since further the index varies in a well-known manner with the

minimum of deviation, the wave-length of the line observed could be computed from the measured minimum of deviation. It appears, however, that two simplifications are permissible which make the determination of wave-lengths in the prismatic spectrum much more convenient. Since it is in practice usually not possible to observe the whole of the visible spectrum without alterations of the adjustments, especially of the focusing of the spectroscope, the accuracy of the interpolation formula will be sufficient if it exactly represents the dispersion for shorter portions of the spectrum, say for differences of $100^{\mu\mu}$ in wave-length. This condition is fulfilled by the simplified formula

$$\lambda = \lambda_0 + \frac{c}{n - n_0},$$

in which a has been placed equal to 1. Further, in place of the index n we may put the directly observed quantities dependent upon it—first the minima of deviation, then the deviations measured with stationary prism, and finally their projections on a plane scale, as in the use of a comparison scale in the spectroscope or as in the examination of photographic spectra. When a spectrogram is measured with a micrometer screw we may accordingly put in the micrometer reading directly for n. Although the use of the new formula has thus been shown to be very simple, yet in practical calculation still further simplicity results from the meaning of the three constants, λ_0, n_0 and c. A moment's reflection shows that n_0 is dependent only upon the zero-point chosen for counting the divisions of the screw or scale, so that n_0 may be termed the zero-point, while c represents the value of the revolution of the screw or of a division of the scale. λ_0 is a constant for each spectroscope, and needs be determined but once for each instrument.

In order to exhibit how accurately the formula satisfied the observations, I give in the following table the results of the measurement of a photograph of the solar spectrum obtained here with a stellar spectrograph. The entire length of the spectrum from F to M was but 48^{mm}. The direct readings of the micrometer screw are given under n, and the wave-lengths of

the measured lines from Rowland's atlas under λ, while the column λ' contains the wave-lengths calculated from the formula

$$\lambda' = 223.28 + \frac{[4.819004]}{404.251 - n}.$$

The constants of this formula were determined from three properly selected lines.

Line	n	λ	λ'	O–C
	R	μμ	μμ	μμ
	17.794	370.96	370.93	+0.03
	23.130	372.71	372.72	—0.01
M	23.323	372.77	372.78	—0.01
	27.928	374.36	374.36	0.00
	29.298	374.85	374.84	+0.01
	32.492	375.95	375.96	—0.01
	33.035	376.15	376.15	0.00
	34.219	376.58	376.57	+0.01
	40.406	378.80	378.81	—0.01
	42.351	379.50	379.53	—0.03
	43.574	379.97	379.98	—0.01
	47.135	381.31	381.32	—0.01
	53.699	383.85	383.84	+0.01
	54.341	384.10	384.10	0.00
	60.555	386.58	386.57	+0.01
	63.696	387.86	387.85	+0.01
	67.834	389.57	389.57	0.00
	69.590	390.32	390.31	+0.01
	70.195	390.58	390.57	+0.01
	74.254	392.30	392.31	—0.01
	75.414	392.82	392.81	+0.01
	81.060	395.30	395.31	—0.01
	82.961	396.17	396.17	0.00
	92.349	400.52	400.53	—0.01
	100.677	404.60	404.59	+0.01
	104.224	406.39	406.38	+0.01
h	111.542	410.19	410.18	+0.01
	119.307	414.39	414.38	+0.01
	129.516	420.21	420.21	0.00
g	133.685	422.70	422.69	+0.01
	140.938	427.17	427.17	0.00
G	146.598	430.80	430.80	0.00
Hγ	151.535	434.07	434.08	—0.01
	157.787	438.37	438.38	—0.01
	160.793	440.50	440.51	—0.01
	193.579	466.80	466.82	—0.02
F	213.523	486.17	486.19	—0.02
b_4	239.565	516.77	516.66	+0.11
b_2	239.991	517.28	517.22	+0.06
b_1	240.820	518.39	518.32	+0.07

Aside from the *b* lines, which do not appear quite sharp on the photograph so that their measures are somewhat uncertain, the whole portion of the spectrum embracing a difference of wave-length of $115^{\mu\mu}$ is represented with absolute accuracy by the formula. The small residuals under *O–C*, as is evident from the irregular distribution of their signs, are to be assigned to errors of observation.

POTSDAM ASTROPHYSICAL OBSERVATORY.

RÉSUMÉ OF SOLAR OBSERVATIONS MADE AT THE ROYAL OBSERVATORY OF THE ROMAN COLLEGE DURING THE FIRST HALF OF 1898.

By P. TACCHINI.

I GIVE below a résumé of the solar observations made here during the first half of 1898. We have enjoyed a fairly good winter, and consequently a considerable number of observations have been made. The results for the spots and faculæ are given in the following table:

1898	Number of days of observation	Relative frequency		Relative size		Number of spot groups per day
		of spots	of days without spots	of spots	of faculæ	
January	25	7.84	0.00	37.8	71.6	2.7
February	22	14.64	0.14	50.1	72.3	2.9
March	20	10.40	0.20	48.5	83.5	2.3
April	18	6.17	0.17	16.3	130.0	1.7
May	26	6.96	0.00	24.9	98.4	1.7
June	21	4.57	0.33	7.7	87.1	1.5

For the prominences we have obtained the following results:

1898	Number of days of observation	Mean number	Prominences	
			Mean height	Mean extent
January	18	2.67	31″.5	1°.3
February	14	2.57	30 .8	1 .2
March	13	2.38	31 .2	1 .4
April	16	3.44	27 .6	1 .1
May	25	1.08	19 .9	0 .8
June	24	3.00	32 .4	1 .1

After the secondary maximum in February the spots showed a progressive diminution, both in area and in the number of groups; but the minimum in June was not so marked as the minimum of November 1897, deduced from nearly the same

number of observations. It should also be noticed that even the period of small solar activity was not without fine groups, like those of February and March.

It appears from a comparison of these results with those of the preceding six months that the prominences, like the spots, have shown a diminution in activity; the minimum in May is especially remarkable.

The following table gives the distribution in latitude of the various phenomena, arranged by quarters in ten-degree zones.

PROMINENCES.

Latitude	First quarter		Second quarter	
90°+80	0.000		0.000	
80 +70	0.024		0.012	
70 +60	0.032		0.012	
60 +50	0.040		0.055	
50 +40	0.032	0.429	0.025	0.418
40 +30	0.040		0.086	
30 +20	0.055		0.068	
20 +10	0.111		0.068	
10 — 0	0.095		0.092	
0°—10	0.167		0.062	
10 —20	0.079		0.098	
20 —30	0.055		0.147	
30 —40	0.095		0.098	
40 —50	0.079	0.571	0.092	0.582
50 —60	0.040		0.069	
60 —70	0.024		0.012	
70 —80	0.016		0.012	
80 —90	0.016		0.012	

FACULÆ.

Latitude	First quarter		Second quarter	
60°+50	0.004		0.000	
50 +40	0.000		0.004	
40 +30	0.018	0.435	0.026	0.373
30 +20	0.091		0.053	
20 +10	0.148		0.117	
10 — 0	0.174		0.173	
0°—10	0.213		0.244	
10 —20	0.165		0.233	
20 —30	0.104	0.565	0.109	0.627
30 —40	0.061		0.026	
40 —50	0.018		0.011	
50 —60	0.004		0.004	

SPOTS.

20°+10	0.068	0.273	0.192	0.341
10 — 0	0.205		0.149	
0°—10	0.295	0.727	0.106	0.659
10 —20	0.432		0.553	

The prominences have been most numerous in the southern zones, as was the case in the preceding quarter; they have been seen in almost every zone, with the maximum south of the equator. Like the prominences, the faculæ had their greatest frequency in the southern hemisphere, with the maximum in the zone (0°—10°); they have extended from 0° to ±60°.

The spots have been confined to the equatorial zone, as in the last semester, and they have resembled the prominences and faculæ in the fact that their frequency was greater in the southern zones. The only eruption, a slight one observed March 18, was confined to the region —6°.4 to —9°.

ROME, September 14, 1898.

THE GREAT NEBULA OF ANDROMEDA.

By E. E. BARNARD.

IN June and July of this year I made a series of measures of the nucleus of the Great Nebula of Andromeda with reference to two 11th magnitude stars—one preceding and the other south.

Following are the measures, the stars being referred to the nucleus:

Nucleus and star *a*

1898	June 27	261°.24	124".85	
	July 5	261 .15	124 .41	
	11	261 .15	124 .47	
	12	261 .35	124 .97	
	18	261 .23	124 .63	
	Sept. 27	261 .28	124 .85	
		261 .23	124 .70	∴ $\Delta\alpha$ (neb. $-*$) $= +0^m\ 10^s.84$
				$\Delta\delta$ $= +0'\ 19''.01$

Nucleus and star *b*

1898	June 27	160°.65	228".31	
	July 5	160 .55	228 .57	
	11	160 .45	228 .47	
	12	160 .52	228 .30	
	18	160 .38	228 .46	
		160. 51	228 .42	∴ $\Delta\alpha$ (neb. $-*$) $= -0^m\ 6^s.70$
				$\Delta\delta$ $= +3'\ 35''.34$

A fainter star was also measured on one date:

Nucleus and star *c*, 14^m

1898	July 5	177°.22	157".87

At each of these measures the nucleus was fairly well seen, though it was not stellar.

It has been my custom, both here and at the Lick Observatory, to frequently examine the great nebula for traces of the Nova of 1885, but nothing has ever been seen of it. During the present measures I carefully looked for the Nova, and on July 11 made the following note:

"Looked carefully for Nova of 1885, with powers 460 and 700, seeing fairly good."

It will be seen, therefore, that I am pretty familiar with the appearance of the nebula.

On September 20, past, a telegram announced the discovery of a starlike condensation in the center of the great nebula by Seraphimoff at Pulkowa.

Later advices seem to show that Seraphimoff's observation was simply a confirmation of the discovery of a starlike body in the great nebula by someone else, presumably Mr. A. A. C. Merlin, British vice-consul at Volo, Greece, who had telegraphed August 29 that he had observed a star near the nucleus of the great nebula.

M. Rayet, of the Lyons Observatory, France, announces in the *Comptes Rendus* for September 26, 1898, that he has verified Seraphimoff's observation and that the change is real.

When the announcement of a change in the nebula was made I had the impression that it must be a reappearance of the Nova of 1885. As soon as darkness permitted, the great telescope was turned on the nebula. It was at once seen that the Nova was not present. It was also evident that no perceptible change had occurred in the nucleus or the nebula itself. Since then this object has been carefully examined a number of times with both the 40-inch and the 12-inch telescopes, and the impression has always been that the nebula is unchanged. The measures of the star *a* on September 27 were made as a confirmation of the previous measures to show that no change had occurred in the position of the nucleus, which appeared the same to me as it did at the previous measures.

The nucleus of this great nebula is so affected by the conditions of seeing, moonlight, etc., that one can readily be deceived by it, if he is not perfectly familiar with its appearance under all conditions. When the air is steady the nucleus is almost stellar, especially with low powers.

If the seeing is not good the nucleus is woolly and faint, and often lost sight of entirely, so that one familiar with it under

these conditions alone would suppose a great change had occurred if he saw the nucleus under the best conditions. Its distinctness is more easily affected than that of a star of the same magnitude for two reasons — first, it is not a stellar point; second, its light would be confused with that of the nebula under conditions of seeing that would scarcely affect a star not situated in a bright nebula.

I think photography would be a more satisfactory test of change than the memory of one not perfectly familiar with the nebula.

From visual observations, I would say that no apparent change has occurred in this great nebula in the past few years.

M. Rayet, in the number of the *Comptes Rendus* referred to, gives his measures of the position of the nucleus and the 11th magnitude star preceding. He also gives the measures of other observers.

Here is the list:

	$\Delta\alpha$	$\Delta\delta$
D'Arrest, - -	$+0^m\ 11^s.26$	$+0'\ 19''.5$
Struve, - -	$+0\ \ 10\ .87$	$+0\ \ 18\ .7$
Vogel, - - -	$+0\ \ 10\ .79$	$+0\ \ 19\ .4$
Rayet, - - -	$+0\ \ 10\ .91$	$+0\ \ 18\ .48$
The means are, -	$+0\ \ 10\ .96$	$+0\ \ 19\ .02$

Dr. R. Engelmann, at Leipzig, in the first part of September 1885, measured the position of the star *a* with reference to the Nova (*A. N.* 2683):

Position angle, $262^\circ.42$ (4) Dist., $109''.72$ (6)

From this, the Nova was nearly on the line from the nucleus to the 11^m star and some $16''$ distant from the nucleus.

Engelmann also measured the position of the nucleus with reference to the Nova:

Position angle, $76^\circ.14$ (8) Dist., $16''.33$ (8)

and hence $\Delta\alpha = +0^m\ 1^s.393$, $\Delta\delta = +0'\ 3''.91$

YERKES OBSERVATORY,
October 21, 1898.

Minor Contributions and Notes

PROCEEDINGS OF THE SECOND CONFERENCE OF ASTRONOMERS AND ASTROPHYSICISTS.

The following abstracts of papers read, or presented by title, at the Second Conference of Astronomers and Astrophysicists have been prepared by the authors.

PERSONAL EQUATION IN TRANSIT OBSERVATIONS.

In the study of personal equation, we learn only what may happen under special conditions, which may possibly never recur, and the labor demanded by the study often seems excessive in view of its very limited results. This furnishes an excuse for the comparatively superficial investigation of the subject ordinarily undertaken, as in the present instance. Still, some conclusions which seem worth recording have been reached by the experience obtained during the last thirty years at Harvard College Observatory. One of these conclusions is that the micrometrical determination of the collimation error of a transit instrument, by means of collimators or of meridian marks, so commonly disagrees with the result for collimation obtained by the adjustment to each other of a series of transits observed with the same instrument, that it becomes unsafe to depend upon the micrometrical determination in the reduction of the observations. While the results obtained by different observers by means of the collimators agree very closely, and while the error thus determined remains nearly constant for considerable periods of time, the results obtained by the same observers from the discussion of series of transit observations give different results, which are neither so accordant nor so constant as those obtained micrometrically. It may be, of course, that the errors indicated by the transits are not really collimation errors; that is, it may be suspected that some other coefficient than the secant of the declination might be employed in the reductions to better advantage. After trial of various coefficients, however, none have been found to produce as much accordance in the results for different stars as can be

obtained with the secant of the declination. It is therefore provisionally assumed that the discrepancies which have been noticed are due to a personal equation in collimation, in addition to the kind of personal equation usually recognized in transit observations, consisting simply in a difference in the absolute time of the observations made by different persons.

Another form of personal equation in transit observations occasions errors of the kind which might be attributed to a rate of the clock at the time of observation differing from the rate found for it by the comparison of the results for different days. But this supposition is excluded when we find on the same evening different rates derived from the transits observed by different persons.

It also happens that differences occur systematically between different observers with regard to stars in northern and southern declinations equally far from the equator; so that the apparent position of the axis of the instrument, as well as the line of collimation, changes with the observer. This phenomenon is more difficult to understand than those previously mentioned.

Personal equation dependent on stellar magnitude has been for some time an admitted fact, and was merely mentioned in the paper here abridged. We have, accordingly, five kinds of personal equation possible in transit observations: (1) in absolute time, (2) in collimation, (3) in the apparent position of the instrument, (4) in clock rate, (5) in the effect of stellar magnitude.

ARTHUR SEARLE.

SOME INVESTIGATIONS RELATING TO ZENITH TELESCOPE LATITUDES.

A series of latitude determinations by the Talcott method was undertaken by the author in the winter of 1895 for the purpose of investigating the effect of barometric and thermometric gradients in the atmosphere upon zenith telescope latitudes. The instrument employed was an excellent portable transit by Bamberg, and the observing programme comprised chiefly pairs of stars culminating at large zenith distances, in the average about 75°, since the effects sought increase with $\sec^2 z$.

The amount by which a stratum of homogeneous air was tipped from its normal horizontal position was determined for each observing night from the isobars and isotherms of the United States Weather Bureau's weather map for 8 P.M., and the resulting disturbance com-

pared with the observed value of the latitude. During the period of observation the inclinations of the strata varied from $-0'.4$ to $+2'.2$, but no obvious relation exists between these inclinations and the observed latitudes. These latter, however, show very plainly a dependence upon the direction of the wind, being a maximum when the wind comes from the direction of the Observatory chimney, distant about thirty meters from the instrument. The amplitude of this apparent variation in the latitude is approximately one second of arc.

A similar series of observations made in the months of May and June, when the chimney was not heated, shows no such variation, and also shows no marked dependence of the latitude upon the inclination of the air strata, although in this series the inclination of the strata varied from $-1'.6$ to $+2'.8$, and the zenith distances of the stars observed were included between 76° and 87°. Observations at this latter zenith distance, although difficult and of diminished precision, are nevertheless eminently satisfactory for the purpose in hand.

The foregoing results suggest that the variation from night to night in the value of the observed latitude depends upon local conditions rather than upon elements which can be read from a general weather map. That perturbing elements of appreciable magnitude are generally present in zenith telescope work, may be inferred from the following table of recent determinations of the constant of aberration made by the Talcott method, and the remainder of the paper was devoted to a discussion of the local surroundings at the several places of observation with reference to determining their probable effect upon the value of the aberration.

Place	Authority	Observed Aberration Constant	Corr.
Hong Kong..........	*A. N.*, 3504	20".38 ± 0".07	?
Hong Kong..........	*A. N.*, 3504	20 .43 0 .06	?
Honolulu.............	*Bull. C. and G. Surv.*	20 .43 0 .03	—
New York............	*A. J.*, 401	20 .46 0 .01	?
Strassburg............	*V. J. S.*, 1895	20 .47 0 .01	?
San Francisco........	*Bull. C. and G. Surv.*	20 .48 0 .03	—
Berlin...............	*A. N.*, 3015.	20 .49 0 .02	—
Naples...............	*Nov. Determ.*, Fergola	20 .53 0 .01 (?)	—
Bethlehem............	*A. J.*, 406	20 .55 0 .01	—
Cape.................	*M. N.*, Dec. 1897	20 .58 0 .01	—
Berlin...............	*A. J.*, 429	20 .61 (?)	—
Hong Kong..........	*A. N.*, 3504	20 .64 0 .08	?

This effect cannot, in any case, be determined quantitatively, but its sign is indicated in the last column of the table, a + indicating that the observed value should be increased to eliminate the effect of local conditions: *e. g.*, at Naples the diurnal range of temperature north of the Observatory (over the land) is greater than that south of the Observatory (over the sea), whereby stars near the zenith are relatively thrown toward the north in the evening and toward the south in the morning, thus producing too great a value of the aberration. Similarly, at the other stations, an unsymmetrical diurnal variation of temperature is supposed to produce an effect upon the observed value of the aberration.

A more detailed exposition of the subject will be published elsewhere.

G. C. COMSTOCK.

STARS OF THE FIFTH TYPE IN THE MAGELLANIC CLOUDS.

Stars having spectra consisting mainly of bright lines belong to Type V. In 1891 the number of these stars known was thirty-three. Three of these were discovered by Wolf and Rayet, six by Copeland, one by Respighi, three by Pickering, and twenty from the Draper Memorial Photographs of the Harvard College Observatory. In May 1897 the number known was sixty-seven. All of these stars lie closely along the central line of the Milky Way, and, although the sky had been equally well covered with spectrum plates from pole to pole, a careful examination of these plates failed to show any of these stars outside of this region.

On May 26, 1897, an examination of two plates taken at Arequipa with the Bruce telescope, showed a group of six of these objects in the Large Magellanic Cloud. Two later plates add fifteen more in the Large Magellanic Cloud, and these, with three others in the Milky Way, and one in the Small Magellanic Cloud, make the total number of these stars, so far known, ninety-two. Thirteen of these were found visually, all the others have been found from the photographs in the Draper Memorial work. The great advantages of photography in the study of stellar spectra is thus again demonstrated.

The presence of so many stars of the Fifth Type in the Magellanic Clouds establishes another connection between them and the Milky Way.

M. FLEMING.

CLASSIFICATION OF THE SPECTRA OF VARIABLE STARS OF LONG PERIOD.

In the classification of the spectra of stars in the *Draper Catalogue* the letter M was assigned to stars having spectra of the third type. Later these stars were subdivided into four groups, Ma, Mb, Mc and Md, of which α Orionis, α Herculis, $-2^\circ 3653$, and o Ceti were given as examples. In the last of these stars the hydrogen lines are bright, and this spectrum has been shown to be characteristic of variable stars of long period; indeed, by this peculiarity in the spectrum about a hundred new variables have been found. A further examination of these spectra shows that they can be further subdivided into eleven groups. A classification was made from an examination of the continuous spectrum, the comparative brightness of the hydrogen lines being also carefully estimated, always assuming the brightness of $H\gamma$ as 10. The first class, of which R Lyncis is the typical star, shows a spectrum resembling α Tauri, having also $H\beta$ and $H\gamma$ strongly bright and nearly equal, while $H\delta$ is barely visible. The last group, of which R Leonis is the typical star, shows a continuous spectrum similar to $-2^\circ 3653$, or a little beyond that star in the classification. Of the bright hydrogen lines in R Leonis $H\beta$ is not seen, $H\gamma$ is barely visible, and $H\delta$ is strongly marked. The other classes form a nearly continuous sequence between these extremes.

M. FLEMING.

VARIABLE STARS IN CLUSTERS.

A systematic search of twenty dense globular clusters by means of photographs made with the 13-inch telescope gives the following results:

Number of stars examined about 18,600.

Number of variables found 501.

In certain clusters no variables have been found; in others very few. The results in four clusters are especially striking.

N. G. C. 5272, Messier 3, out of about 900 stars examined, has 132 variables, nearly 15 per cent.; *N. G. C.* 5904, Messier 5, out of about 900 stars examined, has 85 variables, about 9½ per cent. In *N. G. C.* 7078, 51 stars out of 900, or 6 per cent., and in *N. G. C.* 5139, ω Centauri, 125 out of about 3000, or 4 per cent., have been found variable.

Considerable progress has been made in the discussion of the variables in ω Centauri. The approximate periods and light curves of

105 have been determined. The extreme periods so far found seem to be 475 days in the case of No. 2 and $6^h\ 11^m$ in that of No. 91. Ninety-eight have periods less than 24^h.

These short periods are often characterized by sudden increase in brightness, brief maximum, a relatively slow decrease in brightness, and a considerable interval at or near minimum. On the other hand, in some cases the time of increase is equal to or even greater than that of decrease.

S. I. BAILEY.

MERIDIAN OBSERVATIONS FOR STELLAR PARALLAX.

The reduction of the observations made in the years 1893–1896 at the Washburn Observatory, Madison, Wis., of which a report was given at the conference of last October, is now nearly complete.

The Repsold meridian circle of $12^{cm}.2$ aperture was employed in the well-known but comparatively new method of observing meridian transits of each parallax star with two comparison stars. The observing list of nearly one hundred stars comprised over seventy stars of large proper motion, together with binaries, most of which have considerable proper motion. Screens of fine brass wire were employed to reduce the apparent magnitudes of the brighter stars.

The reduction of the present series of parallaxes to the weighted means of good determinations by previous observers, from the thirty stars available, is $0''.03$. Systematic error might be expected mostly on account of larger differences of magnitude between the parallax stars and their comparison stars. These differences range from -2.0 to $+2.0$, the latter sign indicating that the parallax star was fainter. A graphical representation of all the parallaxes shows some appearance of such an error, and it appears to be within the limit $+0''.065$ for a difference of one magnitude in the positive direction. A plot of the differences from previous determinations, on the thirty stars already mentioned, gives a similar indication. This effect appears more marked in several cases which will require further investigation. Extensive observations were made concurrently with the parallax observations upon independent, miscellaneous stars employing the screens in order to control the personal equation dependent upon magnitude.

A plot of the parallaxes according to the right ascensions of the stars gives no plain evidence of seasonal variation. The same holds true of the differences from previous determinations.

The general means of the probable errors of the parallaxes, from the stars most frequently observed, range from $\pm 0''.032$ to $\pm 0''.045$, the former value for stars of higher declination, the latter for those of moderate southern declination. The corresponding probable errors for the observed difference of right ascension of any two stars vary from $\pm 0''.17$ to $\pm 0''.22$. The probable error of a single observation appears not to be affected by such differences as occur in the right ascensions, declinations, or magnitudes of the stars, but is increased materially upon stars too faint for convenient observation.

A graphical representation was made of the parallaxes of the present series according to the proper motions of the stars, in all cases where they were greater than 1'' with the exception of Groombridge 1830. The plotted points happened to fall into three groups, the apparent centers of which lay on one right line. From this line the following figures for mean parallax corresponding to proper motion are derived :

Proper motion	Parallax	Number of stars
1''.25	0''.10	40
2 .05	0 .14	13
4 .20	0 .28	5

A new series of parallax observations by this method has been begun. The list consists of 121 parallax stars, second-magnitude stars, binaries, and certain stars of larger proper motion. Instead of repeating the customary method of observation, that of recording transits over a fixed reticule, a Repsold transit-micrometer is employed, and the observer holds the image of a star during transit in bisection between a pair of movable threads.

ALBERT S. FLINT.

THE DIRECT GRATING SPECTROSCOPE.

This paper consisted of a description of a new method of using the concave grating for stellar spectroscopy. Preliminary notes, giving full details of the method in question, were published in the ASTROPHYSICAL JOURNAL for March 1898, and in the *Monthly Notices of the Royal Astronomical Society* for March 1898. Since these notes appeared a much larger grating was especially ruled by Professor Rowland, and this paper embodied the results obtained with this new grating. This grating has a ruled surface of $5\frac{3}{4} \times 2$ inches. The radius of curvature

is 1 meter, and the grating is ruled with 7500 lines to the inch. The definition is excellent.

Many original negatives were exhibited, as well as several enlarged glass positives. The experiments with the larger grating showed that a good photograph of the spectrum of Sirius could be obtained in the second order with an exposure of five minutes. In general, it may be stated that the time of exposure for different stars was very much less than had been currently expected.

CHARLES LANE POOR.

THE EFFECT OF ATMOSPHERIC DISTURBANCE ON TELESCOPIC DEFINITION.

During the past year a number of papers have been published showing the effect of different conditions of the atmosphere on telescopic definition. In a discussion of this nature it is important to discriminate between the effects produced by disturbances directly in front of the object-glass and those in the distant atmosphere.

A cylinder of light the size of the object-glass extends through the whole body of atmosphere, and it is extremely probable that abnormal conditions may exist at different distances from the object-glass.

In the latitude of the United States atmospheric waves are constantly propagated from west to east with a velocity from ten to forty miles per hour. Aside from this, near the surface of the Earth there is a bodily translation of air which may move in any direction. Hence it follows that at every instant the wave-front meets a new stratum of atmosphere. Now when we consider the magnitude of atmospheric waves, the area of the wave-front for the largest telescope in use will be infinitely small as compared with area of the disturbed region, and it is reasonable to assume that the wave-fronts will be in the same condition in every part.

As every unit of surface of the object-glass forms an image at the focus, the superimposed images will be of the same nature and the primary images formed by a large object-glass or a small one, will be of the same quality.

Under this conception the disturbance in definition will depend on the magnifying power, and will be entirely independent of the size of the object-glass. This conclusion appears to be borne out by experience. When the definition is bad, reducing the aperture of the 18½ inch refractor of the Dearborn Observatory, does not materially improve the seeing, and I long ago abandoned the use of caps for this purpose.

A good deal has been written regarding the advantages of a small aperture under the condition of imperfect definition. The following considerations I think will show the fallacy of the argument.

If on a night when the definition near the zenith is at its best, and the highest power (925) of the 18½-inch refractor can be used on a double star of $0''.2$ or planetary detail studied with a power of 500, the telescope be pointed at a zenith distance of 45°, it will be seen that the definition is not as good as near the zenith. When it is pointed at 60° zenith distance, a lower magnifying power must be used to secure reasonable definition.

When we have gotten below 70° zenith distance one may do some useful work by employing a lower power, but as we approach the horizon a point will be reached where the definition is too poor for any useful work.

A 6-inch object-glass would be affected in the same manner, but objects below a zenith distance of 60°, where the definition was bad, which were observed with the 18½-inch, would be entirely beyond the reach of the 6-inch.[1]

On nights when the definition is bad near the zenith, the 18½-inch can be used on objects which are beyond the reach of much smaller instruments.

Hence, under any atmospheric conditions we can see more and better with a large telescope than with a small one.

The time during which a large aperture can be used to its full capacity is another question.

At Chicago and Evanston the 18½-inch can be used successfully with the highest power on about 10 per cent. of the observing nights. For 50 per cent. of the time it can be used for observations beyond the reach of much smaller instruments, and in the worst condition of seeing it would not be inferior to a smaller telescope.

G. W. HOUGH.

ON THE SPECTRA OF STARS OF SECCHI'S FOURTH TYPE.

An investigation of the spectra of stars of Secchi's fourth type, upon which I have been engaged, with the assistance of Mr. F. Ellerman, since the beginning of the present year, while still in its preliminary stages, has advanced far enough to yield some interesting results. The spectra of 22 stars of this class, ranging in magnitude from 5.4 to 8.2,

[1] *Vide* Catalogues of double stars discovered with large apertures.

have been photographed with the stellar spectrograph attached to the 40-inch refractor of the Yerkes Observatory. In most cases, on account of the faintness of the stars, a dispersion of one 60° prism has been used, but good photographs of the spectra of 132 and 152 Schjellerup have been obtained with three 60° prisms. The work has been confined for the most part to the region between D and *b*, where Cramer's Instantaneous Isochromatic plates have given very satisfactory results. Some photographs of the upper part of the spectrum, extending as far as λ 4450 have also been secured, and by the aid of the new "Erythro" plates the spectrum of 152 Schjellerup has been photographed in the region extending from D to Hα.

After a study of the photographs I have been led to the conclusion that the spectra of 152 Schjellerup and other stars of this class contain bright lines, in addition to the heavy bands attributed to some form of carbon and numerous dark lines. On account of Dunér's emphatically expressed disbelief in the existence of bright lines in these stars,[1] I have taken special pains to submit the question to a careful test. The true character of these lines seems to me to be demonstrated by the following considerations:

1. The lines are best shown on photographs taken with high dispersion (3 prisms); whereas bright spaces between dark lines would be most conspicuous with low dispersion. This photographic result is confirmed by visual observations made with the 40-inch telescope. With a small direct vision spectroscope the bright lines cannot be seen, but with a dispersion of three prisms the brightest ones have been observed without difficulty in 132 and 152 Schjellerup.

2. Widening the slit does not cause them to disappear, nor, in the case of such lines as λ 5593, does it appreciably diminish the visibility.

3. The intensity of many of the bright lines in the photographs is decidedly greater than that of any part of the spectrum in their vicinity which appears to be continuous.

4. At my request the spectrum of 152 Schjellerup has been observed by Professors Keeler and Campbell with a three-prism spectroscope attached to the 36-inch Lick telescope. Professor Keeler reports that in his opinion "there is little doubt that the spectrum of this star contains bright lines."

The wave-lengths of about one hundred bright and dark lines between *b* and D in the spectrum of 152 Schjellerup have been deter-

[1] *Sur les Étoiles à spectres de la Troisième Classe*, p. 10.

mined from measurements made on several plates. It is perhaps worthy of note that the mean wave-lengths obtained for two of the most intense bright lines, λ 5592.8 and λ 5693.4, agree very closely with the mean wave-lengths of two bright lines in the spectra of the Wolf-Rayet stars, Professor Campbell's values being λ 5593 and λ 5693. Although several other bright lines in fourth type spectra probably fall very close to Wolf-Rayet lines, it is too soon to conclude that these classes of stars are related. With this possibility in mind, however, I requested Mr. J. A. Parkhurst, who had kindly offered his services, to determine the distribution of fourth type stars with reference to the Milky Way. The results of Mr. Parkhurst's investigations indicate that stars having spectra of the fourth type exhibit a decidedly greater tendency to cluster in and near the Milky Way than any other class of stars except those of the Wolf-Rayet type.

A comparison of fourth type (III*b*) spectra with those of the third type (III*a*)[1] fails to show any very striking similarities, though there appear to be certain resemblances, particularly in the region just below *b*, which deserve and will receive further study.

The peculiarities in the spectra of the stars 541 Birmingham (*DM* 38° 3957) and *DM* 59° 2810, which were recognized and discussed by Professor Dunér,[2] are well brought out in the photographs. It has been found possible to arrange the spectra of ten stars in a series, having at one end 155 *b* Schjellerup and at the other *DM* 59° 2810. The differences between contiguous spectra are in most cases very slight, though 541 Birmingham, which comes next to *DM* 59° 2810, is separated by a rather wide gap from the star next above it. 132 Schjellerup comes near the middle of this series. I have as yet no evidence favoring Dunér's hypothesis that fourth type (III *b*) stars are developed from those of the second class (II *a*).

The investigations will be continued as rapidly as possible, but it is necessarily slow, as the 40-inch telescope is available for this work on but one and one-half nights each week, and for the fainter stars exposures of many hours are required. Mr. F. Ellerman has had an important part in the work, as he has made many of the photographs and rendered assistance in the work of reduction. For the last few weeks I have also had the aid of Dr. Frank Schlesinger, Research Assistant

[1] Photographs of the spectra of many stars of the third type were taken for this purpose.

[2] In his invaluable memoir *Sur les Étoiles à spectres de la Troisième Classe.*

during the Summer Quarter, who has done excellent work in measuring photographs and determining wave-lengths.

GEORGE E. HALE.

PORTRAIT-LENS PHOTOGRAPHS.

While connected with the Lick Observatory series of photographs were made of the Milky Way, the various comets, etc., with a six-inch portrait lens. Lantern slides from these pictures were projected on the screen. Some of the subjects were:

The phenomenon of the "old Moon in the new Moon's arms," or the Earth-lit portion of the new Moon. The night part of the Moon, illuminated only by light reflected from the Earth, was clearly shown on the photograph and the details of the lunar surface during the lunar night could be made out distinctly.

The total eclipse of the Moon September 3, 1895. These photographs were made in the hope of showing a possible lunar satellite which might at the time be outside the shadow and fully illuminated.

The photography of meteors. It was found that meteors were rather frequently caught with the photographic plate and the short focus portrait lens. Several photographs of meteor trails were shown which had been obtained at the Lick and Yerkes Observatories.

Photographs of numerous portions of the Milky Way were exhibited showing the cloud structures and the great dark rifts where the substratum of stars seemed to be breaking up. A photograph in the region of Antares seemed to show that the apparently small stars that made up the ground work of the Milky Way in this region were really comparatively small bodies, perhaps very much less then our own Sun in size.

The various comets had been photographed and many extraordinary phenomena, heretofore unknown, were shown on the pictures, some of which were highly suggestive.

The peculiar phenomena shown in the tail of Swift's comet of 1892 could readily be explained by an irregular emission of matter from the nucleus, while the photographs of Brooks' comet of 1893 showed such an extraordinary condition of the tail that some outside influence seemed required to explain the phenomena. It was suggested that the tail of this comet possibly encountered some kind of resisting medium about October 21, 1893, and for some time subsequent to that date. No other explanation seemed satisfactory.

E. E. BARNARD.

A BRIEF ACCOUNT OF THE WORK OF THE BLUE HILL METEOROLOGICAL OBSERVATORY.

Although in the past meteorological observatories have been associated generally with astronomical institutions, yet this union usually is disadvantageous to the former, and the separation is now customary, especially in Europe. In the case of the Harvard and Blue Hill Observatories the circumstances favoring coöperation were exceptional. The former maintained meteorological observations in some detail, while the latter, only twelve miles distant, was admirably located and equipped for this work, which, moreover, it was desired to continue indefinitely and to publish in a permanent series. Accordingly an alliance was formed in 1887, two years after the foundation of the Blue Hill Meteorological Observatory by the writer, whereby his observations and investigations are published annually, or oftener, in the *Annals of the Astronomical Observatory of Harvard College*, at the joint expense of the college and the writer. It is expected that the two institutions will be united ultimately, and with this expectation the summit of the hill on which the Blue Hill Observatory stands has been leased for a long term of years to Harvard College. The writer, however, continues to direct the work at Blue Hill and to defray the whole cost of maintenance. Three assistants are now employed, and the annual expense exceeds $4000.

The Blue Hill Observatory was probably the first in this country to record automatically and continuously all the meteorological elements, of which hourly values have been reduced, printed, and discussed. Local weather predictions were issued for several years until the United States Weather Bureau commenced this service in Boston in 1891. Of various investigations undertaken and published, there may be mentioned the comparison of different forms of anemometers by Mr. Fergusson; the first measurements in America of the heights and velocities of clouds by Messrs. Clayton and Fergusson in 1890–1; and the exploration of the air by means of kites, which was originated at Blue Hill in 1894. An exhaustive discussion by Mr. Clayton of all cloud observations has been published, and the international cloud measurements which were completed last year will soon be printed. At the present time the chief work, apart from the routine observations, is obtaining simultaneous records of the temperature and relative humidity of the air and the velocity of the wind at heights exceeding two miles in the free air by means of kites and near the ground. It is expected that

the final discussion of the kite records will be published by the Smithsonian Institution, which has aided the investigation by a grant from the Hodgkins Fund.

A. LAWRENCE ROTCH.

PHOTOGRAPHIC RESEARCHES NEAR THE POLE OF THE HEAVENS.

If a series of short exposures be made with a photographic telescope pointed at the pole of the heavens, we obtain a negative on which each star gives a series of dots lying approximately in a circular arc corresponding to the diurnal motion. The common center of all such arcs will be the point on the negative representing the pole on the sky. It is evident that photographs of this kind furnish a method of determining the place of the pole with respect to the surrounding stars at the time the negative was made. Moreover, by the intercomparison of the results obtainable from plates made on different dates, it is possible to arrive at a determination of the fundamental constants of stellar astronomy.

The method possesses the advantage of being entirely different in principle from those hitherto in use, and as it promises high precision, it may serve to obtain a useful check upon existing results. The first attempts to use the method practically were made, I believe, at the Harvard College Observatory, by Professor E. C. Pickering, some years ago. The present paper contains, however, the first approximately complete discussion of exact measurements upon plates of this kind. The negatives employed were made at Helsingfors by Professor Anders Donner for the writer. The Observatory of Helsingfors possesses the advantage of being the most northerly of existing observatories provided with adequate photographic telescopes. Photographs of the pole made at Helsingfors are therefore less distorted by refraction than those made elsewhere on account of the greater altitude of the pole at that place. The elimination of refraction effects by computation constitutes one of the greatest difficulties with which the writer has had to contend, so that the importance of a high latitude for researches of this kind is very evident.

The results here presented consist of:

1. The deduction of formulæ suitable for the complete reduction of ordinary polar star photographs, as well as the special kind of plates just described.

2. An application of the formulæ to measures on one plate of six images of each of ten stars, or sixty images in all.

While no entirely definite conclusions can be drawn from material so scanty, enough has been done to show that the new method merits the serious consideration of astronomers. It is the intention of the Columbia Observatory to extend the work so as to include the complete measurement of all the images on two Helsingfors plates made six months apart. A tentative determination of the constant of aberration should thus be possible, and should throw much light on the possibilities of the new method. The details of the work so far accomplished cannot be summarized in this brief notice, but an extended paper is in the hands of the printer, and will, it is hoped, soon be laid before the astronomers.

HAROLD JACOBY.

A PROPOSED CATALOGUE OF NORTH POLAR DISTANCES.

The United States Coast and Geodetic Survey has made it its uniform practice to make careful computations of the mean north polar distances of all stars observed for latitude with a zenith telescope. It is now proposed to publish in the near future the accumulated results of these years of computation in the form of a star catalogue.

The catalogue will comprise about 3500 stars, all brighter than the seventh magnitude. The computations have been made substantially upon the Boss system — the weights and systematic corrections of that system as published in the Northern Boundary Report being used for the catalogues there given and supplemented by special investigations for later catalogues.

In most of the later computations both the polar distance and proper motion have been derived by the method of least squares and the probable error of each will be given in the catalogue.

J. F. HAYFORD.

THE K LINES OF β AURIGAE.

By Antonia C. Maury. (Published in the ASTROPHYSICAL JOURNAL, October 1898.)

THE VARIABLE STAR U PEGASI.

By G. W. Myers. (Published in the ASTROPHYSICAL JOURNAL, October 1898.)

THE ZODIACAL LIGHT.

Fifty years ago, and indeed until within ten or twelve years of the present time, all hypotheses relating to the origin of the zodiacal light were opposed by difficulties which could not satisfactorily be met. The recent progress of our knowledge upon the subject consists chiefly in the removal of the difficulties attending one hypothesis—that which assumes the light to be reflected from meteoric dust forming a part of the solar system. At present, therefore, this hypothesis tends more and more to prevail. It is true that the two principal observers of the zodiacal light, Jones and Heis, were inclined to regard it as a ring encircling the Earth. The obvious objections to this hypothesis are the considerable extent of the light in latitude, which would require the supposed ring to be very near us, and the absence, at the same time, of any decided parallax, or of any obscuration by the shadow of the Earth. The hypothesis, apparently, was suggested by the observations which tended to show that the zodiacal light forms a continuous belt all along the zodiac, with a slight maximum of brightness, commonly called the Gegenschein, in opposition to the Sun. This maximum was the chief difficulty in the way of the meteoric hypothesis now prevalent, for, according to Lambert's theory of phases, which had acquired a fictitious authority by long usage, no such maximum should exist. But accurate photometric observation of asteroids in recent times has made it clear that Lambert's theory is wholly inapplicable to their phases, and if we suppose that meteors have phases like those of asteroids, it appears that light reflected from such bodies should exhibit a maximum at opposition.

Since the time of Jones and Heis, observation of the zodiacal light, in its ordinary and conspicuous form, has been greatly neglected. On the other hand, we have numerous recent observations of Gegenschein, and some which were undertaken in 1893, to be made as nearly as possible simultaneously at widely separated stations, formed a principal subject of the paper of which this is an abstract. It was concluded from the comparison of nearly simultaneous observations made in New England, in California, and in Peru, that no effect of parallax in the observed object could be detected.

Another subject of the paper was the bands permanently visible in and near the zodiac, and probably due to aggregations of faint stars or of nebulous matter, which tend to interfere with exact observation of

the fainter portions of the zodiacal light, including the Gegenschein. Simultaneous and careful drawings of these bands were made by two observers during 1893 and 1895, and plates showing specimens of these drawings were laid before the audience. It appeared from these plates that the agreement between the two observers was sufficient to encourage the belief that the drawings represent real objects, which should be carefully studied by all those who desire to add to our knowledge with regard to the zodiacal light.

ARTHUR SEARLE.

ROTATION OF ASTEROIDS.

The suggestion that if the asteroids were observed photometrically there might be a discrepancy arising from a difference in the reflection from the opposite sides of an asteroid, presented to us at different times in consequence of their rotation, did not originate with me. Prior to my observations discrepancies in the observations had been attributed to this cause as a probable explanation. In the first year of my observation of asteroids, 1887, I found discrepant observations of two asteroids, Nos. 40 and 42, which seemed to me to indicate their probable rotation. There were twelve asteroids more fully observed which showed no such difference. In the following year, 1888, I observed No. 40 much more thoroughly and demonstrated to my satisfaction that no such variation as I had suspected existed in its light. During 1888 and 1889, the number of asteroids which I observed was more than doubled, and no additional asteroid was suspected of such variation. It has not been practicable for me to take up the remaining asteroid, No. 42, until June of the present year. I have now observed it anew, with the result that no such variation appears. Furthermore, upon revising the observations of No. 42 in 1887, by the aid of the constants obtained from the more extended observations of the present year, the hypothesis of rotation is contradicted by those very observations of 1887.

Of course I cannot prove a negative; it is not impossible that some asteroid does exist, observation of which would show large variation from rotation; but all my evidence having been demonstrated to be illusive, and the observation of many new asteroids not yet furnishing additional evidence of it, I think it is safe to dismiss the theory of a photometric change from rotation from further consideration.

HENRY M. PARKHURST.

REMARKS REGARDING THE PARALLAXES OF 61^1 AND 61^2 CYGNI AND THE PROBABLE PHYSICAL CONNECTION OF THESE TWO STARS.

In *Contribution 13* from the Observatory of Columbia University a discussion of the measures of certain photographic plates, made by Dr. Rutherfurd in 1871–1874 for the determination of the parallax 61 Cygni, has been presented by the writer, to whom the measures were entrusted for that purpose by Professor J. K. Rees, Director of the Observatory.

The main result given in that publication is summarized by the lines:

$$\text{Parallax of } 61^1 \text{ Cygni} = +0''.360 \pm 0''.015.$$
$$\text{Parallax of } 61^2 \text{ Cygni} = + \quad .288 \pm \quad .028.$$

The object of the paper prepared for the Astronomical Conference was to suggest what these results signify.

In the first place, consideration was made of whether these two stars are only optically double or are physically connected, forming a binary system. This is a question which has adherents on each side, some claiming that the variations of measures of distance, from Piazzi's time to the present, show an orbital motion; others claiming that such variations are due to accidental or systematic errors of observation. But it was pointed out that the theory of probabilities has been used somewhat more effectively in favor of orbitál motion than has been the better established theory of motion of matter acted on by gravitation. Thus, too often, have the claimants of rectilinear motion been met by the statement that whether the *measures* are conclusive or not, the *chances* are many millions to one against two stars of such unusual proper motion being placed accidentally in such close juxtaposition in line of sight from the Earth at this particular age in the Earth's history.

Then were quoted the conclusions arrived at by Wilhelm Struve regarding the probability of their binary character. The "reasoning in a circle" by Otto Struve from the conclusion of his father back to his hypothesis was used to show that beliefs regarding this matter are based sometimes more on tradition than on logic. And the Spittaler and Zona comets of 1893 were instanced (by the quoting of Professor Barnard in *Sidereal Messenger*, 10, 18) to show how facts sometimes overrule probabilities. Other arguments were also adduced for the purpose of removing many of the traditional beliefs now existing concerning these stars.

While facts are more convincing than probabilities, so also are measures as convincing as logic. It was interesting, therefore, to note that the measures made by Professor Wilsing at Potsdam as discussed by the writer likewise seem to confirm the results deduced from the Rutherfurd plates. This new discussion, given in *Contribution 13*, shows a difference of parallax:

$$(61^1 \text{ Cygni} - 61^2 \text{ Cygni}) = +0''.088 \pm 0''.012.$$

The mean of previous determinations of the parallax of each of these stars give also a difference of:

$$(61^1 \text{ Cygni} - 61^2 \text{ Cygni}) = +0''.082 \pm 0''.013.$$

While admitting, therefore, the great need of further observations, and not denying the possibility of Professor Wilsing's hypothesis that 61^1 Cygni may have a dark companion, it seems that the evidence at present available leaves little room for doubt as to the existence of the difference of parallax detected by the measures of the Rutherfurd plates. And since it is the well-accepted policy of scientific investigators to accept provisionally as true, until disproved (if such should be its ultimate fate), that which has the greatest direct evidence in its favor, we are called on to believe that the two stars of 61 Cygni are too remote from each other to form a binary system. But it is much to be desired that more evidence shall be accumulated to either verify or disprove this conclusion, and thereby throw more light upon the mystery of these interesting, because troublesome, stars.

HERMAN S. DAVIS.

THE PROBABLE RANGE OF TEMPERATURE ON THE MOON.

By Frank W. Very. (See p. 199.)

THE DOUBLE STAR WORK OF THE FLOWER OBSERVATORY, UNIVERSITY OF PENNSYLVANIA.

The object of this communication is to call attention to the performance of the 18-inch equatorial of this Observatory. While there are in this country a number of larger instruments by Alvan Clark & Sons this is the largest so far attempted by Brashear.

The atmospheric conditions probably differ little from those at other points east of the Allegheny Mountains. The diffraction rings are not often seen, in fact on only one occasion has an entirely satisfactory view of these been had. They were then clearly defined and

symmetrically arranged, showing that their customary absence is due to no fault of the glass.

The list of stars observed has been arranged by Mr. Burnham, and included a considerable number of difficult objects.

First will be mentioned very close pairs, not very unequal in magnitude, furnishing a test of the separating power; this, for an aperture of 18 inches, is 0".275 by Dawes' formula. A small list of stars of distance 0".5 and less follows. These have all been clearly separated and measured.

Star	α	δ	ρ	θ	Mags.	Remarks
β 232	0^h 43^m 38^s	49° 59′	0".30	326°	8–8	
A. C. 13	1 1 59	44 34	0 .35	250	8–8	
β 1017	6 6 28	— 2 56	0 .50	160	8–9	
O Σ 234	11 25 24	41 50	0 .30	140	7–8	
O Σ 269	13 28 18	35 46	0 .35	224	7.3–7.7	
β 939	14 7 48	— 7 58	0 .50	146	8–8	
β 1879	14 40 23	10 10	0 .50	145	8–9	
O Σ 285	14 41 42	42 48	0 .30	130	7.5–7.6	
η Cor. Bor.	15 19 6	30 39	0 .44	327	6–6	
Δ 15	16 40 12	43 42	0 .40	340	8–8	
β 814	16 23 9	40 9	0 .24	320	8 + – 8 +	
β 631	17 33 47	— 0 35	0 .40	60	7–7	
O Σ 337	17 44 46	7 16	0 .40	32	7–8	
β 671	20 29 34	62 3	0 .50	330	8–10	(Unequal.)
β 155	20 47 25	50 38	0 .50	37	7½–8	

Second—Close, faint companions to bright stars. These afford a test of the condition of the atmosphere rather than of the telescope. Usually the blurring of the bright star renders a critical test of the glass impossible.

Among other pairs of this class the following have been satisfactorily measured:

Star	α	δ	ρ	θ	Mags.	Remarks
O Σ 38	1^h 57^m 48^s	41° 51′	0".40	120°	5½–7	
Σ 1306	8 59 48	67 37	1 .3	210	5–8.2	
O Σ 235	11 26 42	61 38	0 .70	98	6–8	
Σ 1768	13 33 0	36 48	1 .1	133	5–8½	
β 616	14 27 15	38 50	28 .	100	4–12	
μ^2 Boötis	15 20 42	37 43	0 .85	80	6½–8	
β 1251	17 36 55	16 2	1 .2	70	5.8–10	
β 954	16 50 59	18 38	2 .5	178	5–12	
β 151	20 32 54	14 15	0 .7	359	4–6	
β 733	23 56 54	26 34	0 .7	223	6–10	

Third—Excessively faint stars. Among Burnham's measures with the 18½-inch Clark glass of the Dearborn Observatory are found a number of stars marked 16–14 magnitude. The note "excessively faint" accompanies most of these. In one case is a star whose magnitude is given as 14 which was not measured at all (β 639).

In case of all these pairs where the attempt has been made, including the last mentioned, it has been found practicable to obtain satisfactory measurements.

The measurements at the Flower Observatory were all made by Mr. Eric Doolittle, who has had no previous experience in this class of work. Yet he has been able to see and measure objects as faint as those which so experienced and skillful an observer as Mr. Burnham could reach with the slightly larger Dearborn glass.

No effort has been made to find new pairs, but a number have been picked up in connection with the regular work.

The following are believed to be new:

Star	α	δ	ρ	θ	Mags.	Remarks
1	0^h 4^m 33^s	57° 8′	1″	285°	9.8 – 10.5	Near K 1
2	6 10 6	– 4 52	4½	243	6 – 11	
3	6 50	56 42	4	328	9.5 – 10	
4	8 40	56 38	6½	120	9 – 11.5	$\overline{AB}$
			40	242	12	$\overline{AC}$
5	9 43	58 45	1.8	37	9 – 9.4	
Σ1775	13 37 18	– 3 40	39	178.5	13.5	$\overline{AC}$
β33	15 24 42	–12 35	32	132	11.5	$\overline{AE}$
8	21 23	59 34	7	115	9 – 9.5	
9	21 31	59 11	2	127	8.5 – 8.8	
10	21 33	58 22	10	162	9.5 – 10	
11	21 37	57 43	3	40	9 – 11.5	
12	22 26 30	55 49	3	330	10.8 – 11	
13	22 27 30	56 21	1.4	45	9½ – 10½	
14	22 28 30	56 31	3	235	9.8 – 11	

We expect soon to publish the observations made during the past eighteen months, embracing about 500 pairs.

C. L. Doolittle.

THE POSITION OF THE AREQUIPA STATION OF THE HARVARD COLLEGE OBSERVATORY.

During a temporary residence at Arequipa, through the kindness of Professor Pickering and of Professor and Mrs. Bailey, the latitude and longitude of the station were determined. The chief instrument avail-

able was the special transit instrument devised for use in the prime vertical by the late Professor W. A. Rogers and described at the meeting of the A. A. A. S. in 1892. This instrument could not be adapted to use as a zenith telescope, but proved a very excellent instrument in the prime vertical. Two of Bessel's methods were employed, and the result is

$$\phi = -16^{\circ}\ 22'\ 28.''0 \pm 0''.19.$$

The longitude was obtained by telegraph with a temporary station at Arica, Chile, whose longitude had been determined by officers of the United States Navy in 1883. Their determination of the longitude of Arica was part of an extensive series of longitudes on the west coast of South America through the coast cables. Considerable difficulty was found in getting telegraphic connection between Arequipa and Arica, though there is a land line between the places, which, however, is never used for through messages. It became necessary to use the land line to the seaport Mollendo and to repeat the signal by cable between Mollendo and Arica. Communication was made in this way on seven nights, and proper precautions taken to insure the elimination of the errors of transmission. The result is

$$\Delta\lambda,\ \text{Arequipa — Arica},\ + 4^{m}\ 51^{s}.81 \pm 0^{s}.032.$$

Assuming the longitude of Arica to be $4^{h}\ 41^{m}\ 19^{s}.90$ the longitude is

$$4^{h}\ 46^{m}\ 11^{s}.71 \text{ W. of Greenwich.}$$

WINSLOW UPTON.

RECENT BOLOGRAPHIC RESULTS FROM THE ASTROPHYSICAL OBSERVATORY AT WASHINGTON.

In his absence Secretary Langley authorizes me to state, on behalf of the Astrophysical Observatory at Washington, something with reference to the recent results derived at that Observatory in the bolographic investigation of the infra-red solar spectrum.

By the use of the bolographic method devised by Dr. Langley and developed at this Observatory there have now been found over 700 absorption lines in the solar spectrum between wave-lengths $0^{\mu}.76$ and $0^{\mu}.0$. Of these more than two-thirds have been added in the last eight months, owing to the use of several new pieces of apparatus lately obtained.

It will be recalled that this method essentially consists in the auto-

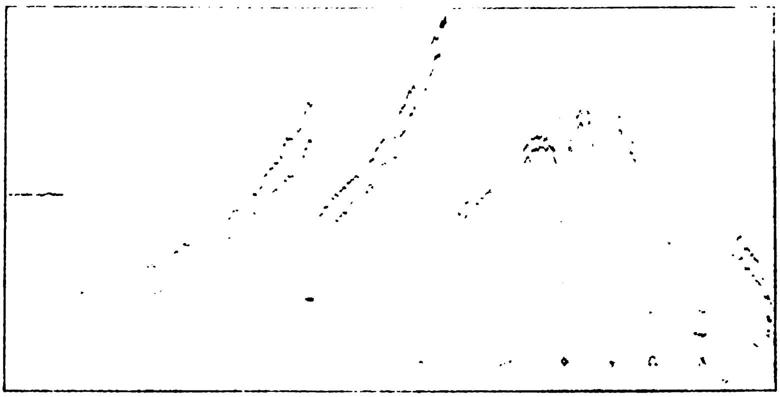
matic production of an energy curve, in which abscissæ correspond to prismatic deviations and ordinates to energy of radiation in the spectrum. Indentations in this curve indicate diminution of energy due to absorption, either from the Sun's gaseous envelope or our atmosphere, and thus give the position of Fraunhofer lines.

It will be readily understood that attention must be paid to exclude errors from the introduction of deflections due to tremor of the ground, fluctuations of the electric current, and to other causes. These are in practice reduced to the smallest possible magnitudes, and such as remain (being variable in place and form) are eliminated from the final results through a comparison and measurement of six or more plates. Such a reduction as this is very arduous, and, in the case of the present results, has entailed about 44,000 separate comparator observations. The accuracy of the mechanical devices by which the abscissæ of the bolographs are governed is extreme, and it is largely due to this that the final values of deviation are not usually in error (in so far as dependent on the bolographic processes) by as much as a second of arc.

The abscissæ of the bolographs, giving as they do the accurate position of the absorption lines, are naturally of the first importance in the present work; but the ordinates, corresponding to intensity of radiant energy, are by no means lost sight of. Indeed, at several regions in the infra-red spectrum there have lately been noticed very marked changes in the absorption. It appears, for instance, that for three months in the year, which are (fortuitously or otherwise) the months in which foliage is put forth, there is a very noticeable decrease in the absorption on the long wave-length side of the band at about $1^{\mu}.4$ called here ψ. Paschen has found this band to be caused by water vapor. If the absorption in the portion affected was due to carbonic acid gas perhaps the phenomenon might be accounted for by the vegetable growth as above hinted.

To illustrate the general form of the energy curve, as well as the agreement between successive bolographs, two superposed curves extending from the F line in the green to the great absorption band Y at about $4^{\mu}.2$ are presented in the accompanying plate. These curves are given on a background of plotting paper whose intervals (reduced in the same ratio as the curves) were originally centimeters, and correspond each to $4'$ of arc in deviation. At one point between F and D one curve was intentionally shifted in the making, as it was

feared to be too near the edge of the photographic plate. The scale of ordinates was altered at three points by changing the slit width. From F to near C it is 1.00; from this point to below A it is 0.50; from



this point to X it is 0.20, and from this point to the close, 1.00. Such curves as these show in the original some 200 "real" deflections. In detail work the scale of deviations is spread out four times, the plate is moved much slower, and as a consequence, many small deflections here rushed over and obliterated are brought out.

C. G. ABBOT.

THE DISTORTION OF PHOTOGRAPHS MADE WITH THE BRUCE 24-INCH PHOTOGRAPHIC TELESCOPE.

A letter was read from Professor H. H. Turner, University Observatory, Oxford, England, giving the provisional results of measurements of a plate taken with the Bruce 24-inch photographic telescope in order to determine the distortion, if any. His conclusions are:

"First—The optical distortion on the plate is quite small, if existent, up to 3° from the center, and perhaps further; that is, the star images are truly projected on a tangent plane with considerable exactness. I have not yet detected any *radial* displacement, though there seems to be a small term depending on the product xy indicating a sort of *strain* of the field.

"Second—Whatever may be the ultimate definite result as regards optical distortion, your plates will give (by due allowance for such distortion which will be quite easy) as good places as we are getting on

the plates for the Astrophotographic Catalogue. The images are perhaps a little larger and more diffused, but are quite easy to measure. This holds for certainly $4° \times 4°$ and probably further.

"Third — If I am right in believing that fear of optical distortion was the motive for rejecting the photographic doublet, the above conclusions show this fear to have been groundless, and the manifest advantages of getting a large field at once prove the doublet to be the right instrument — certainly for charting purposes."

ON A PRACTICAL METHOD OF PHOTOGRAPHING THE SPECTRUM OF THE CORONA IN NUMEROUS DISTINCT REGIONS.

RECENT research on the corona indicates that it is a composite of a triple system of rays or streamers. But it is not known whether the filaments of these systems have the same origin or not. Perhaps the only conclusive method of testing this important question is that of a multiple spectroscope, in which slits are arranged tangentially at both extremities of equatorial and polar diameters; also radially, or approximately so, along the lines of expected streamers. It is proposed to bring the spectra of at least four separate inner coronal regions on a single plate, and of four outer areas on another plate. All on a given plate will therefore have the same exposure and development, and will be approximately comparable. A two-inch image with large aperture will be requisite; and the varying lengths of exposure, together with a maximum number of plates, are easily secured by means of the automatic apparatus of the Japan expedition of 1896.

DAVID P. TODD.

ON AN EFFECTIVE INSULATION OF MERCURIAL HORIZONS.

In regions of frequent and violent seismic disturbance, like Japan, it is well known that shocks which are very destructive at lower levels on the plains, may often fail to be felt in the mountains surrounding them. The reason for this is not difficult to see: as the origin of the earthquake wave is submarine, the vibrations, which readily propagate themselves through the relatively small masses of earth in the plain, become dissipated by inertia of the vaster masses of outlying mountains. Application of this principle to the insulation of a support for a mercurial horizon has led to complete success in checking the tiny undulations of its surface which destroy its capacity for specular reflection. First a zenith collimator of the ordinary type was constructed, in

which the reflected reticle was perfectly visible in a collimating eyepiece. Then several slabs of marble or granite, about three inches thick and a yard square were provided; the number requisite will vary with the degree of disturbance to be overcome, and the perfection of insulation sought. Practically, five or six will be sufficient. They are laid up horizontally, as in strata, but separated from each other by wave-checking material, inserted at three points between each two slabs. Practically, tufts of any material suitable for the ordinary mattress serve this purpose well; but I have obtained the best results by the use of cork-dust in small, tightly-stuffed bags. The top slab of the series is made three or four times the thickness of the intermediate ones, and surmounted on this is the zenith collimator. Of course the slightest touch of the finger to the topmost slab will cause the reflected reticle to disappear; but a nadir observation is wholly secure against interruption by passing freight trains and trolley cars, although in close proximity.

DAVID P. TODD.

ON THE FOUNDING OF ASTRONOMICAL AND OTHER INSTRUMENTS OF PRECISION.

All astronomical and physical instruments work best when provided with (1) an absolutely stable foundation, and (2) a perfectly rigid mounting. These ideal conditions can only be reached approximately. In my experience the first is more often secured than the second. The object of this paper is to show how rigidity of mounting may be secured equally with stability of foundation. This means, of course, a welding or union of foundation and mounting into one. If it is convenient or practicable to build the mounting very massive, gravity alone may make this union sufficiently effective, provided ordinary care is exercised in making the junction at three points only. But in many classes of instruments, massiveness of mounting is wholly impracticable. Eclipse instruments especially belong to this category; they must be light in order to be easy of transportation, but rigidity is equally essential for perfect definition of photographs of the solar corona. As a result of several years' study of this problem, in connection with preparation for several expeditions, I have finally reached a satisfactory construction for effecting a union of mounting and foundation, which appears applicable to instruments in observatories as well. Simply stated, it is a method by which the mounting is enabled to secure a

permanent grasp upon many tons of earth underneath it. First, three holes were dug, as if for setting the posts of a fence; their distance apart is regulated by the area desirable to include as the base of the instrument. Into these holes were set three wrought-iron pipes, about five feet long and three inches in diameter; perhaps two feet of each pipe being left projecting above the surface of the ground. To the bottom of each pipe was screwed a projecting flange, and the hole around both pipe and flange was filled about a foot deep with cement and gravel, thoroughly rammed into position and allowed to harden. The remainder of the holes were then filled as was most convenient. The second point of construction was met by placing three stout timbers or pipes horizontally between the vertical pillars. These were forced into position close to the ground, and spread the pier pipes a trifle outward. Through holes bored near the top of the pier pipes, long rods or bolts were then run, parallel to the timbers below them, thus forming the third essential of the mechanical construction. By tightening up these bolts in the ordinary way, the tendency was to spread the cement bases of the piers outward. But these would yield only slightly, and any degree of tension could be secured by means of the bolts. All these features are shown in the accompanying plate. This highly portable form of mounting was devised and first employed for three mountings for the photographic instruments of the Amherst Eclipse Expedition of 1896, equipped by the liberality of Mr. D. Willis James, of New York, and sent out to Japan in his yacht "Coronet." Such a mounting admits of the most rapid installation, and furnishes a degree of rigidity unusual in the ordinary types.

DAVID P. TODD.

RESULTS OF HELIOMETER OBSERVATIONS ON THE REFRACTION OF RED STARS.

By F. L. Chase.

SWIFT'S COMET (I, 1892).

Professor William H. Pickering presented a paper (not read) on "Swift's Comet, I, 1892." This comet, discovered March 6, was first noticed to have a faint tail, by Finlay, on March 26. As discovered at Arequipa March 29, the tail was well developed. Forty-four photographs were taken of this and used in a recent investigation of its nature. The tail was plainly twofold in character, the inner tail con-

tinuously issuing from the head, and, being at least twenty degrees in length a month after passing perihelion, consisted of two absolutely straight rays inclined at an angle of ten degrees. The outer tail was formed of successive eruptions from the head, later diminishing in angle. The whole effect was to produce multiple tails like the comet of 1744. The photographs showed the inner tail to be of a different character on alternate days, now appearing continuous near the head, and now bifurcated. The explanation is that the comet rotated about a longitudinal axis directed toward the Sun, the rotation period determined being 94.4 hours. This period was determined by the use of the photographs of Professors Barnard and Wolf, and those made at Arequipa by the writer. Bessel observed in Halley's comet, in 1835, "a vibration from side to side," with a period of 110 hours; this was probably a similar rotation of the comet with an analogous period. The changing appearance of both comets being explicable by the hypothesis of rotation, the next endeavor was to find a physical explation of the cause of such rotation. The Sun furnishes a strong electromagnetic field, the comet also, by the electrical induction of the Sun, receives electrical charges, and these charges are, by the gases of the tail, radially projected from the head of the comet, and become in effect so many electrical currents. The result of the motion of these radiating currents, *i. e.*, tails in the magnetic field of the Sun, is the production of a spinning of the comet about a longitudinal axis. Theory would thus produce the rotation demanded by observation, and the rotation would be independent of the direction of the motion of the comet, but north of the solar equator the rotational force is in one direction, and south of it in the other.

It is generally impossible to indicate a point in the tail of the comet; the exception occurs in the plates of April 6, 7, 8, and 10, where a bright detailed structure is receding with increasing velocity of from one million to 25 million kilometers per day. The velocity of recession is thirty-six times the acceleration of gravity, and does not agree with any of the velocities required by Bredichin's hypothesis. The spectrum as given by the objective prism, of the head and the tail, showed five bands probably due to hydrocarbons. A further test of the theory of axial rotation of comets near the Sun would be an observed change of direction or speed of rotation in the solar magnetic field. Another would be the observation of the rotation when the tail is directed approximately toward the observer.

VARIABLE STARS IN CLUSTERS.[1]

THE discovery, by Professor S. I. Bailey, of the presence of large numbers of variable stars in certain globular clusters has already been announced in Circulars Nos. 2 and 18. A systematic search for such variable stars has been made by Professor Bailey and has led to the results given below. In the following table the first column gives the number in the *New General Catalogue* of Dreyer; the second, the Messier number, or, in certain cases, some other designation; the third and fourth, the position for 1900; the fifth column, the approximate number of stars examined; the sixth, the number of plates used in the examination. In the case of certain clusters later investigations have included more plates. The seventh column gives the area, always a circle having for its center the center of the cluster, included in the examination. The eighth column gives the number of variable stars found; the ninth, the maximum distance of any variable from the center of the cluster. In *N. G. C.* 7078 this distance is given as 7′, since a circle having a radius of 7′ includes 50 variables. One variable, perhaps not related to the others, is found at a distance of 10′. The tenth column gives the area of a circle of sufficient radius to include all the variables. The eleventh column gives the proportion of variables to the whole number of stars examined; the twelfth, the number of stars to one variable.

As will be seen by reference to the table, the whole number of stars examined is 19,050, of which 509 are variable. This amounts to one variable in 37 stars, or nearly 3 per cent. It does not follow, however, that clusters in general contain more variable stars than occur elsewhere, for, if we except the four clusters, ω Centauri, Messier 3, Messier 5, and Messier 15, which together contain 393 variables, an average of 7 per cent., the remaining 19 clusters have 116 variables among 13,350 stars, or less than 1 per cent. There is a very striking difference between the results obtained in clusters equally rich in stars, as, for example, between Messier 13, the great cluster in Hercules, where an examination of 1000 stars shows two variables, one in 500, and and Messier 3, where among 900 stars, 132 are variable, one in 7. A common plane of revolution, orbital or axial, of the different systems or individuals of star clusters, and the relation of that plane to the line of sight, might provisionally be suggested as a possible explanation.

[1] *Harvard College Observatory Circular* No. 33.

VARIABLE STARS IN CLUSTERS.

Designation		Position, 1900 R. A.	Position, 1900 Dec.	No. stars examined	No. plates examined	Area examined	No. variables found	Maximum distance	Area of variables	Proportion of variables	No. of stars to one var.
		h. m.	° ′			Sq. Min.			Sq. Min.		
104	47 Tucanae	0 19.6	−72 38	2,000	10	1,257	6	6	113	.003	333
362		0 58.9	−71 23	675	10	314	14	9	254	.021	48
869		2 12.0	+56 41	1,050	10	10,800	1	—	—	.001	1050
884		2 15.4	+56 39								
1904	Messier 79	5 20.1	−24 37	200	6	79	5	3.5	38	.025	40
3293		10 29.6	−57 40	724	10	314	0	—	—	.000	—
4755	κ Crucis	12 47.7	−59 48	555	10	314	0	—	—	.000	—
5139	ω Centauri	13 20.8	−46 47	3,000	10	1,257	125	22	1521	.042	24
5272	Messier 3	13 37.6	+28 53	900	10	1,257	132	17	908	.147	7
5904	Messier 5	15 13.5	+ 2 27	900	10	1,257	85	17	908	.094	11
5986		15 39.5	−37 26	289	12	314	1	1.7	—	.003	289
6093	Messier 80	16 11.1	−22 44	145	5	79	2	2.5	20	.014	72
6205	Messier 13	16 38.1	+36 39	1,000	10	177	2	1.3	5	.002	500
6266	Messier 62	16 54.9	−29 58	960	10	218	26	8	201	.027	37
6397		17 32.5	−53 37	487	5	218	2	7.5	177	.004	244
6626	Messier 28	18 18.4	−24 55	900	9	314	9	7	154	.010	100
6656	Messier 22	18 30.3	−23 59	1,550	10	218	16	8	201	.010	97
6723		18 52.8	−36 46	900	6	314	16	4.5	64	.018	56
6752		19 2.0	−60 8	600	8	218	1	4	—	.002	600
6809	Messier 55	19 33.7	−31 10	440	6	218	2	5	79	.005	220
7078	Messier 15	21 25.2	+11 44	900	10	1,257	51	7	154	.057	18
7089	Messier 2	21 28.3	− 1 16	600	10	218	10	4	50	.017	60
7099	Messier 30	21 34.7	−23 38	275	10	218	3	2.5	20	.011	92
				19,050		20,830	509		4,867		

The periods and light curves of several variables in other clusters have been determined, but the study of those in ω Centauri is well advanced. This cluster may be called the finest in the sky. It lies just within the border of the Milky Way. There are no bright stars near. To the naked eye it appears as a hazy star of the fourth magni tude. It has a diameter of about 40′. The brightest individual stars in this region are between the eighth and ninth magnitude. Over 6000 stars have been counted on one of the photographs, and the whole number is much greater. Only about 3000, however, are sufficiently bright and well separated to serve for comparison in the discovery of variables. Of these 3000, 125 are variable. 150 photographs of the cluster have been taken with the 13-inch telescope, and already 10,000 measures have been obtained, about half of which have been made by Miss E. F. Leland.

Although the results are at present provisional, it is not probable that the final results of the discussion will materially alter the conclusions. Of the 106 variables in ω Centauri whose periods have been determined, 98 have periods less than 24^h. The longest period is that of No. 2, 475^d, the shortest that of No. 91, $6^h\ 11^m$. Three have periods less than 7^h. Of the eight having periods of more than 24^h, two have periods between one and two days, two between two and three days, one of 4 days, one of 15 days, one of 150 days, and one of 475 days.

The largest range in variation is about five magnitudes, and no star has been included whose light changes do not amount to half a magnitude.

The light curves of the 98 stars whose periods are less than 24 hours may be divided into four classes. The first is well represented by No. 74. The period of this star is $12^h\ 4^m.3$, and the range in brightness, two magnitudes. Probably the change in brightness is continuous. The increase of light is very rapid, occupying not more than one-fifth of the whole period. In some cases, possibly in this star, the light remains constant for a short time at minimum. In most cases, however, the change in brightness seems to be continuous. The simple type shown by No. 74 is more prevalent in this cluster than any other. There are, nevertheless, several stars, as No. 7, where there is a more or less well marked secondary maximum. The period of this star is $2^d\ 11^h\ 51^m$, and the range in brightness, one and a half magnitudes. The light curve is similar to that of well-known short-period variables,

as δ Cephei and η Aquilae. Another class may be represented by No. 126, in which the range is less than a magnitude, and the times of increase and decrease are about equal. The period is $8^h\ 12^m.3$. No. 24 may perhaps be referred to as a fourth type. The range is about seven-tenths of a magnitude, and the period is $11^h\ 5^m.7$. Apparently about 65 per cent. of the whole period is occupied by the increase of the light. This very slow rate of increase is especially striking from the fact that in many cases in this cluster the increase is extremely rapid, probably not more than 10 per cent. of the whole period. In one case, No. 45, having a period of $14^h\ 8^m$, the rise from minimum to maximum, a change of two magnitudes, takes place in about one hour, and in certain cases, chiefly owing to the necessary duration of a photographic exposure, there is no proof at present that the rise is not much more rapid.

The marked regularity in the period of these stars is worthy of attention. Several have been studied during more than a thousand, and one during more than five thousand periods, without irregularities manifesting themselves.

A few words may be added in regard to the kind of clusters in which variables have been found. Up to the present time they have not been found in any except dense globular clusters, of which Messier 3, Messier 5, and the great cluster in Hercules may be taken as examples. The number of such clusters within the reach of ordinary instruments is not great. Of the clusters given in the table, *N. G. C.* 104, 362, 5139, 5272, 5904, 6093, 6205, 6266, 6626, 7078, and 7089 may be described as highly condensed; 1904, 5986, 6397, 6656, 6723, 6752, 6809 and 7099 as moderately condensed; and 3293 and 4755 as open clusters. 869 and 884, the clusters in the Sword-handle of Perseus, are little more than regions relatively rich in stars.

The first group of 11 highly condensed clusters having a total of 11,980 stars has 462 variables, or one in 26. The second group of 8 moderately condensed clusters has 46 variables among 4741 stars, one in 103. The two open clusters furnished no variables, and the region of three square degrees around *N. G. C.* 869 and 884, only one.

Thus far the only regions which are found to be especially rich in variable stars are condensed clusters, but even here only in relatively few cases. These dense clusters are commonly called globular, and many of them are such. In some cases, however, as ω Centauri, the form is somewhat elliptical.

N. G. C. 6266 is the most striking example of a highly condensed cluster which is irregular in form. This irregularity is intensified in the distribution of the variables. The cluster is much compressed on the south side. For a distance of 1′ from the center the distribution of the stars seems to be about equal, but if a line be drawn east and west through the center, and the stars within 1′ of this line are omitted, there are 214 stars south and 354 stars north, within 4′ of the central line. In this cluster are 26 variables, of which 19 are north of the central line and 7 south. Excluding those within 70″, there are 15 north and only 1 south.

EDWARD C. PICKERING.

September 17, 1898.

WITT'S PLANET D Q.[1]

ONE of the most important astronomical discoveries of recent years is the finding of the planet D Q, by Witt. This object will come nearer to the Earth than any other similar object, except the Moon. Its minimum distance is about 15 million miles, and the corresponding parallax nearly a minute of arc. To determine the photometric brightness of this object, the stars, —6°5567, —6°5579, —6°5600, —6°5608, and —6°5626 were each measured on five nights with the meridian photometer, eight settings being made each night, with the resulting magnitudes, 7.87, 8.65, 8.71, 8.56, and 9.57, respectively. The probable error of these values varies from ±0.020 to ±0.033. The brightness of the stars, —6°5550, —6°5560, and —6°5588 was later determined differentially from these, with the resulting magnitudes 8.80, 9.89, and 8.37. Mr. Wendell compared the planet on six nights by means of the photometer with achromatic prisms attached to the 15-inch equatorial. The number of settings on each night was 48. On September 5 and 6, 1898, it was compared with —6°5626, on September 9 with —6°5608, on September 12 with —6°5579, and on September 13 and 14 with —6°5588. The resulting magnitudes were 12.19, 11.97, 12.10, 12.01, 12.20, 12.29. Mean, 12.13 ±0.04. As the logarithms of the distances of the Sun and Earth, on the mean date, September 10, were 0.2366 and 9.9115, the magnitude when these distances are both unity becomes 11.39.

[1] *Harvard College Observatory Circular* No. 34.

It is not easy to obtain photographs adapted to determining the photographic brightness of this object, owing to its faintness and rapid motion. With a short exposure the image is very faint, and with a long exposure a trail is obtained which cannot be compared with the circular images of adjacent stars. Measures of the photographic brightness have been made by Mrs. Fleming on plates taken with the 8-inch Draper telescope, and having exposures of about 15 minutes. The planet was not far distant from the variable star T Aquarii and was compared with four of its comparison stars whose photometric magnitudes had already been determined. Plates taken on September 6, 12, 13, 13, 13, 13, 14, 17, 20, and 21, 1898, gave the magnitudes 12.78, 12.75, 12.80, 12.85, 12.78, 12.75, 12.75, 12.72, 12.65, and 12.68. Mean, 12.75 $\pm$0.01. Similar measures of two isochromatic plates taken on September 17 and 20 gave the magnitudes 12.80 and 12.61. Mean 12.70, $\pm$0.08. Since the planet is fainter photographically than photometrically, it is probable that its color, like that of the Sun, is redder than an average star.

Several interesting photometric problems present themselves in connection with this object. First, the approximate diameter may be determined by comparison with the brighter asteroids and satellites, assuming that the reflecting power is the same. Secondly, the great variation in the distance of this object from the Earth will afford an excellent test of the law that the light varies inversely as the square of the distance. The existence of an absorbing medium in the solar system will thus be tested. Thirdly, owing to the proximity of this object to the Earth at opposition, its phase angle will vary by a large amount. It will, therefore, afford an excellent test of the law connecting this angle with the variation in brightness, which has been found by two or three observers independently.

NEBULA IN ANDROMEDA.

A comparison of photographs of the Nebula in Andromeda taken with the 8 inch and 11-inch Draper telescopes on September 20 and 21, 1898, with similar photographs taken in 1893, 1894, 1895, and 1896, fails to show the new stellated appearance recently announced by Seraphimoff of Pulkowa. See also *A. N.*, 147, 223.

EDWARD C. PICKERING.

September 30, 1898.

To the Editors of the ASTROPHYSICAL JOURNAL:

GENTLEMEN: In the ASTROPHYSICAL JOURNAL, Vol. VIII, p. 102, there is an article by Mr. Mitchell on the "Concave Grating," in which I am named as an investigator on the theory of this beautiful apparatus. I beg you to state that I have no merits in this direction. In the *Handbuch der Physik*, I have only published an abstract of some parts of a very complete investigation on the concave grating written by Professor C. Runge, but never published in full. In the *Handbuch* I of course stated that the theory there given is due to Professor Runge and not to myself.

I am, gentlemen,

Yours truly,

H. KAYSER.

BONN, September 25, 1898.

NOTICE.

The scope of the ASTROPHYSICAL JOURNAL includes all investigations of radiant energy, whether conducted in the observatory or in the laboratory The subjects to which special attention will be given are photographic and visual observations of the heavenly bodies (other than those pertaining to "astronomy of position"); spectroscopic, photometric, bolometric and radiometric work of all kinds; descriptions of instruments and apparatus used in such investigations; and theoretical papers bearing on any of these subjects.

In the department of *Minor Contributions and Notes* subjects may be discussed which belong to other closely related fields of investigation.

Articles written in any language will be accepted for publication, but unless a wish to the contrary is expressed by the author, they will be translated into English. Tables of wave-lengths will be printed with the short wave-lengths at the top, and maps of spectra with the red end on the right, unless the author requests that the reverse procedure be followed. If a request is sent *with the manuscript* one hundred reprint copies of each paper, bound in covers, will be furnished free of charge to the author. Additional copies may be obtained at cost price. No reprints can be sent unless a request for them is received before the JOURNAL goes to press.

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THE

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THE PROBABLE RANGE OF TEMPERATURE ON THE MOON. II.

By FRANK W. VERY.

THE opinions which have been hazarded concerning the lunar surface temperature cover an extraordinary range. We find suggestions of something approaching a red-hot state in the middle of the lunar day, or to quote Sir John Herschel's most conservative statement, we learn that "the surface of the full Moon exposed to us must necessarily be very much heated,—*possibly* to a degree much exceeding that of boiling water;" and on the other hand, Ericsson concluded in favor of "perpetual intense cold," and a temperature about 80° F. above absolute zero under full sunshine.

Something of this uncertainty still lingers in even our best text-books. There are few works on astronomy more thoroughly reliable than Professor Young's; but his statements on this subject, while holding a middle ground, are still unsatisfactory; and his conclusion that "it now seems rather more probable that the temperature never rises above the freezing point of water," because measures in the infra-red spectrum of the Moon show "the presence in the lunar radiations of a considerable quantity of (radiant) heat which has a wave-length greater

than that radiated from a block of ice, and is, therefore, probably radiated from something colder than ice," by no means follows, since the point of maximum radiation in the spectrum has been displaced by atmospheric absorption. By choosing radiations on which our atmosphere exercises little absorption, and using only such in the comparison, it can be shown that the effective radiating power of the sunlit Moon is certainly much above that of ice; and any reasonable restoration of the spectral energy curve of the Moon, with allowance for atmospheric absorption, will lead to the same result.

Before proceeding to other evidence in this line, I will describe an experiment designed as a test of the lunar glaciation theories of Ericsson and Peal.

With apparatus arranged for thermal measures in the lunar image, a concave mirror receiving rays from a siderostat and concentrating them on the bolometer, I replaced the siderostat mirror by a surface of snow, and in full sunshine with a clear sky, between 1^h and 2^h P.M., exposed the bolometer to the radiation of sunlit snow by withdrawing a black copper screen placed between the snow and the concave mirror. The angular aperture of the bolometer being but a small part of that of the Moon, the observation amounts to one in the image of a virtual snow Moon, since it is a matter of indifference whether the reflecting surface is inside or outside the Earth's atmosphere, the absorption being the same in either case.

The progressive increase of glass transmission is probably due to increasing depth of water from melted snow, accumulating in the interstices, and coating the ice-particles, thus depleting the solar rays passing through the water of those longer waves not transmitted by glass, in consequence of which the reflected sunbeam, more deficient in these long waves than at the outset, is more readily transmitted by glass.

The surface of the snow was then sprinkled with lampblack, but the wind blew so much of the light carbon away that the resulting surface was not dead black, but gray. The result was a diminution of the reflection from an intensity of 95 to 29; but

the transmission by glass (0.803) remained scarcely changed from the previous final measurement.

TABLE VI.

Temperature of exposing screen, ... $= +5^\circ.3$C.
Temperature of room, ... $= +5^\circ.1$
Outside temperature, ... $= -7^\circ.0$
Snow, at first dry, soon melting in the sunshine.

Radiation from pure white snow, in sunshine	Same through glass	Transmission by glass
div.	div.	
$\left.\begin{matrix}84\\98\end{matrix}\right\}$ 91.0		
	$\left.\begin{matrix}75\\63.5\end{matrix}\right\}$ 69.3	$\frac{69.3}{95.3} = 0.727$
$\left.\begin{matrix}99\\100\end{matrix}\right\}$ 99.5	} 72.3	$\frac{72.3}{99.5} = 0.727$
	$\left.\begin{matrix}75\\75.5\end{matrix}\right\}$ 75.3	$\frac{75.3}{97.8} = 0.770$
$\left.\begin{matrix}96\\96\end{matrix}\right\}$ 96.0	} 76.8	$\frac{76.8}{96.0} = 0.800$
	$\left.\begin{matrix}79.5\\77\end{matrix}\right\}$ 78.3	$\frac{78.3}{94.3} = 0.830$
$\left.\begin{matrix}94\\91\end{matrix}\right\}$ 92.5	} 77.8	$\frac{77.8}{92.5} = 0.841$
	$\left.\begin{matrix}76\\78.5\end{matrix}\right\}$ 77.3	$\frac{77.3}{93.8} = 0.824$
$\left.\begin{matrix}93.5\\96.5\end{matrix}\right\}$ 95.0	} 76.6	$\frac{76.6}{95.0} = 0.806$
	$\left.\begin{matrix}76.5\\75\end{matrix}\right\}$ 75.8	Mean. - 0.791

A concluding measure on fresh white snow in shade gave a negative reading of $-5^{\text{div}}.3$.

It appears from the first series (Table VI) that the solar radiation, reflected from white snow, gives a deflection of about 100 divisions, which is very nearly the same as that produced

with this apparatus by the hottest parts of the Moon when compared with the neighboring sky. Instead, however, of transmitting only 12 to 17 per cent. of this radiation, the same glass which allows this fraction of the lunar rays to pass, transmits from 73 to 84 per cent. of the reflection from white snow.

This experiment alone throws considerable doubt on Mr. Peal's supposition of a lunar surface largely composed of ice and snow, in which circular Maria and walled plains have been melted out by local sources of heat. But it is urged by Mr. Peal that the ancient lunar snows have become darkened by meteoric dust. The second part of the experiment answers the question whether a deposit of some sort of dust could so change the reflecting power of snow as to enable the radiation from it to approach in quality that from the Moon. The answer is that when snow is blackened until its reflective power is less than 30 per cent of that of pure white snow, the reflected solar beam still has identically the same transmission by glass (about 80 per cent.) in either case. We might imagine the meteoric dust to have peculiar reflecting power, but unless the layer were of considerable depth (which I understand is not Mr. Peal's supposition), the character of the radiation could hardly suffer so extraordinary a change from a mere surface deposit of dust, and further facts, yet to be adduced, are conclusive against a day temperature on the Moon consistent with glacial formations.

Ericsson's estimate of a lunar day temperature about 45° C. above absolute zero, rests upon his claim "that the previous temperature of a body exposed to the Sun's radiant heat is immaterial," and that the excess of temperature imparted by the Sun's rays will be the same for a body at the absolute zero as for one already several hundred degrees hotter. (See *Nature*, **34**, 250, 1886.) As no figures are given to support this assertion, it may be said in contradiction that experimental results decide otherwise. For example, Violle obtained constantly diminishing excesses of his Sun thermometer, when the temperature of the enclosing water jacket of his actinometer was raised.

Temperature of water jacket	Excess of Sun thermometer
99°.35	10°.75
107 .4	9 .55
116 .15	9. 25
136 .5	8. 22

The failure to recognize the distinction between radiant power and temperature is responsible for many of the misconceptions connected with this subject. Ericsson would have said that a body at 400° radiates twice as much as one at 200°. Actually these relations are sustained by bodies at 400° and 323° absolute temperature.

I now pass to the measures of lunar radiation contained in my essay on the *Distribution of the Moon's Heat*, published by the Utrecht Society of Arts and Sciences. I have several times been asked by meteorologists and astronomers to give an interpretation of these measures, translating the arbitrary values into thermometric degrees. The desirability of this translation is evident, and the preceding studies of the first part were undertaken in order to approach the subject from the experimental side.

The most modern measures indicate values for the absolute constants of radiation at the freezing point about half as great as those which Pouillet surmised from the experiments of Dulong and Petit. The curve of absolute radiation of lampblack on metal, adopted in this research, rests partly on the measurements of Dr. J. T. Bottomley on the absolute emissivity of sooted copper, and partly on astrophysical considerations which will be described at length in a special paper. The following table gives the adopted values of absolute radiation from the standard surface, in C. G. S. units (see p. 271), for temperatures on the absolute Centigrade scale, derived from this curve.

With the help of this table I can state that—the concave mirror of 721.066$^{\text{sq cm}}$ area occupying $\frac{1}{861}$ of the hemisphere, and the bolometer of some 19$^{\text{sq mm}}$ area, slightly cut down by the

diaphragm at the corners to a working aperture of $0.178^{sq\ cm}$, filling 0.000000288 of a hemisphere at the mirror's focal distance—a deflection of one standard millimeter division of the galvanometer in my measures, represents an actual energy received by the bolometer, corresponding in thermal measure to 0.0000000396 small calories per second (about four one-hundred millionths of a small calory per second). But there is no need of using this constant, except for comparisons of instrumental sensitiveness, and it will be more instructive to state the results of the measurements of lunar surface radiation in terms of the radiation from unit surface of sooted copper to a hemisphere.

TABLE B.

Temperature	Radiation	Temperature	Radiation	Temperature	Radiation
25°	0.0002	200°	0.0033	375°	0.0204
50	0.0004	225	0.0044	400	0.0251
75	0.0006	250	0.0058	425	0.0307
100	0.0009	275	0.0076	450	0.0373
125	0.0013	300	0.0099	475	0.0452
150	0.0018	325	0.0129	500	0.0544
175	0.0024	350	0.0164	525	0.0650

The assumption has been made that radiation from unit angular area of a plane surface is equal in all directions, which is not quite true, especially at low angles, and for polished surfaces of particular substances, but for lampblack the assumed equality is nearly fulfilled.

Much vagueness, and no little misconception exists in writings on radiation through the lack of a consistent nomenclature. Dr. J. T. Bottomley has been very explicit in his description of radiant quantities, and in view of the prevalent looseness of expression, it may be well, in order to fix ideas, to speak, as he does, of radiation as measured in *gram-water-degree-Centigrade units of heat lost per square centimeter per second per degree Centigrade of excess of temperature ;* but this mode of speech, though precise, is exceedingly cumbersome, and indeed intolerable, if it is to be often repeated; and I have therefore ventured to introduce a

new term: ***radim***, from ***radius***, a ray, and the root *im* = image or picture, a unit of radiation, representing a unit quantity of heat, namely one gram-water-degree-Centigrade heat-unit, lost ***as radiation*** per square centimeter of surface per second of time, by a heated body, or transmitted by the ether as an equivalent amount of radiant energy through a normal section of 1 sq. cm. in 1 sec. of time. The term is to be distinguished from the unit of absolute emissivity which includes: (*a*) the quantity of heat lost by radiation from a heated body, measured as ***radiant emissivity*** in radims per degree of difference between its temperature and that of the walls of the enclosure; as well as (*b*) the further quantity of heat transferred by convection and penetration to the surrounding fluid, and eventually imparted to the solid enclosing walls by conduction. This is the ***convective emissivity*** to be measured either in heat-units lost per sq. cm. per sec. per deg. Centigrade excess, or, if desired, in equivalent radims per deg. Centigrade of excess. It ought, however, to be clearly distinguished from component *a*, since it continues to exist, during the state of transference, as molecular thermal motion, and is propagated at a vastly slower speed, while it follows altogether different laws. I trust that the use of this new term may be pardoned in view of its saving of words.

In reducing my lunar observations, temperatures will be stated in absolute Centigrade degrees, and the corresponding radiations to absolute zero in radims. The value of one division of the galvanometer in radims, at the time of these observations, is obtained from measurements on blackened screens. For example, on January 12, 1889, screens at $+52^\circ.0$ C. $= 325^\circ.0$ Absol. and $+9^\circ.7$ C. $= 282^\circ.7$ Absol., gave the differential deflection $24^{\text{div}}.0$. The corresponding differential radiation to a hemisphere is

$$0.0129 - 0.0084 = 0.0045 \text{ radim.}$$

On this night, the factor reducing to standard instrumental conditions, which is needed for comparing observations on different dates, was 0.982, so that the $24^{\text{div}}.0$ represent 23.6 standard divisions, and one standard division represents

$$0.0045 \div 23.6 = 0.000191 \text{ radim}$$

to the hemisphere.

The average sky deflection in the vicinity of the Moon on this night was:

$$22.0 \times 0.982 = 21^{\text{div}}.6 = 0.00413 \text{ radim.}$$

The absolute radiation of the sky was

$$0.0081 - 0.00413 = 0.00397 \text{ radim}$$

the corresponding temperature of the comparison screen being $+7^\circ.7$ C. $= 280^\circ.7$ Absol.; and the effective temperature of the sky at the mean altitude of the Moon ($66^\circ\ 36'$), taken from the new curve (or from Table B), is 214° Absol. $= -59^\circ$ C.

For the actual observations and the first steps in their reduction, reference must be made to the paper cited, where, for comparison of observations made on the same night with various altitudes of the Moon, all galvanometer deflections from the Moon, relatively to the sky, are reduced to the zenith. The factors for this reduction were originally obtained by observation of the apparent transmission of the total moonbeam by the atmosphere, which was found to agree so closely with the apparent transmission of visual rays that no distinction need be made. Let it be well understood that this operation merely corrects for the variation in the apparent transmission of the already sifted moonbeam. The correction for the real absorption by an entire atmosphere, has still to be applied to each reduced zenithal deflection. The reduction to zenith is not made on the sky deflections. Each observation is a differential one, the standards being blackened screens from which an effective temperature of the sky is inferred and made the final datum-level for the night in question. This virtual sky temperature varies greatly from night to night, and from horizon to zenith. It depends upon atmospheric absorption of the bolometer's radiation towards outer space; but for a given set of observations the variation has not been large in any case, and the virtual sky temperature may be classed with instrumental constants in the present reduction.

Table VII gives measurements of radiation with inferred effective temperatures for limited regions of the Moon, made January 12, 1889 when the Moon's age was 11.2 days. They favor those regions where the angle of incidence of the solar rays is large.

TABLE VII.

January 12, 1889.			Moon's Age, 11.2 days.		
No.	Zenithal Deflection	Differential Radiation	True Emission	*t*	*i*
	div.	radim	radim		
1	85.7	0.01637	0.02865	416°	7°
2	59.7	0.01140	0.01995	372	47
3	27.4	0.00523	0.00915	292	74
4	56.8	0.01085	0.01899	366	70
5	58.7	0.01121	0.01962	370	70
7	32.3	0.00617	0.01080	308	74
8	33.9	0.00647	0.01132	312	74
9	74.7	0.01427	0.02497	400	35
10	64.7	0.01236	0.02163	382	35
11	77.7	0.01484	0.02597	404	22
12	82.6	0.01578	0.02762	412	10
13	76.7	0.01465	0.02564	403	32
14	72.7	0.01389	0.02431	396	37
15	86.6	0.01654	0.02895	418	7
16	82.6	0.01578	0.02762	412	17
17	78.7	0.01503	0.02630	406	18
18	81.6	0.01559	0.02728	411	17
19	54.8	0.01047	0.01832	362	70
20	54.8	0.01047	0.01832	362	70
21	36.8	0.00703	0.01230	321	66

Table VIII is for the full Moon, April 15, 1889, and favors those regions of maximum radiant power, over which the Sun is shining nearly vertically.

The first column contains a reference number which, with the date, identifies the observation. The second column gives the average radiation from a special region of the Moon, referred to the neighboring sky, and stated in galvanometer divisions, reduced to the standard instrumental condition, and corrected for the variation of atmospheric absorption which results from changing altitude, each observation being reduced to the value which might be expected after the rays have passed normally through an

atmosphere of average composition and 760^{mm} pressure. The third column gives the excess of radiation over that of the sky in absolute measure, and is obtained by multiplying the galvanometer deflection by the constant, $1^{div} = 0.000191$ radim. The fourth column is obtained by multiplying the third by 1.75, and refers the lunar radiation to that of a comparison surface at absolute zero with abstraction of intervening atmosphere, and at the same time eliminates that portion which is simply reflected solar radiation. The factor, 1.75, is an approximate average value whose derivation is given below (p. 275). The column headed *t* in Tables VII and VIII, gives the temperature of the lunar surface at the angle *i* from the subsolar point, and is inferred from the curve of absolute radiation for blackened copper (Table B, p. 270). This curve has been obtained by a combination of physical and astrophysical methods whose description I must defer. It is sufficient to say that it has responded very satisfactorily to a variety of tests, and is believed to be a close approximation to the truth.

TABLE VIII.

April 15, 1889.			Moon's Age, 15.7 days.		
No.	Zenithal Deflection	Differential Radiation	True Emission	*t*	*i*
	div.	radim	radim		
1	113.3	0.02164	0.03787	452°	7°
2	91.5	0.01748	0.03059	425	49
3	84.2	0.01608	0.02814	414	57
5	93.6	0.01788	0.03129	427	53
6	104.0	0.01986	0.03476	441	26
7	114.4	0.02185	0.03824	453	20
8	110.2	0.02105	0.03684	448	36
9	95.9	0.01832	0.03206	430	53
10	102.0	0.01948	0.03409	438	31
11	102.0	0.01948	0.03409	438	33
12	110.2	0.02105	0.03684	448	34
13	112.3	0.02145	0.03754	451	13
14	89.7	0.01713	0.02998	422	57
15	93.8	0.01792	0.03136	427	57
16	88.7	0.01694	0.02965	421	49
17	95.9	0.01832	0.03206	430	49
18	94.8	0.01811	0.03169	429	49
19	116.4	0.02223	0.03890	455	7

The derivation of the factor for reflection at the lunar surface and absorption by the Earth's atmosphere shall now be given.

Observations of the transmission of the moonbeam by glass enable us to distinguish very closely between the reflected and emitted portions. My measures were made in winter and early spring, and winter measures give a ratio of lunar radiation transmitted by glass to radiation absorbed by glass = $\frac{1}{6}$. The glass used in my measures absorbed 23 per cent. of solar radiation, and transmitted 2 per cent. of radiation from a low temperature source, such as a boiling Leslie's cube. We must therefore increase the transmitted part by 21 per cent. to restore the proportion of reflected rays, and this gives:

$$\text{percentage of reflected rays} = \frac{121}{7} = 17.3.$$

The transmission of extreme infra-red rays by the Earth's atmosphere being larger in winter than the mean value — let us say 0.48 instead of the mean value 0.40 — and the mean transmission of the total moonbeam being one-half, while the atmospheric transmission of solar rays is 0.67, we get for the ratio outside our atmosphere:

$$\frac{17.3 \times \frac{3}{2}}{82.7 \times \frac{10}{4.8}} = \frac{1}{6.62},$$

and for the percentage reflected $\frac{100}{7.62} = 13.1$. This falls a little below the percentage for luminous rays. An albedo may be deduced from Zöllner's measures by dividing the theoretical ratio of the lunar disk to the hemisphere, supposed to reflect with albedo of unity to a central point, by the observed ratio of sunlight to moonlight, giving

$$\frac{97300}{618000} = 0.157.$$

Zöllner himself, assuming that the marked increase of light at the full is due to the roughness of the Moon's surface, applied a correction for light specularly reflected from crystalline facets,

and from elevations of every sort at an average angle of 52° with the surface of the sphere, getting 0.174 for the Moon's albedo.

The reflecting powers of various rocks for light run about like this:

White quartzite,	0.25
Clay shale,	0.16
Diorite, or dark slate,	0.09

Dark areas on the Moon may average a luminous albedo of 0.09. The surface of the Moon is about equally divided between dark and bright areas, and a mean albedo of 0.23 would be indicated for the bright parts, if 0.16 were adopted for the whole.

We need not assume that total reflected and luminous reflected rays are in identical proportions, although the difference is not very great; and as it has been found that long waves are less perfectly reflected by the Moon than short waves, there need be no hesitation in accepting for the combined lunar reflection of all sorts of solar rays, at least from regions within 60° of the subsolar point, a fraction as small as 1/6. It will be shown presently that marginal zones do not reflect uniformly in all directions, and that a larger fraction than 1/6 is required for them.

In illustration of the conditions of measurement, suppose that the bolometer at 300° Absol. is radiating to space at absolute zero, through the atmosphere. Its outgoing radiation of 0.010 radim is cut down to 0.005 radim by absorption, which is, in effect, as though the body of air had a like radiating power, and a temperature of 235°. Let a region of the Moon possessing a virtual temperature of 360° Absol., which corresponds to a radiating power to absolute zero (due in this case to combined reflection and emission) of 0.018 radim, be confronted with the bolometer. The lunar radiation reduced to 0.009 radim by absorption, combined with the effective radiation of the air, 0.005, is opposed to the potential 0.010 of the bolometer, and the balance stands thus:

Sky		Moon
± 0.000	$+0.018 \times \frac{1}{2} =$	$+0.009$
$+0.005$		$+0.005$
-0.010		-0.010
-0.005		$+0.004$

Radiation of Moon compared with sky:

$$+0.004 - (-0.005) = +0.009.$$

This observed radiation, multiplied by 2, and less $\frac{1}{8}$, gives the proper radiation of the Moon:

$$\frac{0.009 \times 2 \times 7}{8} = 0.01575 \text{ radim},$$

corresponding to a genuine effective temperature of 346° Absol

It will be seen that temperatures deduced by this method rest on the value adopted for reflection of the combined bundle of rays from the Sun at the Moon's surface. This depends further upon the agreement of results, derived from the selective absorption of glass, with others depending on a comparison of the energy-curves of the solar and lunar spectra, together with estimates of atmospheric absorption in different regions of the spectrum, derived by various and somewhat complex methods, which also, however, now exhibit a fair degree of accordance. In applying the method to different lunar regions, it must be remembered that the fraction $\frac{1}{8}$, given here for lunar reflection, is a mean value for central regions. Its variation must also be considered.

The percentage of reflected rays, as measured by the bolometer over somewhat wide areas after absorption by the atmosphere, has been found:

(*a*) smaller in dark regions than in bright: dark reflections = 50 to 70 per cent of bright;

(*b*) smaller in dry cold weather than in moist and hot; thus, in general, smaller in winter than in summer:

Apparent reflection (extreme dry) = 50 per cent. of extreme wet,
Apparent reflection (mean winter) = 70 per cent. of mean summer;

(*c*) about 20 per cent. smaller in normal diffusively reflected and emitted radiation from low Sun areas than would be inferred, on the assumption of uniform reflection and absorption, from measures of the combined normal radiation in regions where the Sun is high;

(*d*) especially large—perhaps two to three times the usual proportion—in a narrow crescent Moon, where the rays suffer a grazing reflection at a large angle of incidence, the emission under a corresponding angle of emission being small.

Local variations of temperature on the Moon occur and are connected with the reflecting power. Thus bright regions reflect much, absorb little, and must be colder than dark regions, when the combined reflected and absorbed radiations for regions of similar insolation are identical. In general, such identity cannot hold after absorption by our atmosphere, the emanations from the dark parts being richer in absorbable rays, and thus appearing colder.

Regions near the subsolar point in *Oceanus Procellarum*, on January 17, 1889, gave deflections of 87 to 91$^{\text{div}}$. At the same time the bright highlands on the central meridian in 0° to 30° S. latitude, and far from the theoretical heat-center, gave 95 to 98$^{\text{div}}$. The coefficient for passing from dark to bright regions is therefore at least

$$96.5 \div 89.0 = 1.084.$$

From many comparisons under similar conditions of incidence, it appears that the total radiation of bright regions is 11 per cent. greater than the combined reflection and emission of dark regions. However, at a large angle of incidence, there is strong reflection at the same large angle of reflection, but diminished diffusive reflection at angles near the normal to the surface, as in reflection from the zone just within the terminator at first and third quarters; but dark areas do not reflect to the same extent, and normally they absorb and radiate more than bright in this position. (See "Prize Essay on the Distribution of the Moon's Heat," p. 37.) This is one reason, and possibly the principal one, why the phase curve for moonlight falls below

that for total radiation at the first and third quarters.[1] With a phase-angle of 70° from full Moon, I found a total heating effect 27.4 per cent. of that at the full, a day and a half after first quarter; and 37.5 per cent., a day and a half before third quarter, while Zöllner obtained 14.4 per cent. for the corresponding ratio of light. Evidently, the transmission by glass ought to be smaller at first and third quarters than at the full, and this is found to be the case. In fact, the relative radiating power of dark and bright regions near the terminator is reversed at these epochs, as just noted. I find also that measures of transmission by glass for the total moonbeam may frequently be arranged in a series according to their distance from full Moon, the transmission and inferred reflection being greater at the full, but I do not wish to insist too strongly upon evidence of this sort, because variations of atmospheric moisture produce equally great fluctuations. The following summer values with somewhat similar atmospheric conditions, are selected from the Allegheny measures:[2]

Transmission by glass:

23.3	per cent.	day of full Moon.
21.5	"	one day from full Moon.
18.6	"	three days from full Moon.
19.0	"	three days from full Moon.
15.2	"	seven days from full Moon.

A lampblack surface radiates nearly equably in all directions, but other substances have diminishing radiant and absorbent powers, and increased reflection, as the angle of the rays with a normal to the surface grows larger. Thus glass, which gives off 93 per cent. of the amount of radiation emitted by lampblack, provided the inclination of the emitted rays is nearly normal, radiates:

[1] The considerable area occupied by shadows at these phases partly accounts for the diminution of light. The shadows may radiate somewhat freely, being warmed from surrounding regions in sunshine.

[2] "The Temperature of the Moon," by S. P. Langley, assisted by F. W. Very. *Mem. Nat. Acad. of Sci.* 4, 143, 145, 146, 147, 160.

86.4 per cent. at 60° from normal.
77.5 " at 70° from normal.
57.3 " at 80° from normal.

There is little change up to an angle of 60°, but beyond this the diminution of radiation is rapid. I found a similar diminution in the radiation from the subsolar point on the Moon as it departed from the center of the disk, but the relative dimensions of bolometer and lunar image did not permit the following of the effect to those large angles of emission in the narrow foreshortened margin near the limb, where it might be expected to be greatest; and the observations, being necessarily made on different dates, are not entirely comparable. With approximate correction for atmospheric condition, and reduction to standard brightness, these measures of subsolar radiation are as follows:

Angle of subsolar point from center of lunar disk		Observed radiation	Corrected radiation
4°	(bright region)	115 div	115 div
11	(dark region)	96	107
23	(bright region)	118	118
46	(dark region)	86	95
53	{34° from subsolar point reduced by mean curve}	79	92
61	(dark, air hazy)	61	85

The observations are neither numerous enough nor good enough to serve for deducing a mean curve of radiation according to the angle of the subsolar point from the center; but in obtaining a mean curve of lunar radiation as it varies with the inclination (i) of the solar rays to a normal at the lunar surface, I have profited by the knowledge of these probable changes, and have tried to avoid error by employing only central or nearly central positions.

From such measures, given in Table IX, the curves of the variation of lunar radiation along the Moon's equator in the lunar morning and afternoon have been prepared. These individual central measures have been supplemented, in drawing the final curves, by ratios derived from the mean phase curve

TABLE IX.

Date, 1889	i	Position	Period of lunar day	Zenithal defl'ction	Differential radiation	True emission	t	Sky deflection	Differential radiation of sky	Temp. of screen	Effective temperature of sky
				Div.	Radim	Radim		Div.	Radim		
January 23	102°	15° from central	Night	0.0	0.00000	0.00000	0°	—25.7	0.00491	281°	197° = — 76°C.
February 7	76	Central	A.M.	18.9	0.00361	0.00632	258	—20.2	0.00386	274	206 = — 67 C.
January 23	75	Central	P.M.	21.7	0.00414	0.00725	272	—25.7	0.00491	281	197 = — 76 C.
January 23	67	20° from central	P.M	43.7	0.00835	0.01461	337	—25.7	0.00491	281	197 = — 76 C.
April 10	61	Central	A.M.	40.1	0.00766	0.01341	329	—20.2	0.00386	289	238 = — 35 C.
January 12	46	Central	A.M.	59.7	0.01140	0.01995	372	—21.6	0.00413	281	214 = — 59 C.
January 17	38	27° from central	P.M.	85.9	0.01641	0.02872	417	—23.3	0.00445	285	216 = — 57 C.
April 15	31	27° from central	P.M.	102.0	0.01948	0.03409	438	—25.1	0.00479	287	213 = — 60 C.
April 13	23	Central	A.M.	106.6	0.02036	0.03563	448	—33.3	0.00636	285	164 = —109 C.
April 15	16	20° from central	A.M.	112.3	0.02145	0.03754	451	—25.1	0.00479	287	213 = — 60 C.
April 15	4	Central	P.M.	114.9	0.02195	0.03841	453	—25.1	0.00479	287	213 = — 60 C.

given in my essay, which represents the summation of many measures distributed over the lunar surface, eliminating local peculiarities. Thus the principal part of the radiation at first and third quarters comes from the zone 40° to 60° from the subsolar point. The mean radiation at third quarter is to that at first as 1⅓ is to 1, giving the following average equatorial afternoon values by derivation from morning measures:

i	A.M.	P.M.
60°	1.33 × 41	= 54.5
50°	1.33 × 56	= 74.5
40°	1.33 × 74	= 98.4

The column headed *t* in the following table gives the concluded effective lunar temperature on the absolute Centigrade scale at the angle *i* from the subsolar point.

It is hardly possible at present to give a closer approach to a categorical statement of the lunar surface temperature than is contained in Table IX and the accompanying curves. If interior conduction is more rapid than has been here virtually assumed, midday surface temperatures may be lower, but retention of heat must then be greater.

I cannot emphasize too strongly the significance of the observations which prove that a few hours' deprivation of sunshine, as in a lunar eclipse, reduces the proper lunar radiation from its normal value at full Moon to less than 1 per cent., while in less than a day after the Sun has set, the night surface of the Moon gives no radiation sensible to our instruments. The Earth's atmosphere transmits probably at least 50 per cent. of extreme infra-red radiation such as must come from a body at very low temperature, and the measuring apparatus, according to the radiation curve, ought to be able to distinguish between radiators 25° apart, even near absolute zero. Hence, unless the Moon has an atmosphere capable of retaining these long etherwaves, the night temperature of the Moon can hardly exceed 50°, or at most 100°, on the absolute Centigrade scale. But the rapidity with which the Moon's surface cools in the afternoon shows that there is no such strongly absorbent atmosphere, and

the extremely low temperature of the lunar night is as nearly demonstrated as anything incapable of direct observation can be.

In the light of the preceding observation, one other mode of computing the lunar temperature may be used as a check. The Sun sends both Earth and Moon a radiation of 0.0500 radim, of which the Moon retains about ⅞ or 0.0438 radim, radiating its heat again from a surface virtually but little greater than twice the receiving section. Hence the mean proper radiation of the day side of the Moon is not much below 0.0219, corresponding to a mean temperature of 383° Absol. for blackened copper.

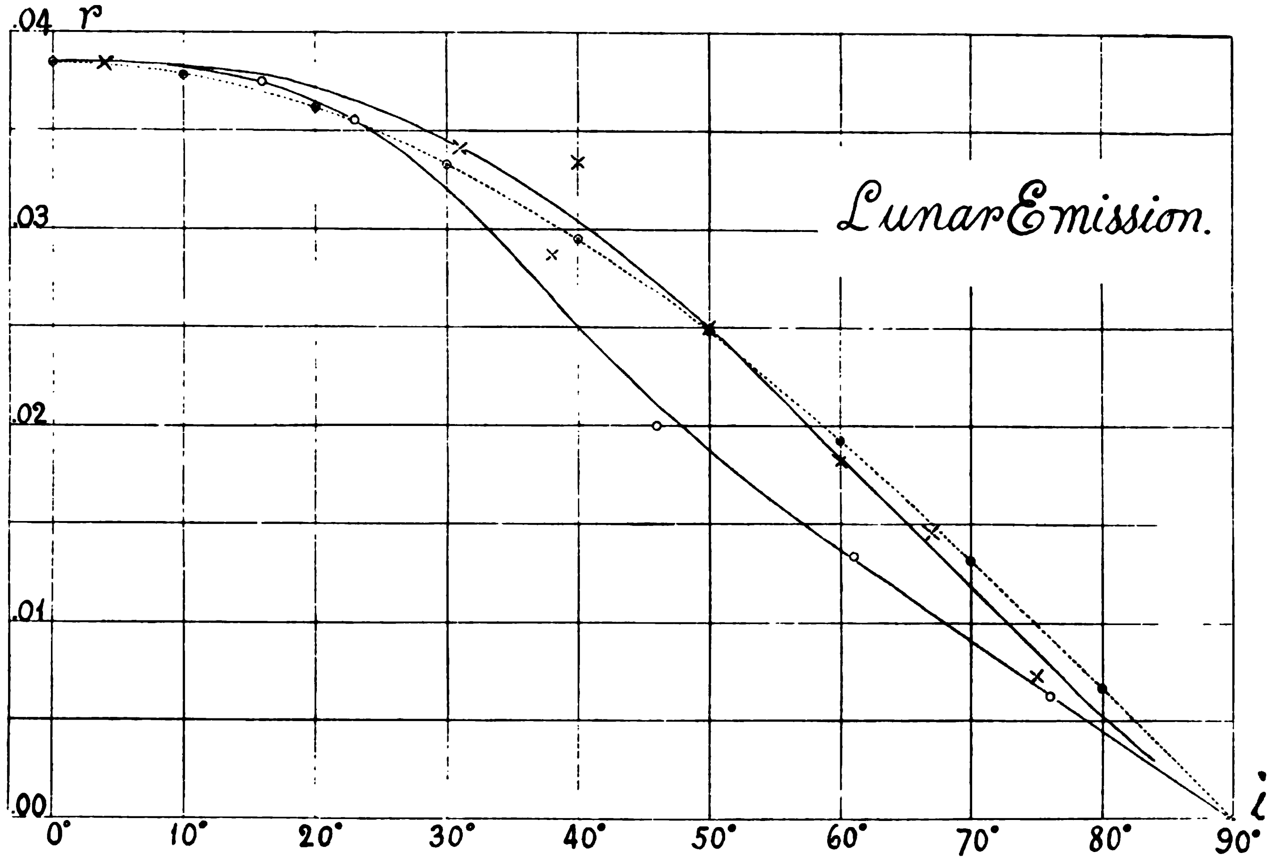
The Earth, by reason of its atmosphere, keeps its acquired temperature, and radiates both by night and by day through a surface four times the area of the absorbing section. If its reflecting power were the same as that of the Moon, its proper radiation would be 0.011 radim, corresponding to a mean temperature of 310° Absol. = +37° C. in blackened copper. The actual mean temperature of the Earth is about 15° C., and the reflective power must therefore exceed that of the Moon, which is known to be the case from other considerations. If the Earth reflects 0.3 of the Sun's rays, the theory agrees with the facts; and it seems to me that this warrants the hypothesis that blackened copper is a fairly typical radiator for comparison with the Earth, the temperatures and emissions just given having been read from the curve of absolute radiation for this substance. But blackened copper may not be as good a choice foı comparison with the Moon, where surfaces of dry rock take the place of water and vegetation, both of which agree well with lampblack as radiators. The surface temperature imparted by absorbed radiation, depends upon the capacity of the absorbing substance for heat, on the rapidity with which it conducts heat, and the consequent depth to which heat penetrates, and the mass of substance through which it is distributed, as well as upon the rate of reception and emission of radiation. If the irradiated body be very cold, its subsurface temperature-gradient may be so steep that the rate at which heat is conducted from the surface to the interior, forms a considerable fraction of the rate at which heat is supplied by

the radiant energy annihilated. In this case the surface temperature will be lower than that which the same radiation is capable of maintaining after the subsurface gradient is filled up; but if, after the heat capacity of the body has been satisfied, the received radiation falls off in intensity, a reversed flow of heat from the interior may preserve the surface temperature above the point which unaided radiation is capable of producing with the actual emissivity. In this way the solid substance of a nearly airless planet may exercise a retaining power for heat, and distribute temperatures over a wider interval, moderating excesses. The immobility of a solid prevents this mechanism from being anything like as effective as that of the Earth's atmosphere, which preserves the arctic regions from the cold of space during many months of darkness; but it is sufficient to give a very notable difference in temperature between the lunar morning and afternoon, and to make the diurnal temperature curve a little flat-topped.

TABLE X.

i	Radiation a. m.	$r_0 \cos i$	Radiation p. m.	Temperature	
				a. m.	p. m.
	radim	radim	radim		
0°	0.0385	0.0385	0.0385	454°	454°
10	0.0382	0.0379	0.0383	453	453
20	0.0365	0.0362	0 0372	447	450
30	0.0320	0.0333	0.0345	430	440
40	0.0250	0.0295	0.0304	400	424
50	0.0188	0.0248	0.0250	365	400
60	0.0137	0.0193	0.0188	331	365
70	0.0091	0.0132	0.0120	292	318
80	0.0045	0.0067	0.0052	227	240
90	0.0000	0.0000	0.0005	0	75

In Table X and the accompanying plate, r_0 being the maximum lunar radiation under a vertical Sun, i the angle of incidence of solar rays and also the angular distance of any region on the Moon, from the subsolar point, the ordinates of the curve, $r_0 \cos i$, are proportional to the insolation. Dots in circles indi-


Lunar Emission.
r
.04
.03
.02
.01
.00
i
0°
10°
20°
30°
40°
50°
60°
70°
80°
90°

cate this curve. Circles show the morning curve of emission, abscissae being angles from the subsolar point, and the afternoon observations of emission are denoted by crosses.

Table X gives only average values for every 10° of the subsolar angle.

The morning curve falls much below the curve of insolation, reaching its greatest actual, though not its greatest percentage departure, between the third and fourth day after lunar sunrise. Evidently heat is being stored up. But in the early part of the lunar afternoon, and for nearly three days after lunar midday, the curve of emission slightly exceeds that of insolation, following it closely, however, in the latter part of the afternoon, when cooling is very rapid, temperatures below the freezing point being reached twenty-four hours before sundown.

There may be local differences of heat-retaining power. Finger-like extensions of the isotherms beyond the lines of symmetry have been noted. But, on the whole, the peculiarities are not numerous, and no such extraordinary divergences from isothermal symmetry as our Earth exhibits, can find place on the Moon.

One other evidence of heat-retention may be mentioned. Measures of lunar radiation made all around the limb at full Moon show a slight preponderance at the poles. The incidence of the solar rays being the same in every case, and the nature of the surface not essentially different, we must infer that the poles are hotter because the Sun has been shining longer upon them.

The mean emission of the daylight hemisphere of the Moon has been estimated by the aid of the solar constant (p. 283) as something less than 0.0219 radim, admitting a reflection of one eighth of the solar rays at the Moon's surface; but the zone, 60° to 90° from the subsolar point, reflects more than this, hence the estimate must be reduced. We are now in a position where we can give a value from the lunar measures. Taking the products of the mean radiations by the areas of the zones in Table X, we have:

Zone	0°–10°	0.00584
	10 – 20	0.01696
	20 – 30	0.02587
	30 – 40	0.03050
	40 – 50	0.03055
	50 – 60	0.02727
	60 – 70	0.02117
	70 – 80	0.01297
	80 – 90	0.00451
	Mean emission =	0.01952 radim

If the hemisphere were uniformly heated, this mean radiation would correspond to a mean temperature of + 97° C.

In conclusion, it seems to me reasonably certain that a large part of the Moon experiences daily great vicissitudes of temperature. Its rocky surface at midday, in latitudes where the Sun is high, is probably hotter than boiling water; and only the most terrible of Earth's deserts, where the burning sands blister the skin, and men, beasts, and birds drop dead, can approach a noontide on the cloudless surface of our satellite. Only the extreme polar latitudes of the Moon can have an endurable temperature by day, to say nothing of the night, when we should have to become troglodytes to preserve ourselves from such intense cold. Yet great as is the midday heat on the Moon, it might be even greater, if an atmosphere existed at all comparable with ours in density; and possibly the comparative absence of atmosphere which has been regarded by some as an insuperable barrier to life, may be the one condition needed for the preservation of some sort of life from destruction by the burning heat.

PROVIDENCE, R. I.,
August 1898.

PHOTOGRAPHS OF COMET I, 1898 (BROOKS), MADE WITH THE CROSSLEY REFLECTOR OF THE LICK OBSERVATORY.

By James E. Keeler.

Although the three-foot reflector presented to the Lick Observatory by Mr. Crossley was set up in its present place some time ago, it has been little used, chiefly for lack of the necessary assistance in managing it. I have recently, through the appointment of several Fellows from the astronomical department of the University, at Berkeley, been able to bring this instrument into active service, and have employed it for a number of different purposes. I shall reserve an account of its performance and capabilities for a future paper, and shall limit the present note to a description of some photographic observations of the comet discovered by Mr. Brooks on October 20.

For photographing stars and nebulæ the reflector is provided with a double slide plate-holder and guiding eyepiece of the form devised by Dr. Common. The precision with which stars can be followed with this apparatus leaves nothing to be desired. But for guiding the instrument in photographing a comet there is at present no better arrangement than a four-inch finder, of eight and one-half feet focal length, attached to one of the four iron standards which form the "tube" of the telescope. It will readily be understood that the results of guiding a three-foot telescope with a four-inch finder are not entirely satisfactory. In photographing a comet the trails left by the stars in their relative motion consist of a great number of irregular zigzag lines, the departures of which from the mean position, or axis of the trail, roughly measure the disparity in the optical powers of the two telescopes.

With the assistance of Mr. H. K. Palmer, Fellow in Astronomy at the Lick Observatory, I obtained photographs of Brooks' comet on eleven consecutive nights, from November 4 to Novem-

ber 14, inclusive. At first it was possible to give an exposure somewhat exceeding one hour in length; but the comet was moving rapidly southward, and the limits set by the end of twilight on the one hand, and the construction of the mounting of the telescope on the other, soon narrowed, so that by November 14 the greatest possible exposure was reduced to thirteen minutes. The observations were then discontinued.

The best photograph was obtained on November 5 with an exposure of $1^h 10^m$. On this plate the extreme diameter of the coma is 0.25 inch=4'.1. A very narrow straight tail extends from the center of the head to a distance of 1.4 inch, or 23'. In appearance the comet closely resembles Comet b, 1894 (Gale), as photographed by Barnard.[1]

A photograph made by Mr. Palmer on the night of November 3, with the Willard six-inch portrait lens, closely resembles this photograph, though the scale is of course very much smaller. The exposure was $1^h 17^m$. The tail as shown on the plate is about 45' long.

At the end of each exposure with the Crossley telescope the driving-clock was stopped, and the stars were allowed to trail on the plate for about two minutes, in order to obtain reference lines for the measurement of position angles. No perceptible trail was left by the comet.

The tail is shown best on the photograph of November 5. It is much shorter and less distinct on my other negatives, which received shorter exposures, and is barely visible on plates which were exposed for less than thirty minutes. The position angle of the tail was measured on six plates, with the following results, which may have some interest in connection with theories of the physical constitution of comets. In the table r is the computed position angle of the radius vector of the comet, and t is the position angle of the tail.

On November 6 several photographs were taken with exposures of different lengths, instead of giving all the available time to one long exposure. These plates showed a very small, almost

[1] *A. and A.* 13, 421, 1894.

stellar nucleus, which was easily photographed with an exposure of four minutes—the shortest that was tried.

Date	R. A.	Dec.	r	t	$r-t$
	h m s	° ′	° ′	° ′	° ′
1898, Nov. 5^d 7^h	17 34 53	+17 12	53 55	48 37	+5 18
Nov. 7 7	17 41 56	14 21	54 49	52	+2 49
Nov. 8 7	17 45 7	12 32	55 40	52½	+3 10
Nov. 9 7	17 48 0	10 51	56 19	53	+3 19
Nov. 10 7	17 50 30	9 19	56 51	55	+1 51
Nov. 12 7	17 55 3	+ 6 21	57 53	60	−2 7

The uncertainty of guiding, to which I have referred, affects, of course, the impression of the comet on the plate, as well as the star trails. Certain irregularities in the outline of the head and in the shape of the nucleus, which appear on some of the plates, are in general attributable to this cause. There is reason to believe that other appearances of the kind are real. The effect of imperfect guiding on the image of the comet can be ascertained by a study of the star trails, in which all errors of pointing are accurately registered, and in one or two cases some other explanation of the observed irregularities seems to be required.

On the negative of November 10, obtained with an exposure of 50^m, the head of the comet is made up of two clearly separated nebulous masses, surrounded by the nearly circular coma. The centers of these masses are 1^{mm} apart, and the line joining them is nearly at right angles to the general direction of the star trails. The short zigzag lines which make up the latter are, however, practically all included within a strip $\frac{1}{8}^{mm}$ wide, and there is no evidence in the trails that the errors in the pointing of the telescope were grouped around two distinct and slightly different values. The guiding was neither better nor worse than usual.

The appearance described above is therefore not explicable as a result of imperfect guiding. I am inclined to believe that the division of the nucleus was real. It was not certainly visible with the twelve-inch and thirty-six-inch refractors; but it is well known that short exposure and forced development of a photo-

graph greatly exaggerates contrasts of light and shade, and in this case the differences in the brightness may have been so slight as to escape direct visual observation. The separation of the head of a comet into distinct masses is not an unusual phenomenon, and it is one which causes considerable embarrassment to the observer with the micrometer. In this case the separate masses were not marked strongly enough to be visible, and they were symmetrically situated with respect to the center of the coma, so that no appreciable effect on measures of position which may have been made at the time is to be apprehended.

On the photographs of November 11 there was no separation of the nucleus to be seen.

These preliminary results show that the Crossley reflector only needs a sufficiently powerful guiding telescope to become an efficient instrument for the photography of comets, and as soon as possible the necessary apparatus will be provided.

THE VARIABLE VELOCITY OF o LEONIS IN THE LINE OF SIGHT.

By W. W. CAMPBELL.

MY measures of four spectrum plates of *o* Leonis ($\alpha = 9^h\ 36^m$, $\delta = +10^\circ\ 21'$) give the following velocities with reference to the solar system :

1898 — March	22,	$V = +66^{km}.7$	per	second.
Oct.	31,	$V = -30 \pm$	"	"
Nov.	1,	$V = -23 \pm$	"	"
Nov.	4,	$V = +40.0$	"	"

It will be seen that the velocity varies between wide limits, probably in a comparatively short period of time. The extreme range observed thus far is about 97^{km} per second.

All the photographs were obtained under extremely poor atmospheric conditions. The second and third photographs are unsuitable for accurate measurement, and they will not be used in future discussions of the motion.

There is some evidence that the spectrum consists of two superposed spectra.

LICK OBSERVATORY,
Nov. 5, 1898.

THE VARIABLE VELOCITY OF χ DRACONIS IN THE LINE OF SIGHT.

By W. W. CAMPBELL.

THE velocity of χ Draconis ($\alpha = 18^h\ 23^m$, $\delta = +72°\ 42'$) in the line of sight is variable, having a range of about 40^{km} per second. The first three photographs secured gave little indication of a variable velocity. The first evidences of variation were detected by Mr. Wright while developing the plate of October 25. The velocities determined to date are given below:

Mt. Hamilton M. T.	Velocity	Mt. Hamilton M. T.	Velocity
1898, July 25,	$+45^{km}.6$	1898, November 1,	$+11^{km}.9$
September 5,	+46 .0	November 5,	+11 .3
September 19,	+42 .6	November 12,	+10 .7
October 24,	+14 ± [1]	November 18,	+10 .6
October 25,	+16 ± [1]	December 3,	+15 ± [1]
October 26,	+14 .5	December 7,	+18 .3

The observations seem to indicate a period of between five and six months. At the present time the velocity is rapidly increasing. The spectrum does not appear to be composite.

The period of *o* Leonis, recently announced, is very close to 14½ days. The range is about 112^{km}.

The period of η Pegasi seems to be about 2¼ years.

LICK OBSERVATORY,
December 9, 1898.

[1] The plates of October 24, October 25, and December 3 are poor, and the approximate results assigned to them were obtained from hasty and incomplete measures of a few lines. While they fall very close to the velocity curve furnished by the other observations, it is not intended to use them in subsequent discussions.

ON THE CONSTITUTION OF GASEOUS CELESTIAL BODIES.

By A. Ritter.

EDITORIAL INTRODUCTION.

Between the years 1878 and 1883 a series of eighteen papers by A. Ritter, of Aachen, appeared in *Wiedemann's Annalen* under the title, "Researches on the height of the atmosphere and the constitution of gaseous celestial bodies." These seem to have received hardly the attention they deserve, and at the suggestion of one of our associates we have translated one of the most important papers of the series. This is the sixteenth paper, from Bd. XX of the *Annalen*, and it is of especial interest from its bearing on the question of stellar classification. It will be seen that the author, reasoning on purely physical grounds and assumptions, arrives at a classification which includes stars with rising temperature as well as those with falling temperature. Thus he infers that the red stars of class III a have not reached the acme of their brilliance, and that those of class III b (Secchi's fourth type) are nearing extinction after having run their course of stellar evolution.

Many assumptions and hypotheses are made in the series of papers that have not been substantiated or have been disproved by more recent researches, but the author frankly and clearly states what are assumptions and what are facts of observation.

The fundamental assumption is the principle of the neutral equilibrium of a gaseous atmosphere. This is stated as follows: "When a vertical motion is given to a small mass of air at any point in the atmosphere otherwise at rest, it expands as it rises, and its temperature consequently falls. The slow rise of the small mass of air will be *uniform* motion if the impelling force at every point exactly balances gravity. This condition will be

fulfilled if the temperature of the rising air is always the same as that of the air into the neighborhood of which it comes.

"As no transfer of heat occurs at equal temperatures, the condition of adiabatic expansion of the rising air will in this case be also fulfilled. A uniform upward motion and adiabatic expansion then require the common condition that the vertical temperature gradient of the air at rest shall coincide with the adiabatic gradient of the rising air. This condition being satisfied, the atmosphere will be in neutral equilibrium.

"This may be regarded in a sense as the normal state of the atmosphere, for temporary instability always develops motions (storms, etc.), which lead to neutral equilibrium, and the relatively high temperature of the Earth's surface prevents a long continuance of stable equilibrium. In general the neutral state is to be regarded as the final condition which will always result from any initial condition whatever, provided the different strata of air fully and freely mingle with each other." (*Wied. Ann.* **5**, 405–406.)

The term "condition-line" (*Zustandslinie*) frequently employed by the author is defined as follows: "At the separate points of a straight line drawn from the center to the surface of a gaseous celestial body, its matter is in different conditions; and we may assume that the condition varies continuously along that line if the body is composed of an elemental substance having the properties of a perfect or ideal gas. Since a definite point of the temperature surface corresponds to each definite condition of the gas, the law of the space-rate of change of condition may be geometrically represented by a continuous line. This we shall call the condition-line of the body. On the hypothesis of neutral equilibrium the condition-line of a celestial object in an entirely gaseous state is an adiabatic curve." (Bd. **5**, 547.)

We have retained the numbering of paragraphs and equations given in the original article, for the convenience of reference to earlier papers of the series. To assist further in this respect the following list of Ritter's papers is appended:

REFERENCES TO RITTER'S ARTICLES IN "WIEDEMANN'S ANNALEN."

Numbers	Sections	Equations	Volume	Pages
I	1–7	1–41	5	405–425
II	8–11	42–80	5	543–558
III	12–14	81–133	6	135–144
IV	15–17	134–175	7	304–317
V–VI	18–26	176–245	8	157–183
VII	27–29	246–281	10	130–143
VIII	30–33	282–324	11	332–344
IX	34–36	325–358	11	978–997
X	37–40	359–393	12	445–465
XI	41–44	394–449	13	360–377
XII	45–48	450–498	14	610–634
XIII	49–55	499–584	16	166–192
XIV	56–60	585–593	17	322–342
XV	61–63	594–596	18	488–508
XVI	64–68	597–637	20	137–160
XVII–XVIII	69–76	638–659	20	897–927

§ 64. HEAT RADIATION OF AN ADIABATIC SPHERE OF GAS.

The adiabatic condition-line of an ideal gaseous sphere represents an increase of temperature with increasing depth below the surface, and in the immediate proximity of the surface the temperature increment dT corresponding to the depth increment dz can be calculated from the formula

$$\frac{dT}{dz} = \frac{T_0}{ar}, \tag{597}$$

where r denotes the radius of the sphere, T_0 the temperature of its center, and a a constant depending on the value of the ratio of specific heats at constant pressure and constant volume, $k = \frac{c_p}{c_v}$. The latter can be represented geometrically by the surface F bounded by the gravitation curve (§ 31, Fig. 7). For $k = 1.41$ (§12 and §31), a becomes 2.4, and in general a can be computed from the approximate empirical formula

$$a = 0.5 + \frac{0.4}{k - 1.2} \tag{598}$$

For small values of the ratio $\frac{z}{r}$ the temperature at the depth z below the surface can be computed from the formula

$$\frac{T}{T_0}=\frac{z}{ar}, \tag{599}$$

and since under an adiabatic condition the temperatures vary as the $(k-1)$-th powers of the densities (Eq. 309), we obtain for the ratio of the corresponding densities

$$\frac{\gamma}{\gamma_0}=\left(\frac{z}{ar}\right)^{\left(\frac{1}{k-1}\right)}. \tag{600}$$

The mass of the surface layer of thickness z, per unit of surface, is

$$\mu=\int_0^z \gamma\, dz, \tag{601}$$

and if we substitute the value for γ obtained above and perform the integration, we shall obtain for this surface layer

$$\mu=\left(\frac{k-1}{k}\right)\gamma_0\,(ar)^{-\left(\frac{1}{k-1}\right)}\,z^{\left(\frac{k}{k-1}\right)}. \tag{602}$$

The above equations hold not only for the given conditions, but also for any other adiabatic condition of equilibrium whatever, into which we can imagine the gaseous sphere to be brought, and for a given value of μ we shall obtain for the thickness of a surface layer of this mass a smaller value, the greater the density of the sphere is assumed to be. If the radius were to decrease to the value $\frac{r}{n}$, the density of the center would become $n^3\gamma_0$, and the temperature of the center $n_0 T$ (from § 8). Under this new adiabatic condition of equilibrium, the depth of the surface layer at which the mass-intensity would again assume the value μ would be calculated from the equation

$$\mu=\left(\frac{k-1}{k}\right)n^3\gamma_0\left(\frac{ar}{n}\right)^{-\left(\frac{1}{k-1}\right)}z_1{}^{\left(\frac{k}{k-1}\right)}. \tag{603}$$

On equating the two expressions now found for μ we obtain as the ratio of s and s_1

$$\frac{s}{s_1} = n^{\left(\frac{2-3k}{k}\right)}. \qquad (604)$$

The temperature T_1 corresponding to the value s_1, can be computed from equation (599) by applying it to the new condition of equilibrium, whence

$$\frac{T_1}{n\,T_0} = \frac{s_1}{a\left(\frac{r}{n}\right)}. \qquad (605)$$

Therefore

$$\frac{T_1}{T} = n^2 \frac{s_1}{s}, \qquad (606)$$

and after substituting the value of $\frac{s_1}{s}$ from the expression above we obtain

$$\frac{T_1}{T} = n^{\left(\frac{2-k}{k}\right)}. \qquad (607)$$

Since the law of the increase of temperature and density with increasing depth below the surface remained unchanged during the transition to the new condition of equilibrium, the ratio of the maximum temperatures T_1 and T here found indicates at once in what ratio the mean temperature of the surface layer of mass μ has increased during the change of the sphere to the denser and hotter condition.

Under the assumption of a sufficient density of the gaseous sphere, we may assume that the thickness of the surface layer whose particles send out heat radiations directly into empty space, constitutes a comparatively small fraction of the whole radius of the sphere. If we further make the provisional assumption (reserving a correction to be later applied) that the mass-intensity of this radiating layer is independent of its mean temperature, and possesses one and the same value μ for all conditions of the sphere, then the above quantity can at the same time be considered as the ratio in which the mean temperature of the radiating surface layer has increased during contraction,

or more briefly, as the ratio in which the surface temperature of the gaseous sphere has increased.

According to Stefan's researches the intensity of heat radiation varies approximately as the fourth power of the absolute temperature of the surface. Accordingly we get for the ratio of the heat radiation per unit of surface

$$\frac{q_1}{q} = \frac{T_1^4}{T^4} = n^{\left(\frac{8-4k}{k}\right)}. \tag{608}$$

Since the surfaces vary as the squares of the radii, the ratio of total quantities of heat radiated per unit of time is

$$\frac{Q_1}{Q} = \frac{1}{n^2}\frac{q_1}{q} = n^{\left(\frac{8-6k}{k}\right)}. \tag{609}$$

This equation shows that $Q_1 = Q$ when $k = \frac{4}{3}$, and that Q_1 is less than Q when k is greater than $\frac{4}{3}$. Since the adiabatic condition of equilibrium of a gaseous sphere can only be maintained permanently when k is greater than $\frac{4}{3}$, as was shown in § 33, the last case only comes under consideration. For an ideal hydrogen sphere k would be 1.41, and for this case the following numerical values obtain:

$n = 1$	2	10	100
$\frac{T_1}{T} = 1$	1.336	2.621	6.868
$\frac{q_1}{q} = 1$	3.2	47	2226
$\frac{Q_1}{Q} = 1$	0.8	0.47	0.22

If we should place $k = \frac{5}{3}$ we should get:

$n = 1$	2	10	100
$\frac{T_1}{T} = 1$	1.15	1.585	2.512
$\frac{q_1}{q} = 1$	1.74	6.3	39.8
$\frac{Q_1}{Q} = 1$	0.436	0.063	0.00398

These numerical values are based upon the inaccurate assumption that the mass intensity of the radiating surface layer remained unchanged during the transition of the sphere to the hotter and denser condition. Since the absorptive power of a stratum of gas in all probability increases with rising temperature, smaller values than the above should be used for the mass-intensity, and hence for the mean temperature, in order to make allowance for this fact. In actuality, therefore, the quantity of heat radiated per unit of time will decrease with gradual condensing of the sphere more rapidly than found above.

Since we may assume that the quantity of light emitted will diminish simultaneously with the quantity of heat radiated, it follows that a decrease in the luminosity of a star may accompany an increase of the interior heat. A falling off in light, therefore, by no means premises a previous decline of temperature as a necessary condition. A decrease in the light of a star may indeed occur while the interior heat and the surface temperature are still increasing.

The Sun is at present in a condition to elude an answer to the inquiry whether its mean temperature is still increasing or already decreasing. In any case, however, the Sun has already attained a condition of density such that the fulfillment of the above assumption of a comparatively slight depth of the radiating surface layer is hardly to be doubted. From the results of the above investigation we may therefore reason with a high degree of probability that under any circumstances—whether the Sun's temperature be increasing or decreasing—the total yearly heat radiation from the Sun is at present already on the decline, therefore that the Sun is to be considered as a star gradually beginning to disappear, or as a star that has already entered the stage of diminishing light intensity.

[The investigation of § 11 necessarily led to the opposite result, because the assumption was there made that with the rise of the interior heat of a gaseous celestial body, the temperature of radiation—or the mean temperature of the particles whose radiations are sent directly out into space—increases in the same

proportion as the mean temperature of the whole body. The investigation just made shows that this assumption and the conclusions based upon it must be regarded as inapplicable to the case of the Sun in its present condition of density.]

§ 65. HYPOTHESES AS TO THE LAW OF CONTRACTION OF THE SUN.

The potential of an ideal gas ball of mass S in a condition of adiabatic equilibrium can be computed (§ 32) from the equation

$$\mathfrak{U}=\left(\frac{3k-3}{5k-6}\right)NrS, \tag{610}$$

in which N denotes the force of gravitation per unit of mass at the surface layer. The mechanical equivalent of its interior heat has (§ 20) the value

$$U=\frac{\mathfrak{U}}{3k-3}. \tag{611}$$

The potential $\mathfrak{U}$ can be regarded as the mechanical work the forces of gravitation would perform if the radius decreased from ∞ to r. We may regard this work as resolved into the two components U and $\mathfrak{U}-U$, of which the first is used for producing the interior heat, and the second for developing the heat radiated outward during contraction. This latter quantity of heat has therefore the value

$$W=A(\mathfrak{U}-U)=A\left(\frac{3k-4}{3k-3}\right)\mathfrak{U} \qquad \text{or} \tag{612}$$

$$W=\left(\frac{3k-4}{5k-6}\right)ANrS. \tag{613}$$

If $N=N_0$ when $r=r_0$, we have from the law of gravitation

$$Nr^2=N_0r_0^2, \tag{614}$$

and after substituting this value for N, the above equation takes the form

$$W=\left(\frac{3k-4}{5k-6}\right)\frac{AN_0r_0^2S}{r}. \tag{615}$$

By differentiating with respect to the time t, we obtain for

the quantity of heat sent outward from the whole sphere in each unit of time

$$\frac{dW}{dt} = -\left(\frac{3k-4}{5k-6}\right)\frac{A N_0 r_0^2 S}{r^2} \cdot \frac{dr}{dt}. \tag{616}$$

The average quantity of heat given out by each kilogram of mass is therefore

$$w = -\left(\frac{3k-4}{5k-6}\right)\frac{A N_0 r_0^2}{r^2} \cdot \frac{dr}{dt}. \tag{617}$$

If σ denotes the amount by which the radius diminishes per unit of time, for instance per year, then we may place

$$\sigma = -\frac{dr}{dt} \tag{618}$$

and from the above equation we get

$$(619) \quad \sigma = \left(\frac{5k-6}{3k-4}\right)\frac{w r^2}{A N_0 r_0^2}, \qquad \sigma_0 = \left(\frac{5k-6}{3k-4}\right)\frac{w_0}{A N_0}. \quad (620)$$

In applying these equations to the Sun (where the quantities with subscript zero are taken to apply to its present condition) we accordingly make $N_0 = 27.4$, $w_0 = \frac{1}{3}$, and find for σ_0 and k the following corresponding values:

$k =$	1.41	$\frac{5}{3}$
$\sigma_0 =$	$94075^m.$	$48083^m.$

From equations above we obtain

$$\frac{\sigma}{\sigma_0} = \frac{w r^2}{w_0 r_0^2}. \tag{621}$$

Since the quantities of heat given out per unit of mass vary as the total quantities radiated, we place (609)

$$\frac{w}{w_0} = \left(\frac{r}{r_0}\right)^{\left(\frac{6k-8}{k}\right)}, \tag{622}$$

and if at the same time we substitute for σ its value given in equation (618), we shall get an equation which we can put in the form

$$dt = -\frac{dr}{\sigma_0}\left(\frac{r}{r_0}\right)^{\left(\frac{8-8k}{k}\right)}. \tag{623}$$

The time in which the Sun's radius decreased from r to r_0 can therefore be computed from the equation

$$t = -\frac{r_0^{\left(\frac{8k-8}{k}\right)}}{\sigma_0} \int_r^{r_0} r^{\left(\frac{8-8k}{k}\right)} dr, \tag{624}$$

or

$$t = \frac{k\, r_0}{(7k-8)\,\sigma_0} \left\{ 1 - \left(\frac{r_0}{r}\right)^{\left(\frac{7k-8}{k}\right)} \right\}. \tag{625}$$

If we now first put $k = 1.41$ and hence $\sigma_0 = 94075^{\mathrm{m}}$., this equation assumes after substitution of $r_0 = 688\,000\,000^{\mathrm{m}}$ the form

$$t = 5\,514\,312 \left\{ 1 - \left(\frac{r_0}{r}\right)^{1.326} \right\}. \tag{626}$$

For corresponding values of $\frac{r_0}{r}$ and t, we thus get

$\frac{r_0}{r} = 1$	$\frac{1}{2}$	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{215}$
$t = 0$	3 315 060	5 254 154	5 502 036	5 509 864.

Since the radius of the Earth's orbit is about 215 times the present solar diameter, it follows—on the assumptions made here—that about 5.5 million years ago the solar radius was of the same size as the radius of the Earth's orbit.

The premise of the above conclusion was that the depth of the radiating surface layer was always small in comparison with the radius of the gas ball. It therefore now remains to inquire whether that condition can also be regarded as approximately satisfied under a 215-fold magnification of the Sun's radius.

If we take $\mu = 1\,000\,000^{\mathrm{kg}}$,—which is equivalent to supposing that a gaseous layer whose mass-intensity amounts to more than a million kilograms per square meter is almost completely opaque to heat rays—we shall obtain the following results for $\frac{r_0}{r}$ and $\frac{z}{r}$, after substituting in equation (602) the values $a = 2.4$, and $\gamma = 31416^{\mathrm{kg}}$, corresponding as above to $k = 1.41$:

$\frac{r_0}{r}=$	1	$\frac{1}{2}$	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{215}$
$\frac{s}{r}=$	0.019625	0.029365	0.07488	0.28574	$0.44588.$

This table shows that the above condition cannot be regarded as sufficiently fulfilled for the larger values of r, especially for $r=215r_0$, so that the values of t found above require a correction on this account, making them larger than we have found; for those values presuppose that the radius of the radiating spherical shell, or the mean radius of the radiating surface-layer, was always nearly as large as the whole radius r, while in fact at the time when $r=215r_0$, that radius was considerably smaller.

Equation (626) and the table calculated from it show, however, that the value of t increases with r the more slowly as the radius r is greater; and since the above mentioned correction chiefly affects the values of t corresponding to the larger values of r, the absolute values of t will only be slightly altered (even if the differences of the values of t may suffer comparatively large changes), since the correction can never amount to more than a relatively small fraction of the whole absolute value.

According to the above table we shall find, on the assumption that the radius of the radiating spherical surface was always as large as the radius of the whole surface, that the solar radius decreased from the size $215r_0$ to $10r_0$ in 255 710 years. Should we make instead the assumption that the radiating surface was constantly half the whole surface (an assumption that would in any case give too great a value for the time, namely, 511 420 years, or twice as many as before) we should obtain for the whole interval in which the Sun's radius contracted from $215r_0$ to its present size $t=5\ 765\ 578$ years (instead of 5 509 864). Hence even in this case we should get a value only about 4.6 per cent. larger than that found originally. In this way we may easily convince ourselves that the original values of t are to be regarded as sufficiently close approximations, in spite of the imperfect fulfillment of our assumption.

If we should use $k=\frac{5}{3}$ instead of 1.41, to which would then

correspond $a=1.35$, $\sigma_0=48083^{m}$, $\gamma_0=8568^{kg}$, we should obtain from equation (625) the value

$$t=6\,504\,000\left\{1-\left(\frac{r_0}{r}\right)^{2.2}\right\}, \tag{627}$$

and repeating our calculation with these data we should arrive at the following table:

$\frac{r_0}{r}=$	1	$\frac{1}{2}$	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{215}$
$t=$	0	5088730	6462955	6503741	6503952
$\frac{z}{r}=$	0.00338	0.005877	0.021339	0.134637	0.248384.

This last row of figures shows that also in this case the values of t calculated directly from (627) can be regarded without hesitation as close enough approximations. Since the mechanical theory of heat teaches that $k=\frac{5}{3}$ is the largest possible value of $\frac{c_p}{c_v}$, it follows from the table that the time during which the Sun's radius contracted from $215r_0$ to its present size could at most have amounted to about 6.5 million years.

We should arrive at still smaller values if we took account of the fact that recent observers[1] agree in finding the quantity of heat radiated yearly from the Sun at least 50 per cent. greater than the amount given by Pouillet, from which the value $w_0=\frac{4}{3}$ was assumed. If we shall therefore place $w_0=2$, we should get for the interval for either value

$$k=1.41 \text{ or } k=\tfrac{5}{3},$$
$$t=3\,673\,241, \text{ or } t=4\,335\,968 \text{ years.}$$

The above results were deduced from the supposition that the Sun had a spherical shape throughout the whole interval t, and that the whole mass constantly preserved a purely gaseous condition. For the last part of the time, for instance during the period of solar contraction from $2r_0$ to r_0, this supposition is perhaps sufficiently justified; but in the earlier times, especially

[1] LANGLEY, The Mount Whitney Expedition. *Nature*, 1882; *Wied. Ann.* 19, 384, 1883.

when the Sun's radius extended to the Earth's orbit, we must nevertheless consider that conditions differed considerably from those supposed here.

To obtain a more accurate value of t we should first have to take into consideration the fact that the motion of rotation of the Sun must in early times have caused a considerable departure from the spherical shape; and on the other hand that at that time the mass-intensity and the mean temperature of the radiating surface layer may possibly have been considerably modified by the occurrence of processes of condensation.

The possibility does not therefore appear *a priori* excluded that the introduction of that correction would produce a comparatively large change in the entire value of t, even if we may suppose that the correction (as in the case of the correction above applied to z) probably would be required for only a relatively small part of the whole interval—as for instance the time in which the solar radius decreased from $215 r_0$ to $2 r_0$ amounted by the last table to only about 22 per cent. of the whole value of t.

For these reasons we cannot give to the maximum value $t=$ 4 336 000 years found above the significance of a superior limit for the age of the Earth, the less in fact since the original assumptions must still be regarded as hypotheses imperfectly satisfied. Nevertheless it seems permissible to conclude from the above investigation that the actual age of the Earth must be far less than the estimates of some geologists, who place it at hundreds of millions of years.

§64. HYPOTHESES AS TO THE CONSTITUTION OF THE WHITE STARS.

After substituting in (599) the value of z from (602) we obtain for the maximum temperature of the radiating surface layer, or for the temperature of its inner bounding surface,

$$T = T_0 \left\{ \frac{\mu k}{\gamma_0 (k-1) a r} \right\}^{\frac{k-1}{k}} \tag{628}$$

On applying this same equation to the case of another sphere of the same constitution, with the only approximately accurate assumption that the mass-intensity of the radiating surface is independent of its temperature, we find as the ratio of the two boundary temperatures

$$\frac{T}{T'} = \frac{T_0'}{T_0}\left(\frac{\gamma_0' r'}{\gamma_0 r}\right)^{\frac{k-1}{k}} \tag{629}$$

If S and S' are the masses of the two spheres, the ratio of the temperatures of their centers will be (by §12 and §31)

$$\frac{T_0}{T_0'} = \frac{S}{S'} \cdot \frac{r'}{r}, \tag{630}$$

and since the ratio of the boundary temperatures of the radiating surface layers coincides with the ratio of their mean temperatures, the latter will have the value

$$\frac{\tau}{\tau'} = \frac{S}{S'} \cdot \frac{r'}{r}\left(\frac{\gamma_0' r'}{\gamma_0 r}\right)^{\frac{k-1}{k}}, \tag{631}$$

which may be briefly called the ratio of the surface temperatures. If further δ and δ' denote the mean densities of the two spheres, we can place

$$(632) \qquad \frac{\gamma_0}{\gamma_0'} = \frac{\delta}{\delta'} \text{ and } \frac{S}{S'} = \left(\frac{r}{r'}\right)^3 \cdot \frac{\delta}{\delta'}. \tag{633}$$

Using these expressions (631) takes the form

$$\frac{\tau}{\tau'} = \left(\frac{S}{S'}\right)^{\frac{k+1}{3k}} \cdot \left(\frac{\delta}{\delta'}\right)^{\frac{2-k}{3k}}. \tag{634}$$

If we premise similar physical and chemical properties for the substances composing the Sun and stars, this equation can serve to answer the question—Under what circumstances can the surface temperature of a star exceed the present surface temperature of the Sun? where the expression surface temperature is always to be understood as above explained.

The numerical values for the two exponents of the above equation are:

$k = \frac{4}{3}$	1.41	$\frac{5}{3}$
$\frac{(k+1)}{3k} = \frac{7}{12}$	0.57	$\frac{8}{15}$
$\frac{(2-k)}{3k} = \frac{1}{6}$	0.14	$\frac{1}{15}$.

This table shows that the exponent of the mass-ratio is almost independent of k and is always slightly more than 0.5. Since our assumption of independence of absorption from temperature would anyhow lead to too high values of these exponents, we can in an approximation place it equal to 0.5, whence the following proposition:

The surface temperatures of two stars of equal densities are to each other nearly as the square roots of their masses.

If S is placed equal to S', the ratio of the surface temperatures is

$$\frac{\tau}{\tau'} = \left(\frac{\delta}{\delta'}\right)^{\frac{2-k}{3k}} \tag{635}$$

The values of the exponents of the density-ratio given in the table indicate that in the cases chiefly coming under consideration — with the larger values of k — the surface temperature can be considered as nearly independent of the density, since a significant rise of the surface temperature would only occur under a very great increase of density.

Thus for the ratio $\frac{\tau}{\tau'} = 2$, the ratios of increase of density would be

for $k = \frac{4}{3}$	1.41	$\frac{5}{3}$
$\frac{\delta}{\delta'} = 64$	144	32768.

For the present central density of the Sun we should get (from § 31)

for $k = \frac{4}{3}$	1.41	$\frac{5}{3}$
$\delta_0' = 81\,400$	31 416	8 568

The comparison of these two tables shows that the assumption of a comparatively slight power to rise for the surface tempera-

ture would predicate a possibility of the increase of its central density beyond the range of probability, whatever the value taken for the ratio of the specific heats. It is therefore decidedly improbable that the surface temperature of the Sun will increase significantly in the future. We may perhaps with greater probability assume that it is already on a slow decline.

The above values for the present central density of the Sun were deduced on the supposition of an unlimited validity of Mariotte's law, and they can be called the "theoretical" in contrast to the "actual" central densities. It is highly probable that the actual is less than the theoretical central density, so that we infer that the Sun long ago passed the stage of development at which the approximate validity of Mariotte's law ceases in consequence of the gradual increase of the density. If, however, the actual central density is less than the theoretical, then the actual density of the surface layer must be greater than the theoretical, and hence the actual surface temperature must be less than the theoretical because the depth from which heat is directly radiated into empty space will be less as the density of the surface layer is greater.

Since the departure from Mariotte's law grows with increasing density, the difference between the actual and the theoretical surface temperature must have been less in earlier times than now (as on account of less density the departure was also less), and hence the possibility is not excluded that then the actual surface temperature was higher than now. In any case the possible differences between the present surface temperature and a maximum value perhaps reached then are comprised within narrow limits. At no period could the actual surface temperature have been greater than the theoretical. For the latter, however, the table of § 64 gives us smaller values the further back we follow the change of state of the Sun. We may therefore state the result of this inquiry as follows: *The surface temperature of the Sun was never very much higher, and in future can never be very much higher than it is at present.*

From the two last propositions it follows that the surface

temperature of a star can be markedly higher than the present surface temperature of the Sun only when the star's mass exceeds the Sun's mass. For a slight difference of temperature the higher value for the star could also, it is true, be explained on the supposition that the Sun has already passed the culmination of its surface temperature, and that the star in question has not yet reached the present density of the Sun. But we may assert in any case that such a difference of temperature admits of a much simpler explanation on the assumption of a greater mass of the star, and that this assumption opens a much wider range for the explanation of high surface temperatures than that of a lesser density of the star, inasmuch as according to the first of the above propositions, a double surface temperature would correspond to a fourfold mass.

Since a rise of the surface temperature of a body at white incandescence requires an increasing predominance of the blue and violet rays in the spectrum, we may infer that those stars whose light appears white or bluish-white in contrast to the yellowish light of the Sun, in consequence of the predominance of the more refrangible rays, (Sirius, Vega) possess a higher surface temperature than the Sun.[1] From the hypotheses here proposed this conclusion then follows:

The masses of the white stars are greater than the mass of the Sun.

§ 67. HYPOTHESES AS TO THE CONSTITUTION OF THE RED STARS.

A gaseous sphere in adiabatic equilibrium, of mass equal to that of the Sun, and of radius one hundred thousand times that of the Sun, would have such a slight mean density that even the rays emitted from the center would suffer only a vanishingly small absorption, inasmuch as the total mass in the layer which such a ray would have to penetrate would amount to only a few hundred kilograms per square meter.

The quantity of heat radiated per unit of time under this

[1] Cf. H. C. VOGEL, "Resultate spectralphotometrischer Untersuchungen," *Berliner Berichte*, Sitzung vom 21. October 1880.

condition of density can depend solely upon the temperature, increasing with a rise of temperature. Under the contraction consequent upon giving out heat the interior heat of the sphere increases in inverse proportion to the radius, and hence during the contraction the quantity of heat radiated per unit of time must also increase so long as the mean density is low enough to approximately satisfy our first condition.

If the mean density later becomes greater so that the direct radiation into space extends inward just to the center, then at this epoch a central nucleus will begin to develop from the particles of which no heat rays will be able to reach out to empty space. With further contraction the radius of this nucleus, thus protected from loss by direct radiation, will at first increase; but later, when the difference between the radius of the nucleus and the whole radius—or depth of the radiating surface layer—amounts to only a small fraction of the whole radius, the radius of the nucleus must again decrease, because the whole radius continually diminishes.

It was shown in §64 that during this last period of development the quantity of heat radiated per unit of time continuously decreased, and since during the first period the radiation at least at first increased, it follows that at some intermediate point the quantity of heat radiated per unit of time must have reached a maximum.

It was shown in the previous paragraph that, although the theoretical surface temperature would constantly increase during this latter period, in consequence of the departure from Mariotte's law the actual surface temperature must begin to fall again after passing a certain culmination point.

The results of §65 indicated that a star always reaches the culmination of its heat radiation earlier than the culmination of its surface temperature. The first corresponds to the epoch of greatest brightness or light-intensity, and the second to the epoch when the color representing the total impression of the emitted light shifts toward the violet end of the spectrum, and when the light appears bluish-white if the star's mass is sufficient. The

Sun perhaps passed the first culmination point at the time when its radius was about the size of the radius of the Earth's orbit, while the last point of culmination corresponds to a condition which in all probability differs but little from the present state.

After passing the first point the luminous intensity decreases while the surface temperature is still rising. With the passage of the second point the surface temperature in turn begins to fall and the color begins to approach the red end of the spectrum again. A star which emits bluish-white light at the culmination of its surface temperature must have appeared red at an earlier epoch, when its surface temperature was far below its maximum value; and similarly it will necessarily appear red once more at a later time, when after passing its maximum the surface temperature has again fallen considerably.

If we divide the stars into three classes according to their surface temperatures, assigning with Vogel[1] the white stars to the first class, the yellow to the second, and the red stars to the third class, we should have to distinguish between two groups or subdivisions within the third class. According to the theory here advanced, we should put in the first subdivision those red stars whose surface temperatures have not yet reached the culmination point; in the second all those red stars whose surface temperatures have already passed their culmination. Since we may assume from §65 that a star always reaches its luminous culmination much earlier than that of its surface temperature, the stars belonging to the first subdivision will necessarily possess predominatingly slight density and high luminous intensity, while those of the second subclass will have great density and slight luminous intensity.

If we may further believe that the development of chemical compounds is favored by greater density in conjunction with relatively low temperature, then we should expect in the spectra of the older and denser stars of the latter subclass the occurrence of the absorption lines or bands which indicate the presence of chemical combinations in the surface layer.

[1] *A. N.*, 84, 113–123, 1874.

The question might therefore now be raised whether Vogel's subclasses III a and III b (Secchi's III and IV) may not be identical with the two subclasses proposed here on theoretical grounds, and also whether we may not conclude from the peculiar band spectrum of class III b—as well as from the fact that no stars of this group have been observed which are brighter than the fifth magnitude—that these stars belong to those older, denser, and fainter stars which have already passed the culmination of their surface temperature. The reply to these questions must be left to the further researches of spectrum analysis.

As for stars of the third class we might perhaps also distinguish between two subclasses for each of the first and second classes, namely—that of the relatively young stars which have not yet reached the maximum of surface temperature, and that of the older stars which have already passed it. In any case we may infer that the nature of the spectrum of a star depends not only upon the temperature but also upon the density of its surface layer, which increases with age; and that the spectra of two stars of the same surface temperature can be different if their surface layers have unequal densities. Since according to §65 the surface temperature of a star also depends upon its mass, the spectrum must depend simultaneously on the mass and the relative age of the star. The reply to the inquiry whether the supposition of variations in mass and in age would be sufficient for a satisfactory explanation of the varieties of spectra thus far observed must again be left for future spectroscopic researches.

§68. CONJECTURAL CAUSES OF THE PAUCITY OF RED STARS.

If we exchange the two limits of integration in equation (624), we shall obtain for the interval in which the solar radius would decrease from r_0 to r

$$t = \frac{k r_0}{(7k-8)\sigma_0} \left\{ \left(\frac{r_0}{r}\right)^{\frac{7k-8}{k}} - 1 \right\}, \tag{636}$$

which, after substituting $k=\frac{5}{3}$ and $\sigma_0=48^{m}$, becomes

$$t = 6\,504\,000 \left\{ \left(\frac{r_0}{r}\right)^{2.2} - 1 \right\}.$$

For example we should obtain from this $t=23.4$ million years as the time required for the solar radius to contract to one half its present size. According to equation (627), under the same conditions, the interval in which the solar radius diminished from $2r_0$ to r_0 would be about 5.1 million years. Hence the whole duration of the contraction from the size $2r_0$ to $\frac{1}{2}r_0$ would be about 28.5 million years or about twenty times the interval of contraction from $215r_0$ to $2r_0$, which would be by (627) only about 1.4 million years.

Even if the theoretical values calculated in this way must for the reasons given in §66 be regarded as rough approximations, which can claim only an extremely slight degree of accuracy and may differ widely from the true values, nevertheless we may be permitted to infer from the above results that in early times,—for instance when the solar radius was one hundred times as large as at present,—change of condition and decrease in volume of the Sun's mass took place with a vastly greater rapidity than at present, when the condition is nearly stationary.

We may also conclude that the duration of the period in which the Sun as a star had a greater brightness than at present was very short in comparison to the period in which it had and will continue to have a brightness differing only slightly from its present value.

The point of culmination of heat radiation of a star represents a state of relatively rapid transition; the point of culmination of surface temperature, however, represents a state in which the star remains for a comparatively long time.

In equation (636) we should get $t=\infty$ for $r=0$, *i. e.*, if the Sun possessed the properties of a perfect gas, its volume would shrink to zero only after an infinitely long period, and its luminosity would continue forever. Since however an unlimited possibility of increase of density is out of the question we must assume that after a definite finite though very long period the Sun will reach a point at which its luminosity must entirely cease; and

the same must be assumed for every star. In this sense we may speak of a definite period of apparition of a star, just as of a comet or meteorite, even if many hundreds of millions of years may elapse between the first appearance and the final extinction.

The whole period of apparition of a star is on our theory divided into three parts by the culmination of heat radiation and the culmination of surface temperature. During the first interval the quantity of heat radiated per unit of time is constantly on the increase. At the beginning of this interval—when the star was still in the nebular condition—the change took place very slowly on account of the very slight radiation of heat, but later it became very rapid as with increasing density the maximum of brightness was approached. During the second interval the radiation of heat is constantly on the decrease, changing at first rapidly, and later more slowly as the surface temperature approaches its maximum. During the third period both radiation and surface temperature are in steady decline, but the condition changes very slowly throughout the whole period.

The very protracted period of slow genesis is therefore followed by a very short middle interval of rapid changes of condition and this in turn is succeeded by another very protracted period of slow extinction. If we then divide the stars into three classes according to age corresponding to these three stages of development, we shall assign to the first class *A* those stars still in the nebular phase of development; to the second class *B* those in the transient stage of greatest brilliance; and to class *C* those stars which have already entered into the long period of slow extinction. It should be noted in this classification that we refer to relative and not absolute age, since a star of slight mass passes through the successive phases of its development more rapidly than a star of greater mass.

Since the class *B* represents a phase of relatively very brief duration we should expect from the laws of probabilities that the number of the stars simultaneously in this stage would be very small in comparison to the whole number of stars, and also to the number of stars in class *C*.

According to our hypotheses the class *B* would include the younger stars of class III, that is, the red stars which have not yet reached the culmination of their surface temperature. The class *C* would comprise classes I and II and also the older stars of class III which have passed the culmination of surface temperature and have reached a point approximating total extinction.

We cannot however expect with certainty that these last stars should have numerous representatives among the stars at present actually visible, on the one hand because their faintness may have largely caused them to escape observation, and on the other hand because it is quite possible that the whole system of stars is still in a period of relative youth, so that only a few stars of slight mass have already reached the condition immediately preceding extinction.

The results of spectroscopic observations may be regarded as favorable to our hypotheses, inasmuch as Vogel[1] found in a zone of 3702 stars examined 2165 of class I, 1240 of class II, 288 of class III a, and 9 of class III b.

[1] *Publicationen*, Potsdam, Bd. III, 1883.

Minor Contributions and Notes.

ON THE PRESENCE OF HELIUM IN THE EARTH'S ATMOSPHERE, AND ON ITS RELATION TO THE KINETIC THEORY OF GAS.

We have evidence that there is within the Earth a widespread and as yet unexhausted store of helium, from its presence having been detected in many of the hot springs in distant parts of the world which have been examined. It thus became known that this element is, and no doubt has been for ages filtering from that store into the atmosphere. It comes to the atmosphere through hot springs in appreciable quantities, and possibly everywhere through the soil in smaller quantities.

It is also known that helium which reaches the atmosphere uncombined must remain helium, since this gas does not combine with other elements even under circumstances more favorable than those that prevail in the atmosphere.

Helium, which is probably uncombined, has now been detected as a constituent of atmospheric air in the course of the astonishingly delicate experiments which are being made in Professor Ramsay's laboratory in University College, England; but it is found to bear an exceedingly small proportion to the heavier constituents.

That there must be helium in our atmosphere, and that the quantity present must be very small if the helium is uncombined, had been indicated by the Kinetic Theory of Gas (see this Journal for January 1898, p. 30). The recent discovery is accordingly a confirmation in this instance of the deductions which can be made from the kinetic theory.

The kinetic theory indicates that molecules drift away at the boundary of our atmosphere from all its gaseous constituents; but that the number is so small in the case of gases with the density of nitrogen and oxygen that it would probably take many billions of years to effect any observable reduction of the quantity of these gases; whereas with gases whose density is below a certain limit (which differs from one planet to another, and which on the Earth lies somewhere between 2

and $7\frac{1}{2}$) the drifting away will be sufficient to drain off any stock of these gases altogether in a moderate number of years, unless there be some persistent source of supply. When, as in the case of helium, there is such a source of supply, the gas which will be found in the atmosphere at any one time will be that which is making its way upwards by diffusion, from the bottom of the atmosphere, where it enters, to the top where it escapes.

It is possible that a still more minute trace of uncombined hydrogen will be found if the delicacy of the experiments can be carried much further.

Since physicists are now able to detect in atmospheric air the presence of a constituent which is estimated to occupy only a fraction of a millionth of the whole volume, it may not be out of place to call attention to the fact that besides the gaseous constituents of our atmosphere there is also in the air which is accessible to us a minute quantity of carbon and even traces of other solid elements, owing to the swarms of micro-organisms that are everywhere present. Some of these can be seen by the microscope, but those that are beyond its reach must be vastly more numerous. It should be remembered in this connection that a spore that is only the hundredth of a micron across has still sufficient space within that small volume for something like a million of chemical atoms. Now the smallest interval at which two objects can be seen as two with the best microscope most carefully handled is about twenty times the diameter of that spore.

G. JOHNSTONE STONEY.

LONDON, October 11, 1898.

THE PURKINJE PHENOMENON AND THE SPECTRUM OF THE ORION NEBULA.

SINCE returning from India I have read Professor Scheiner's article[1] concerning my observations of variations in the Orion Nebula spectrum. Professor Scheiner did not succeed in confirming my observations, and declared that "the whole appearance recorded by Professor Campbell is nothing more nor less than the 'Purkinje Phenomenon.'"

Such an extremely positive statement as this is evidently unwarranted for several reasons.

[1] This JOURNAL, April 1898, 295–298.

1. It is pure assumption, since Professor Scheiner did not prove it by observations bearing either directly or closely upon the subject.

2. The assumption is erroneous. Professor Runge has shown (this JOURNAL, 8, 32–36) by observations bearing directly on the question at issue, that *less than a twofold variation* is explained by the Purkinje Phenomenon, whereas the apparent variation observed in the nebular spectrum by myself and by Messrs. Schaeberle, Runge, Aitken, and Wright was from twenty-five to thirtyfold.

3. Professor Scheiner's criticism is based very largely on the assumption that I did not take proper precautions to eliminate the effects of physiological error—an assumption that is erroneous. My published observations show plainly that such precautions were taken, and I venture to say a more careful reading of them would have rendered this entire discussion unnecessary.

In *Astronomy and Astro-Physics* for May 1894, 13, 388, while discussing the relative brightness of the second ($\lambda 4959$) and third ($H\beta$) nebular lines for the Trapezium region, I wrote: "As Dr. Huggins has pointed out, 'the second line suffers in apparent brilliancy from its nearness to the brightest line.' With low dispersion it seems considerably fainter than the $H\beta$ line, even in the vicinity of the Trapezium. But when the very bright first line is covered with a heavy micrometer wire, the second line is seen to be fully as bright as the third. *This point was further tested by using two gratings in the first, second, and third orders, with which the second and third lines become very faint and widely separated. By narrowing the slit until the lines were rendered almost invisible, the second line was seen with certainty to be a very little brighter than the third line.*"

The sentences which I have italicized show that the observer was on his guard against physiological effects. By reducing the brightness of the lines until they became "very faint," and later until they "were rendered almost invisible," the second line was seen to be "a very little brighter than the third line," but much fainter than the first line. In a faint outlying region I found the third line to be at least five times as bright as the first line (Professor Runge's estimate was ten times), and the second line was invisible. The use of two gratings in three orders should be a sufficient precaution against peculiarities of the gratings. My notes show that throughout the grating observations, both with wide and with narrow slits, the second line was estimated to be brighter than the third.

This would seem to be sufficient proof that the observer not only realized the difficulty of making accurate visual estimates, ***but that he took the proper precautions to eliminate the very effects which Professor Scheiner assumed he did not eliminate,*** as well as other subjective effects which Professor Scheiner did not mention.

Further evidence as to the observer's state of mind is afforded on page 494 of my same paper. Concerning the nebula *DM.* — 12°1172 I wrote: "The relative intensities of the light in the three disks (corresponding to three nebular lines) were estimated at 10 : 3 : 7. A wedge photometer of increasing darkness was moved over the eyepiece at right angles to the line joining the three disks; and the fact that the disk at 4861 ($H\beta$) disappeared before the disk at 5007 did, proves that the latter is the brighter."

This nebula is near the Orion Nebula, and the photometer was used on the lines in both. It confirmed the results on the Orion Nebula previously obtained with the grating. The wedge was of neutral tint and its absorption curve would probably be no steeper at $H\beta$ than the absorption curve for the prism.

My observations may be repeated with surprising ease by those who have suitable apparatus; but one who uses a very short collimator, a relatively long view telescope, a weak prism, and a high-power eyepiece will deservedly fail. An observation of this kind made with an inefficient instrument is worth even less than a stellar parallax determined with a ring micrometer.

In asking Professor Scheiner for his estimate of the relative intensities of the lines in the vicinity of star Bond 734 — with which he disclaims power from nature to comply -- I was asking him to use the method and notation which are sufficiently common, and which he himself employed in his book on *Spectrum Analysis*. For example, he wrote "Für den Orionnebel findet Vogel als Helligkeitverhältniss der vier Linien 10, 5, 8, 1" (Scheiner's *Die Spectralanalyse*, p. 247).

W. W. CAMPBELL.

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